KAON–NUCLEON SCATTERING IN AN EXTENDED CLOUDY BAG MODEL

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We describe low-energy kaon–nucleon scattering by generalizing the cloudy bag model (CBM) to SU(3) chiral symmetry. We restrict our attention to $I = 0$, s-wave scattering. We obtain a reasonable fit to both the $\Sigma\pi$ cross section in the region of the $\Lambda^*(1405)$ and low-energy kaon–nucleon scattering.

The cloudy bag model (CBM) has had considerable success in describing low-energy pion–nucleon scattering [1]. In this letter we use a chiral SU(3) × SU(3) extension of the cloudy bag model to describe kaon–nucleon scattering near threshold. The original version of the CBM has the mesons coupling to the baryons at the bag surface. Here we use an alternative version [2] obtained by a chiral rotation of the quark wave functions, in which the pions couple throughout the bag volume. This gives much faster convergence for the sum over intermediate quark states [3], and also yields the current algebra results in a transparent manner [2], although it does introduce the complication of a contact term.

In the present letter we ignore backward-going lines and crossed meson lines (the Chew–Low series). From our experience in pion–nucleon scattering [1] this is expected to be a reasonable first approximation. We restrict our attention to $I = 0$ s-wave $\bar{\mathrm{K}}\mathrm{N}$ scattering, including the $\bar{\mathrm{K}}\mathrm{N}$ and $\pi\Sigma$ channels. We allow for one excited baryon state which we take to be an SU(3) singlet (with a quark excited to a 1p level). Thus in our model the $\Lambda^*(1405)$ could in principle be either a pure quark state or a $\bar{\mathrm{K}}\mathrm{N}$ bound state depending on the parameters. In practice we find that both aspects play a role with the $\bar{\mathrm{K}}\mathrm{N}$ bound state being the more important.

The kaon–nucleon and kaon–nucleus field has been recently reviewed by Dover and Walker [4]. An outstanding problem in kaon–nucleon physics is the difference in the sign [5–8] of the real part of the scattering length obtained from kaon–nucleon scattering and from the kaonic hydrogen energy shift. Analysis of the kaon–nucleon scattering yields a negative sign [6] while the kaonic hydrogen data indicate a positive sign [7]. Attempts to explain this discrepancy using potential models [8] have not been successful.

Kumar and Nogami [9] suggest that it is possible to get a positive sign for the scattering length from the interference between a pole term and the background producing a zero in scattering amplitude near threshold. Our model has a similar zero, but at a much

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higher energy and it is probably associated with the $\Lambda^*(1670)$ resonance.

It is interesting that the $\Lambda^*(1405)$ stands out both in the non-relativistic quark models [10] and in the bag model [11] as one state that is hard to fit. It is usually predicted to be too high. In our model the $\Lambda^*(1405)$ occurs much below the bare quark state mass. However, with one bare quark state (not two) we get not just the $\Lambda^*(1405)$ but also a second resonance which may well represent the $\Lambda^*(1670)$.

For a pedagogic review of the SU(2) X SU(2) version of the cloudy bag model we refer to ref. [12]. The straightforward extension to chiral SU(3) X SU(3), with volume coupling, yields the lagrangian:

$$\mathcal{L}_{\theta_{\mathrm{v}}} = [\mathrm{i}\bar{\mathrm{q}}\not{D}\mathrm{q} - B]\,\theta_{\mathrm{v}} - \tfrac{1}{2}\bar{\mathrm{q}}\mathrm{q}\delta_{\mathrm{s}} + \tfrac{1}{2}[\mathrm{D}_\mu\boldsymbol{\phi}]^2 + (1/2f)\bar{\mathrm{q}}\gamma^\mu\gamma_5\,\boldsymbol{\lambda}\cdot\mathrm{q}\,[\mathrm{D}_\mu\boldsymbol{\phi}]\,\theta_{\mathrm{v}}\,. \tag{1}$$

