Phenomenology of $f_0(980)$ photoproduction on the proton at energies measured with the CLAS facility

M. L. L. da Silva[1] and M. V. T. Machado[2]
[1]*Instituto de Física e Matemática, Universidade Federal de Pelotas Caixa Postal 354, CEP 96010-090 Pelotas, RS, Brazil*
[2]*High Energy Physics Phenomenology Group, GFPAE IF-UFRGS Caixa Postal 15051, CEP 91501-970 Porto Alegre, RS, Brazil*
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In this work we present the results of a theoretical analysis of the data on photoproduction of the $f_0(980)$ meson in the laboratory photon energy between 3.0 and 3.8 GeV. A comparison is done to the measurements performed by the CLAS Collaboration at the JLab accelerator for the exclusive reaction $\gamma p \to p f_0(980)$. In the analysis the partial S-wave differential cross section is described by a model based on the Regge approach with reggeized exchanges and distinct scenarios for the $f_0(980) \to V\gamma$ coupling considered. It is shown that such a process can provide information on the resonance structure and production mechanism.

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I. INTRODUCTION

The spectroscopy of the low mass scalar mesons, like the $f_0(980)$ one, is an exciting issue in hadronic physics and is still an unsettled question. The topic is quite controversial as the mass spectrum ordering of low-lying scalar mesons disfavors the usual quark-antiquark picture and a window is opened to a prolific investigation of the topic. For instance, in the past years the low mass states $J^{PC} = 0^{++}$ have been considered quark-antiquark mesons [1], tetraquarks [2], hadron molecules [3], glueballs, and hybrids [4,5]. Such a conflicting interpretation comes from the fact the situation is complex in low energies where quantitative predictions from QCD are challenging and rely mostly on numerical techniques of lattice QCD. Nowadays, theoretical analysis consider also the gluonic degrees of freedom. This glueball resonance with no quarks has nonexotic quantum numbers and cannot be accommodated within quark-antiquark nonets [6]. In this context, the photoproduction of exotic mesons [7] can be addressed using arguments based on vector meson dominance where the photon can behave like an $S = 1$ quark-antiquark system. Therefore, such a system is more likely to couple to exotic quantum number hybrids. This sort of process could provide an alternative to the direct observation of the radiative decays at low energies. Along those lines, the GlueX experiment [8] is being installed and its primary purpose is to understand the nature of confinement in QCD by mapping the spectrum of exotic mesons generated by the excitation of the gluonic field binding the quarks.

The gluonic content of mesons could be directly tested in current accelerators in the case where a clear environment is available. In the limit of very high energies the exclusive exotic meson production in two-photon and Pomeron-Pomeron interactions in coherent nucleus-nucleus collisions can be easily computed. In these cases, the photon flux scales as the square of the charge of the beam, Z^2, and then the corresponding cross section is highly enhanced by a factor $\propto Z^4 \approx 10^7$ for gold or lead nuclei at the RHIC and LHC, respectively. A competing channel, which produces a similar final state configuration, is the central diffraction process that is modeled, in general, by the two-Pomeron interaction. For instance, in Ref. [9] the cross sections for these two channels are contrasted in the production of glueball candidates like the low-lying scalar mesons. The cross sections were sufficiently large for experimental measurement, and the event rates can be obtained using the beam luminosity [10] for the LHC. It produces 5×10^5 events for $f_0(1500)$ mesons in the two-photon channel whereas the integrated cross section for an exclusive diffractive process is around 500 μb. The central diffractive production of mesons $f_0(980)$ and $f_2(1270)$ at the energy of the CERN-LHC experiment on proton-proton collisions was further investigated in Ref. [11]. The processes initiated by quasi-real photon-photon collisions and by central diffraction processes were also considered. The main motivation is that the ALICE Collaboration has recorded zero bias and minimum bias data in proton-proton collisions at a center-of-mass energy of $\sqrt{s} = 7$ TeV. Among the relevant events, those containing double gap topology have been studied and they are associated with central diffractive processes [12]. In particular, central meson production was observed. In the double gap distribution, the K_s^0 and ρ^0 are highly suppressed while the $f_0(980)$ and $f_2(1270)$ with quantum numbers $J^{PC} = (0,2)^{++}$ are much enhanced. Such a measurement of those states is evidence that the double gap condition used by a large ion collider experiment at CERN LHC (ALICE) selects events dominated by the double Pomeron exchange. The main predictions of Ref. [11] are the exclusive diffractive cross section for $f_0(980)$ being $d\sigma(y=0)/dy \simeq 27$ μb and its production cross section in two-photon reactions giving $\sigma_{\gamma\gamma} = 0.12$ nb at $\sqrt{s}_{pp} = 7$ TeV.

