Damping and mode-coupling for low-frequency electromagnetic waves in a dusty plasma with dust charge fluctuation

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Abstract.
Using a kinetic description to analyze wave propagation in dusty plasmas, taking into account the fluctuation of the charge of the dust particles due to inellastic collisions with electrons and ions, we consider the case of propagation of waves exactly parallel to the external magnetic field and Maxwellian distributions for the electrons and ions in the equilibrium, and investigate the phenomena of mode coupling which may occur due to the presence of the dust.

Keywords: dusty plasmas, dust charge fluctuations, Alfvén waves
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In a recent publication we have used a kinetic description to analyze the propagation of electromagnetic waves in dusty plasmas, taking into account the dust charge fluctuation due to inelastic collisions with electrons and ions [1], incorporating many details which have appeared in previous publications [2, 3, 4]. In the present paper we return to the subject in order to show mode coupling phenomena which may occur due to the presence of dust particles with variable charge. As a preliminary exposure about this novel feature, we consider a situation with the same general parameters utilized in the case of Ref. [1], and extend the range of parallel wave numbers investigated, as well as the range of values of the dust density.

We consider a plasma in a homogeneous external magnetic field $\mathbf{B_0} = B_0\, \mathbf{e_z}$, taking into account the presence of spherical dust grains with constant radius a and variable charge q_d. The dust grains are charged due to the capture of electrons and ions during inelastic collisions between these particles and the dust particles. As in our previous work, the emphasis is on the study of low-frequency waves, assuming immobile dust particles and therefore excluding modes which can arise from the dust dynamics [1]. The formalism is appropriated for $|\Omega_d| << \omega_{pd} < \omega$.

Using these approximations, and assuming Maxwellian distributions for electrons and ions, the dispersion relation for parallel propagating electromagnetic waves can be written as follows [1]

$$\frac{q^2c^2}{v_A^2 z^2} = 1 + \sum_{\beta} \frac{\eta_\beta^2}{\sqrt{2} q u_\beta z} Z(\hat{\zeta}_\beta^s) \; , \tag{1}$$

where Z is the plasma dispersion function, $\hat{\zeta}_\beta^s = (z + s r_\beta + i\tilde{\nu}_\beta)/(\sqrt{2} q u_\beta)$, $\beta = e, i$, for

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electrons and ions, and where $s = \pm 1$. $z = \omega/\Omega_i$, $q = k_\parallel v_A/\Omega_i$, $u_\beta = v_{T\beta}/v_A$, $\eta_\beta = \omega_{p\beta}/\Omega_i$, $r_\beta = \Omega_\beta/\Omega_i$, $\tilde{\nu}_\beta = \nu_\beta/\Omega_i$, where $v_A = B_0/\sqrt{4\pi n_{i0} m_i}$ is the Alfvén velocity, and where $\omega_{p\beta}$ and Ω_β are, respectively, the angular plasma frequency and the angular cyclotron frequency of particles of species β. The quantity $\tilde{\nu}_\beta$ is the average value of the collision frequency in momentum space, which in terms of dimensionless quantities is written as $\tilde{\nu}_i = 2\sqrt{2\pi}\varepsilon\gamma\tilde{a}^2 u_i(1+\chi_i)$, $\tilde{\nu}_e = 2\sqrt{2\pi}\varepsilon\gamma\tilde{a}^2 u_e e^{\chi_e}$, where $\chi_i = Z_d/\tilde{a}$, $\chi_e = -\chi_i/\tau_e$, $\varepsilon = n_{d0}/n_{i0}$, Z_d is the dust charge number, and where $\gamma = \lambda^2 n_{i0} v_A/\Omega_i$, $\lambda = e^2/T_i$, $\tilde{a} = a/\lambda$, $\tau_e = T_e/T_i$, with λ being the classical distance of minimum approach.