Here q is the quark field (with colour indices suppressed) and $\boldsymbol{\phi}$ is the meson-octet field. The energy density of the vacuum is B, θ_{v} is a step function which vanishes outside the bag, and δ_{s} is a surface delta function which reduces to $\delta(r-R)$ for a static spherical bag. The $\boldsymbol{\lambda}$ are the SU(3) Gell-Mann matrices and f the meson-octet decay constant. Consistent with the CBM philosophy we expand the lagrangian in powers of $1/f$ and we obtain the interaction hamiltonian which to order f^{-2} has two terms. The first-order term is:

$$H_1 = -(\theta_{\mathrm{v}}/2f)\bar{\mathrm{q}}\gamma^\mu\gamma_5\boldsymbol{\lambda}\mathrm{q}\cdot\partial_\mu\boldsymbol{\phi} \tag{2}$$

which couples the $\pi\Sigma$ or $\bar{\mathrm{K}}\mathrm{N}$ to the bare Λ^*. The second-order term is:

$$H_2 = (\theta_{\mathrm{v}}/4f^2)\bar{\mathrm{q}}\gamma^\mu\,\boldsymbol{\lambda}\cdot\mathrm{q}\,\boldsymbol{\phi}\times\partial_\mu\boldsymbol{\phi}, \tag{3}$$

which is a contact term or four-point interaction.

As usual [12] we project this hamiltonian onto the space of non-exotic baryon bags. Then neglecting all diagrams with backward-going lines and crossed meson lines, we obtain a "potential":

$$V_{\alpha\beta} = \langle\alpha|H_1|\Lambda^*\rangle(E-m_0)^{-1}\langle\Lambda^*|H_1|\beta\rangle + \langle\alpha|H_2|\beta\rangle \tag{4}$$

and solve the corresponding Lippmann–Schwinger equation. The states $\langle\alpha|$ and $\langle\beta|$ stand for the various $\bar{\mathrm{K}}\mathrm{N}$ and $\pi\Sigma$ states. The $\langle\Lambda^*|$ indicates the bare Λ^*, a pure quark state, and m_0 its bare mass. The Λ^* is treated on the same level as the N and Σ. We assume it to be in a $(70,1^-)$ representation and a SU(3) singlet [11,13]. For the propagator in the Lippmann–Schwinger equation we use $G(k) = (E - E' - W + \mathrm{i}\epsilon)^{-1}$, where $E'^2 = k^2 + M^2$ is the energy of the propagating baryon and $W^2 = k^2 + m^2$ the energy of the corresponding meson.

Because of the form-factor at the vertices, arising from the finite size of the baryons, the renormalizations are all finite and there is no need to explicitly eliminate m_0 and the bare coupling constants in terms of renormalized ones. In principle we should have a complete set of quark intermediate states rather than just the Λ^* in eq. (4). For the volume coupling we believe these other states give us a relatively small contribution and this has been confirmed in the case of pion–nucleon scattering [3].

It now just remains to calculate the matrix elements needed in eq. (4). The quark wave functions are well known [13]. We begin with the matrix elements for H_1. Rather than evaluating them directly it is useful to do an integration by parts. Using the Dirac equation and the quark boundary condition this yields for massless quarks:

$$\begin{aligned}\langle BM(k)|H_1|\Lambda^*\rangle &= -(\lambda_{\mathrm{B}\Lambda^*}/2f)C^{i_{B3}\,i_{M3}\,0}_{i_B\,i_M\,0}\{N_{\mathrm{s}}N_{\mathrm{p}}/[(2\pi)^3 2\omega_M(k)]^{1/2}\} \\ &\quad\times\Big(2R^2 j_0(\omega_{\mathrm{s}}R)j_0(\omega_{\mathrm{p}}R)j_0(kR) \\ &\quad - [\omega_{\mathrm{s}} - \omega_{\mathrm{p}} + \omega_M(k)]\int_0^R \mathrm{d}x\, x^2[j_0(\omega_{\mathrm{s}}x)j_0(\omega_{\mathrm{p}}x) \\ &\quad + j_1(\omega_{\mathrm{s}}x)j_1(\omega_{\mathrm{p}}x)]j_0(kx)\Big),\end{aligned} \tag{5}$$

where $B(M)$ stands for $\mathrm{N}(\bar{\mathrm{K}})$ or $\Sigma(\pi)$, $\lambda_{\mathrm{N}\Lambda^*} = \sqrt{2}$ and $\lambda_{\Sigma\Lambda^*} = \sqrt{3}$.