As far as the low energy regime is concerned, in Ref. [13] we studied the photoproduction of the $a_0(980)$, $f_0(1500)$, and $f_0(1710)$ resonances for photon energies relevant for the GlueX experiment at $E_\gamma = 9$ GeV using current ideas on glueball and $q\bar{q}$ mixing. The meson differential and integrated cross sections were evaluated, and the effect of distinct mixing scenarios were investigated. Although large backgrounds were expected, the signals could be visible by considering only the all-neutral channels, that is, their decays on $\pi^0\pi^0$, $\eta^0\eta^0$, and $4\pi^0$. The theoretical uncertainties were still large, with $f_0(1500)$ being the more optimistic case.

The $f_0(980)$ photoproduction is measured via the most sizable decay modes which are $\pi\pi$ and $K\bar{K}$. In this mass range was expected an interference of the P-wave from the decay of $\phi(1020)$ and the S-wave from the decay of the $f_0(980)$ resonance. This interference is discussed in Refs. [14,15] where the $\pi\pi$ and $K\bar{K}$ photoproduction was performed close to the $K\bar{K}$ threshold. The photoproduction of $f_0(980)$ was investigated in a unitary chiral model [16]. Here, we investigate the photoproduction of meson state $f_0(980)$, and distinct scenarios for the $f_0(980) \to V\gamma$ coupling are considered. The scenarios discussed in this paper consider the $f_0(980)$ as a tetraquark or as a ground state nonet. We focus on the S-wave analysis on the forward photoproduction of $\pi^+\pi^-$ on the final state. The theoretical formalism employed is the Regge approach with reggeized ρ and ω exchange [13]. This assumption follows from Regge phenomenology where high-energy amplitudes are driven by t-channel meson exchange. This paper is organized as follows: in the next section we present the relevant scattering amplitudes and how they are related to the differential cross sections. In the last section numerical results are presented and parameter dependence is addressed. A comparison to the CLAS data for direct photoproduction of $f_0(980)$ is also done [17]. Finally, the conclusions and discussions follow at the end of Sec. III.

II. MODEL AND CROSS SECTION CALCULATION

We focus on the S-wave analysis of nondiffractive $f_0(980)$ photoproduction and its decay on the $\pi^+\pi^-$ final state. According to the Regge phenomenology, one expects that only the t-channel meson exchanges are important in such a case. The ρ and ω reggeized exchanges are to be considered in the present analysis. To obtain mass distribution for the scalar $f_0(980)$ meson, one represents it as relativistic Breit-Wigner resonance with energy-dependent partial width. The differential cross section for the production of a scalar with invariant mass M and its decay to two pseudoscalars, masses m_a and m_b, can be written as

$$\frac{d\sigma}{dt\,dM} = \frac{d\hat{\sigma}(t, m_S)}{dt}\frac{2m_S^2}{\pi}\frac{\Gamma_i(M)}{\left(m_S^2 - M^2\right)^2 + (M\Gamma_{\text{Tot}})^2}, \tag{1}$$

where $d\hat{\sigma}/dt$ is the narrow-width differential cross section at a scalar mass $M = m_S$ and $\Gamma_i(M)$ being the pseudoscalar-pseudoscalar final state partial width, which can be computed in terms of the SPP coupling g_i. A note is in order at this point. Although the main decay of the $f_0(980)$ is $\pi\pi$, this state resides at the $K\bar{K}$ threshold. Therefore, following Ref. [7] we use the Breit-Wigner parametrizations obtained in the analysis of ϕ radiative decays [18]. In such a case, the Breit-Wigner width takes the form