Eq. (1) is solved numerically. In Fig. 1(a) we observe the real part of the normalized frequency, z_r, for the two roots obtained from Eq. (1) for each of the signs $s = 1$ and $s = -1$, for five values of ε in the range between 0.0 and 5.0×10^{-6}. There are two curves with positive values of z_r, describing waves with positive phase velocities. The uppermost curve is obtained with $s = 1$, and corresponds to the so-called *whistler branch* (W), while the lower curve is obtained with $s = -1$ and corresponds to the branch identified with circularly polarized waves (CP) propagating along the ambient magnetic field. These curves have already appeared in our previous work [1] and are easily recognized from well-known textbooks [5]. There are symmetrical solutions propagating in negative direction, obtained respectively with $s = 1$ (the upper curve, closer to the axis) and $s = -1$ (the lower curve). For small values of q, $q \leq 0.2$, the two branches of waves propagating in a given direction collapse together in a single branch known as the branch of the Alfvén waves.

In Figs. 1(b) and (c) we see the corresponding imaginary parts z_i. The W branch appears in Fig. 1(b), while Fig. 1(c) shows the branch of CP waves. Fig. 1(b) shows that the damping rate for the W branch is negligible in the absence of dust ($\varepsilon = 0$), but becomes significant in the presence of even a small amount of dust, specially in the range of small values of q. Fig. 1(c) shows that the waves in the branch of CP waves feature some damping even in the absence of dust, for q approaching $q = 1$. This damping is due to wave-particle resonance and is known as the ion-cyclotron damping, which occurs for $|\omega| \simeq \Omega_i$. Fig. 1(c) also shows that this damping tends to be dominated by the damping due to the presence of the dust, even for the relatively small dust density which has been considered in the case of Fig. 1 [1].

Fig. 2(a) to (c) display the values of z_r as a function of ε, for three values of q and both signs $s = +1$ and $s = -1$, considering the same parameters utilized for Fig. 1, but considering ε up to 2.0×10^{-4}, much above the maximum value of ε considered for Fig. 1. Panels (a) to (c) of the figure depict, respectively, the cases of $q = 0.1$, 0.2, and 0.3. Other values of q are not shown for the sake of space economy. According to what we have learned from Fig. 1, for $\varepsilon = 0$ the uppermost and lowermost curves for z_r shown in panels (a) to (c) of Fig. 2, identified by a bold line, correspond to the values of z_r for the W branch obtained for $s = +1$ and $s = -1$, respectively. The other two curves in each panel, identified by a thin line, pertain to the other mode, the CP branch.

It is seen from Fig. 2(a) that for $q \simeq 0.1$ the two roots corresponding to the W branch display decreasing values of $|z_r|$ for increasing dust density, up to a point near $\varepsilon \simeq 1.3 \times 10^{-5}$, and then show $|z_r|$ increasing linearly with ε. It can be shown that the point of minimum value of $|z_r|$ changes with q, being close to $\varepsilon = 0.7 \times 10^{-5}$ for $q = 0.05$, 1.3×10^{-5} for $q = 0.1$, and 1.9×10^{-5} for $q = 0.15$ [1]. On the other hand, the

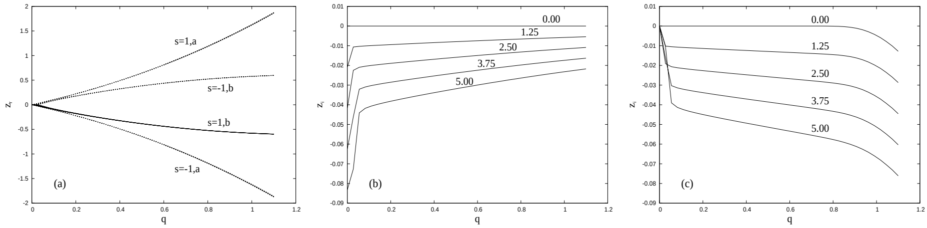

FIGURE 1. (a) z_r for the two roots obtained using $s = 1$ and for the two roots obtained using $s = -1$, as a function of q, for five values of ε. $B_0 = 1.0 \times 10^{-4}$ T, $T_i = 1.0 \times 10^4$ K, $n_{i0} = 1.0 \times 10^9$ cm^{-3}, $Z_i = 1.0$, $m_i = m_p$, $T_e = T_i$. and $a = 1.0 \times 10^{-4}$ cm. The values of ε utilized are $\varepsilon = 0.0$, 1.25×10^{-6}, 2.50×10^{-6}, 3.75×10^{-6}, and 5.0×10^{-6}. (b) z_i for the positive propagating upper root appearing in Fig. 1(a), obtained with $s = 1$, as a function of q. (c) z_i for the positive propagating lower root appearing in Fig. 1(a), obtained with $s = -1$, as a function of q.