The energies $\omega_{\mathrm{s}} = 2.04/R$ and $\omega_{\mathrm{p}} = 3.81/R$ refer to the ground state and first excited p-wave quark state, respectively. The quark state normalizations N_{s} and N_{p} are well known [12]. For surface coupling we would have just the first term in the curly brackets in eq. (5).

In contrast to scattering through the Λ^*, scattering through the contact term is not pure $I = 0$. Thus we must project out the $I = 0$ part. For the case of s-wave scattering the spatial part of the covariant derivative in H_2 does not contribute, so we just quote the result coming from the time derivative for $I = 0$:

$$\langle BM(k)|H_2|B'M'(k')\rangle$$
$$= -(\lambda_{BB'}/2f^2)C^{i_{B3}i_{M3}0}_{i_B i_M 0}\, C^{i_{B'3}i_{M'3}0}_{i_{B'} i_{M'} 0} N_s^2$$
$$\times\{[\omega_M(k)+\omega_{M'}(k')]/(2\pi)^3[2\omega_M(k)2\omega_{M'}(k')]^{1/2}\}$$
$$\times \int_0^R dx\,x^2[j_0^2(\omega_s x)+j_1^2(\omega_s x)]j_0(kx)j_0(k'x)\,, \quad (6)$$

where $\lambda_{NN} = 3/2$, $\lambda_{\Sigma\Sigma} = 2$ and $\lambda_{N\Sigma} = \lambda_{\Sigma N} = \sqrt{6}/4$.

Let us now consider the results for $\bar{K}N$ and $\pi\Sigma$ s-wave $I = 0$ scattering. The threshold are taken to be 1332.1 and 1432.6 MeV for the $\pi\Sigma$ and $\bar{K}N$ channels, respectively.

In our calculation the contact term plays a very important role, giving (by itself) a bump in the $\pi\Sigma$ elastic cross section around 1390 MeV and producing a large scattering amplitude near the $\pi\Sigma$ threshold. As a consequence the results are very sensitive to 10% variations in the value of the meson decay constant (f). In fact, there is not one decay constant but two, with f_π (93 MeV) being some 20% lower than f_K (112 MeV) [14]. However, at the present stage we prefer to take the interaction given in eq. (3) in the limit of exact SU(3) symmetry as a guide, without specifying how the symmetry is broken in the real world. In that case it seems reasonable to tolerate some phenomenological variation in f in the region of 100 MeV. Happily the best fit values, namely 110 MeV with $R = 1.1$ fm and 120 MeV with $R = 1.0$ fm do satisfy this criterion. Finally we note that the bare mass of the Λ^* is also treated as an adjustable parameter, but it turns out that the results are insensitive to its exact value.

In fig. 1 we compare our results with the $\pi\Sigma$ mass distribution given by Chao et al. [15] for $\pi^- p \to (\pi\Sigma)^0 K^0$. The theoretical curves are obtained by multiplying the $\pi\Sigma$ cross section by a phase space factor and an arbitrary normalization factor chosen to get roughly the right height for the distribution. The solid curve corresponds to $R = 1.0$ fm, $f = 120$ MeV, and the dashed one to $R = 1.1$ fm and $f = 110$ MeV. In both cases the bare mass is 1600 MeV. In the present model the Λ^* is largely a $\bar{K}N$ bound state, since most of the contribution to this resonance comes from the contact term. It is due to the strong attraction of the contact term that the $\Lambda^*(1405)$ resonance occurs almost 200 MeV below the bare quark state mass.

It must be pointed out that the present model has not only the $\Lambda^*(1405)$ resonance but also a resonance at a higher energy presumably corresponding to the $\Lambda^*(1670)$, although the present model is not sufficiently accurate at such high energies to make more than qualitative statements. [It leaves out the $\eta\Lambda$ channel and the wide $\Lambda^*(1800)$.]