$$\Gamma(M) = \frac{g_{\pi\pi}^2}{8\pi M^2}\sqrt{\frac{M^2}{4} - M_{\pi\pi}^2} + \frac{g_{K\bar{K}}^2}{8\pi M^2}\left[\sqrt{\frac{M^2}{4} - M_{K^+K^-}^2} + \sqrt{\frac{M^2}{4} - M_{K^0\bar{K}^0}^2}\right], \tag{2}$$

where we set the following parameters: $M = 984.7$ MeV, $g_{K\bar{K}} \equiv g_{K^+K^-} = g_{K^0\bar{K}^0} = 0.4$ GeV, and $g_{\pi^+\pi^-} = \sqrt{2}g_{\pi^0\pi^0} = 1.31$ GeV for the scalar meson $f_0(980)$ considered here. However, when we consider the $f_0(980)$ cross section below the $K\bar{K}$ threshold, the total width cannot be written as in Eq. (2). Thus, in this case we should use the Flattè formula [19] when computing our numerical results in the next section.

Let us proceed: the reaction proposed here is $\gamma p \to p\, f_0(980)$. Within the Regge phenomenology the differential cross section in the narrow-width limit for a meson of mass m_S is given by [13]

$$\frac{d\hat{\sigma}}{dt}(\gamma p \to pM) = \frac{|\mathcal{M}(s,t)|^2}{64\pi\left(s - m_p^2\right)^2}, \tag{3}$$

where $\mathcal{M}$ is the scattering amplitude for the process, s and t are usual Mandelstan variables, and m_p is the proton mass. For the exchange of a single vector meson, i.e., ρ or ω, one has

$$\begin{aligned}|\mathcal{M}(s,t)|^2 = &-\tfrac{1}{2}\mathcal{A}^2(s,t)\big[s(t-t_1)(t-t_2) \\ &+ \tfrac{1}{2}t\big[t^2 - 2\left(m_S^2 + s\right)t + m_S^4\big]\big] \\ &- \mathcal{A}(s,t)\mathcal{B}(s,t)m_p s(t-t_1)(t-t_2) \\ &- \tfrac{1}{8}\mathcal{B}^2(s,t)s\left(4m_p^2 - t\right)(t-t_1)(t-t_2).\end{aligned} \tag{4}$$

where t_1 and t_2 are the kinematical boundaries

$$\begin{aligned}t_{1,2} = \frac{1}{2s}\Big[&-\left(m_p^2 - s\right)^2 + m_S^2\left(m_p^2 + s\right) \\ &\pm \left(m_p^2 - s\right)\sqrt{\left(m_p^2 - s\right)^2 - 2m_s^2\left(m_p^2 + s\right) + m_S^4}\,\Big],\end{aligned} \tag{5}$$

and where one uses the standard prescription for Reggeizing the Feynman propagators assuming a linear Regge trajectory $\alpha_V(t) = \alpha_{V0} + \alpha'_V t$ for writing down the quantities $\mathcal{A}(s,t)$ and $\mathcal{B}(s,t)$:

$$\begin{aligned}\mathcal{A}(s,t) &= g_A\left(\frac{s}{s_0}\right)^{\alpha_V(t)-1}\frac{\pi\alpha'_V}{\sin[\pi\alpha_V(t)]}\frac{1 - e^{-i\pi\alpha_V(t)}}{2\,\Gamma[\alpha_V(t)]}, \\ \mathcal{B}(s,t) &= -\frac{g_B}{g_A}\mathcal{A}(s,t).\end{aligned} \tag{6}$$

It is assumed nondegenerate ρ and ω trajectories $\alpha_V(t) = \alpha_V(0) + \alpha'_V t$, with $\alpha_V(0) = 0.55\,(0.44)$ and $\alpha'_V = 0.8\,(0.9)$ for ρ (ω). In Eq. (6) above, one has that $g_A = g_S(g_V + 2m_p g_T)$ and $g_B = 2g_S g_T$. The quantities g_V and g_T are the vector-nucleon-nucleon (VNN) vector and tensor couplings, and g_S is the γVN coupling. For the ωNN couplings we have set $g_V^\omega = 15$ and $g_T^\omega = 0$ [13], and for the ρNN couplings we used $g_V^\rho = 3.4$ and $g_T^\rho = 11$ GeV^{-1}. The $SV\gamma$ coupling g_S can be obtained from the radiative decay width through [20]

$$\Gamma(S \to \gamma V) = g_S^2\frac{m_S^3}{32\pi}\left(1 - \frac{m_V^2}{m_S^2}\right)^3. \tag{7}$$