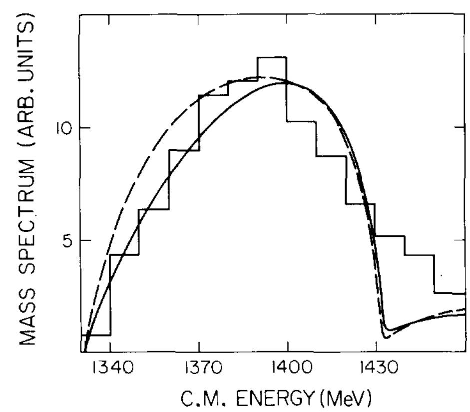

Fig. 1. A plot of the final-state $\pi\Sigma$ mass distribution. The histogram is the data from Chao et al. [15]. The theoretical curves are calculated as discussed in the text, with an arbitrary normalization. The parameters are $R = 1.0$ fm, $f = 120$ MeV (solid curve) and $R = 1.1$ fm, $f = 110$ MeV (dashed curve). The bare mass (m_0) is taken as 1600 MeV.

It is interesting to compare the cross sections with the experimental results. Unfortunately the experimental data do not separate the s-waves from higher partial waves and, in the $K^- p$ elastic scattering, the $I = 0$ from the $I = 1$ contribution. For this reason the

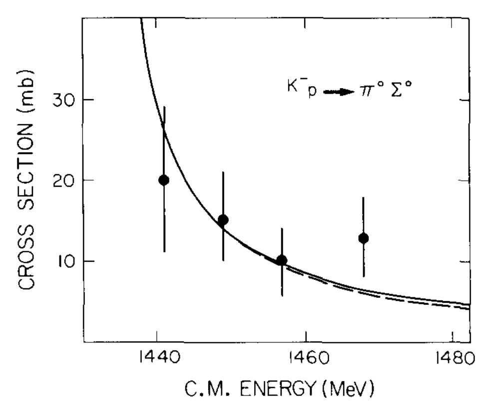

Fig. 2. The $I = 0$ s-wave $K^- p \to \pi^0\Sigma^0$ cross sections for the sets of parameters. The same convention is used as in fig. 1. The data are from ref. [16].

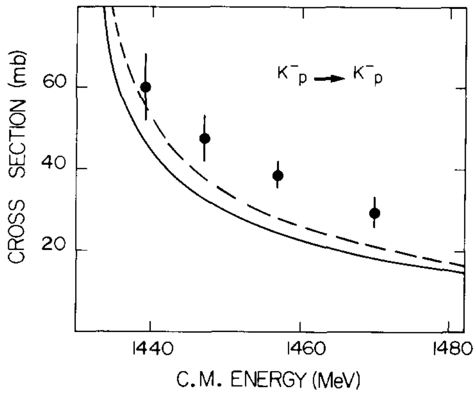

Fig. 3. The curves are the $I = 0$ s-wave K^-p elastic cross sections. The convention is the same as in fig. 1. The data, taken from ref. [16], refer to the $I = 0 + I = 1$.

comparison can be made only qualitatively. Potential calculations [16] indicate that below about 250 MeV/c the $\bar{K}N$ elastic and absorptive cross sections are predominantly s-wave. In fig. 2 we plot the $I = 0$ $K^-p \to \pi^0\Sigma^0$ cross section for the same set of parameters discussed above. The curves, which are very insensitive to the parameters, are in fairly good agreement with the data.

In fig. 3 we plot the $I = 0$ K^-p elastic cross section. The experimental data plotted are not $I = 0$ but rather $\frac{1}{2}[\sigma(K^-p \to K^-p) + \sigma(K^-p \to \bar{K}^0 n)]$. The fits are reasonable, particularly if one takes account that the data have a small $I = 1$ contribution (see ref. [16]).

The $I = 0$ $\bar{K}N$ scattering lengths for the two sets of parameters are:

$$a = -1.16 + 1.44\mathrm{i}\ \mathrm{fm}$$

with $R = 1.0$ fm, $f = 120$ MeV,

and

$$a = -1.36 + 1.55\mathrm{i}\ \mathrm{fm}$$

with $R = 1.1$ fm, $f = 110$ MeV.

For comparison the value obtained by Martin [6] from scattering using dispersion relation constraints is $-1.70 + 0.68\mathrm{i}$ fm. Although the real part of the scattering length is smaller in our calculation, it is still negative – in apparent contradiction with the K^-p atomic data. On the other hand, the K^-p amplitude calculated here undergoes a very rapid variation below threshold which may resolve the problem. This certainly deserves to be investigated further.

In conclusion, we note that in spite of the large mass of the kaon the SU(3) CBM seems to be a good starting point for describing low energy $\bar{K}N$ scattering.

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