This model was applied to $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ mesons which are considered as a mixing of $n\bar{n}$, $s\bar{s}$, and glueball states [21]. In this case their radiative decays into a vector meson are expected to be highly sensitive to the degree of mixing between the $q\bar{q}$ basis and the glueball. In Ref. [13] three distinct mixing scenarios were considered. The first one is the bare glueball being lighter than the bare $n\bar{n}$ state; the second scenario corresponds to the glueball mass being between the $n\bar{n}$ and $s\bar{s}$ bare state; and finally the third one is where the glueball mass is heavier than the bare $s\bar{s}$ state. The numerical values for the widths having effects of mixing on the radiative decays of the scalars on ρ and ω can be found in Table 1 of Ref. [13]. This way it is clear that the width is strongly model dependent, and different approaches can be taken into account. For instance, we refer the work in Ref. [22], where the decays of a light scalar meson into a vector meson and a photon, $S \to V\gamma$, are evaluated in the tetraquark and quarkonium assignments of the scalar states. The coupling now reads

$$\Gamma(S \to \gamma V) = g_S^2 \frac{\left(m_S^2 - m_V^2\right)^3}{8\pi\, m_S^3}. \quad (8)$$

The different nature of the couplings corresponds to distinct large-N_c dominant interaction Lagrangians. In the next section we compare those approaches discussed above for the direct $f_0(980)$ photoproduction in the CLAS energies.

III. RESULTS AND DISCUSSIONS

In what follows we present the numerical results for the direct $f_0(980)$ photoproduction considered in the present study and the consequence of the tetraquark and quarkonium assignments of the scalar states discussed in the previous section. The results presented here will consider five distinct scenarios, three of them assuming that $f_0(980)$ is a quarkonium and two assuming that $f_0(980)$ is a tetraquark. In scenarios 1, 2, and 3 the $f_0(980)$ will be interpreted as a ground-state nonet and in scenarios 4 and 5 as a tetraquark. The g_S coupling can be obtained from the radiative decay width in Table I using Eq. (7) for scenario 1 and Eq. (8) for the remaining scenarios. The values for scenario 1 in Table I were extracted from Refs. [7,20] and from Ref. [22] for the remaining ones. The radiative decay in scenarios 3 and 5 have considered the f_0 resonance as a quarkonium and a tetraquark, respectively, including vector meson dominance, as discussed in Ref. [22].

The partial S-wave differential cross sections for the $f_0(980)$ are presented in Fig. 1 at $E_\gamma = 3.4 \pm 0.4$ GeV and integrated in the $M_{\pi\pi}$ mass range 0.98 ± 0.04 GeV. As a general picture, the typical pattern is a vanishing cross section towards the forward direction (it does not appear in the plot as we are showing the region $|t| \geqslant 0.4$ GeV^2) due to the helicity flip at the photon-scalar vertex and the dip at $-t \approx 0.7$ GeV^2 related to the reggeized meson exchange. The scenarios 1 and 4 are represented by the solid and dot-dashed lines, respectively. Both fairly reproduce the trend of the CLAS data. The scenario 2 is denoted by the dashed curve. In

TABLE I. The widths $\Gamma(S \to \gamma V)$ for the radiative decays of the scalar meson into vector mesons $V = \rho\,(\omega)$ in units of keV. These results are extracted from Ref. [7] for scenario 1 and from Ref. [22] for the remaining scenarios.

Scenario	$f_0(980) \to \gamma V$
1	83 (9.2)
2	69880 (6730)
3	3.3 (0.61)
4	1005 (463)
5	3.1 (3.4)

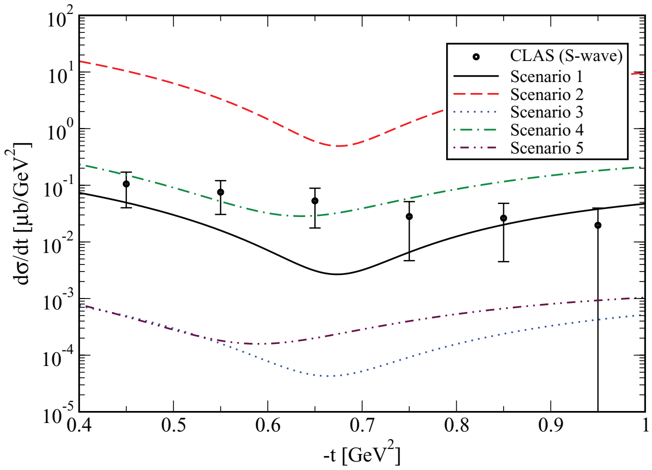

FIG. 1. (Color online) The S-wave differential photoproduction cross sections for $f_0(980)$ photoproduction as a function of momentum transfer squared at CLAS experiment energy $E_\gamma = 3.4$ GeV. The statistical/systematic error bars for CLAS data [17] were summed in quadrature.

this case the result overestimates the CLAS data points by a factor of 50. The scenarios 3 and 5 are represented by the dotted and dot-dot-dashed lines, respectively. Now, the results underestimate the data by the same factor.

The several theoretical predictions were compared to the CLAS analysis at Jefferson Lab [17], where the $\pi^+\pi^-$ photoproduction at photon energies between 3.0 and 3.8 GeV has been measured in the interval of momentum transfer squared $0.4 \leqslant |t| \leqslant 1.0$ GeV^2. There, the first analysis of S-wave photoproduction of pion pairs in the region of the $f_0(980)$ was performed. The interference between P and S waves at $M_{\pi\pi} \approx 1$ GeV clearly indicated the presence of the f_0 resonance. As a final comment on the compatibility of theoretical predictions and experimental results, the scenarios 1 and 4 fairly describe the data (they are parameter-free predictions as we did not fit any physical parameter). Moreover, the CLAS data have no

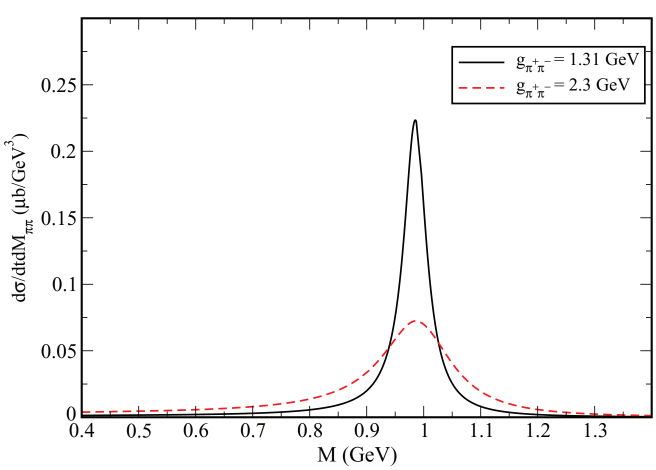

FIG. 2. (Color online) S-wave $\pi^+\pi^-$ invariant mass distribution at $E_\gamma = 3.4$ GeV, $|t| = 0.55$ GeV^2. The results stand for $g_{\pi^+\pi^-} = 1.31$ GeV (solid curve) and $g_{\pi^+\pi^-} = 2.3$ GeV (dashed curve). In both cases, the value $g_{K\bar{K}} = 0.4$ is considered.

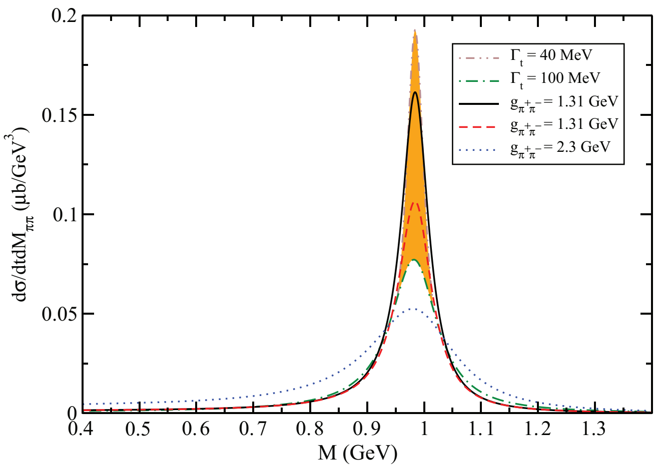

FIG. 3. (Color online) Analysis of the S-wave $\pi^+\pi^-$ invariant mass under several considerations on the $f_0(980)$ partial width (see text).

indication of a minimum as predicted by the reggeized models. Here, we have two possibilities: there is no data point in the dip region (around $|t| \simeq 0.7$ GeV^2) to confirm the reggeized exchange prescription or some additional contribution, i.e., background effects or interference, is missing. The case seems to be the the first option based on the reasonable description of an S wave by a nonreggeized meson exchange [15] as presented in Fig. 3 of Ref. [17].

As a complementary study, we also investigate the invariant mass distribution predicted by the theoretical models, taking the scenario 1 as a baseline. Another way to calculate the mass distribution is to use the branching fractions for the strong decay of $f_0(980)$ associated with the Breit-Wigner width for $f_0(980)$ to $\pi\pi$. In what follows two possibilities for the branching fractions will be used [7,23]:

$$\mathcal{B}[f_0(980) \to \pi\pi] = 85\% \tag{9}$$

and

$$\mathcal{B}[f_0(980) \to \pi^+\pi^-] = 46 \pm 6\%. \tag{10}$$

On the other hand, it is possible to use the experimental value for the total width of $f_0(980)$ which is in the range of 40 to 100 MeV [24]. With this assumptions it is not necessary to calculate the $f_0(980) \to K\bar{K}$ width appearing in Eq. (2).

In Fig. 2 the S-wave $\pi^+\pi^-$ invariant mass distribution at $E_\gamma = 3.4$ GeV and $|t| = 0.55$ GeV^2 is shown. For the theoretical analysis we take the scenario 1. One considers the coupling $g_{K\bar{K}} = 0.4$ and two possibilities for the coupling $g_{\pi\pi}$. The first one is $g_{\pi\pi} = 1.31$ GeV (solid curve) presented in Ref. [7], and the second case $g_{\pi\pi} = 2.3 \pm 2$ GeV (dashed curve) is given in Ref. [25]. The Flattè formula is used to obtain the $f_0(980)$ total width [19]. The results present a strong dependence of the mass distribution on the $g_{\pi\pi}$ coupling.

In Fig. 3 we repeat the previous analysis using now the experimental values $\Gamma_{\text{tot}} = 40$ to 100 MeV for the $f_0(980)$ total width [24]. It can be also obtained by branching ratios for $f_0(980)$ into pions associated with the Breit-Wigner formula. The dot-dot-dashed and dot-dashed lines represent the invariant mass distribution for $\Gamma_{\text{tot}} = 40$ and $\Gamma_{\text{tot}} = 100$ MeV, respectively. In this case, $\Gamma_{\pi^+\pi^-} = 0.46\Gamma_{\text{tot}}$ following Eq. (10). The solid and dotted lines represent invariant mass distribution for $\Gamma_{\text{tot}} = \Gamma_{\pi\pi}/0.85$ following Eq. (9) and where $\Gamma_{\pi\pi}$ is given by Breit-Wigner formula. In the result represented by the dashed line $\Gamma_{\text{tot}} = \Gamma_{\pi^+\pi^-}/0.46$ following Eq. (9) and $\Gamma_{\pi\pi}$ is given by Breit-Wigner formula. As indicated in Fig. 2 there is a strong dependence on the coupling constant $g_{\pi\pi}$. An interesting dependence on branching ratios is observed too.

In summary, we have studied the photoproduction of $f_0(980)$ resonance for photon energies considered in the CLAS experiment at Jefferson Lab, $E_\gamma = 3.4 \pm 0.4$ GeV. It provides a test for the current understanding of the nature of the scalar resonances. We have calculated the differential cross sections as function of effective masses and momentum transfers. The effect of distinct scenarios in the calculation of the coupling $S \to V\gamma$ was investigated. This study shows that we need to known more precisely the radiative decay rates for $f_0(980) \to \gamma V$ which are important in the theoretical predictions. Our predictions of the cross sections are somewhat consistent with the experimental analysis from the CLAS Collaboration, at least for scenarios 1 and 4. The present experimental data are able to exclude some possibilities for the $S \to V\gamma$ coupling. We show also the large dependence on the model parameters as $g_{\pi\pi}$ values and branching fractions.

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