two roots corresponding to the CP waves propagating in positive and negative directions, which are well separated for $\varepsilon = 0$, feature mode-coupling for increasing dust density. Fig. 2(a) shows that the point of mode-coupling aproximately corresponds to the point of minimum value of z_r for the W branch.

In Fig. 2(b) we see the case of $q = 0.2$. As in the case of $q = 0.1$, the two roots corresponding to the W branch display decreasing values of $|z_r|$ for increasing dust density, up to a certain value of ε. However, they meet the curves corresponding to the branch of the CP waves, something that didn't happen for smaller values of q, for the parameters utilized. After this meeting point, at $\varepsilon \simeq 2.7 \times 10^{-7}$, the figure shows $|z_r|$ nearly constant with further increase of ε, instead of growing as in panel (a). The two roots corresponding to the CP waves propagating in positive and negative directions still feature mode-coupling for increasing dust density, as in the case of panel (a). The point of coupling for these waves increases as compared to the case of panel (a), being close to $\varepsilon = 2.4 \times 10^{-5}$.

Fig. 2(c) shows similar features occurring for $q = 0.3$. The comparison between panel (b) and panel (c) shows that the coupling point occurs for larger value of epsilon, for larger q. By considering larger values of q, it can be shown that the coupling points occur for larger and larger values of ε, for increasing values of q, and that the points of coupling, between forward and backward propagating CP waves and between W and CP waves, are progressively separated with the increase of q.

In Figs. 2(d) to (f) we see the corresponding imaginary parts. For $q = 0.1$, Fig. 2(d) shows that the damping of both modes, which is negligible for $\varepsilon = 0$, increases linearly with the dust density, up to nearly the value of ε where the real parts of the two branchs show the closest approximation, in Fig. 2(a). Beyond this point, the damping rates of the two modes are separated one from the other. The damping of the waves in the W branch continues to increase almost linearly with the dust density, while the damping of the waves in the branch of CP waves is gradually reduced for increasing values of ε. For $q = 0.2$ and $q = 0.3$, Figs. 2(e) and 2(f) show behavior qualitatively similar, but with an important difference. For these values of q after the occurrence of mode coupling, it is the branch identified with the cyclotron mode at low ε that displays increasing growth

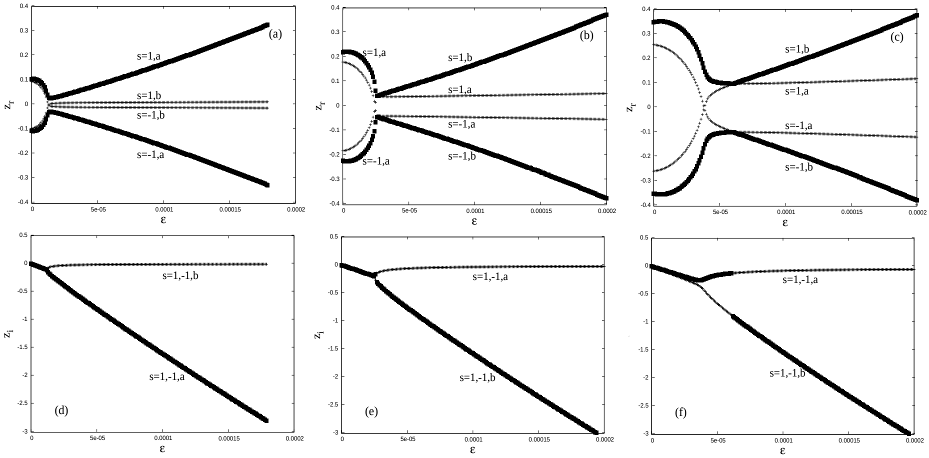

FIGURE 2. z_r as a function of ε, for ε between 0.0 and 2.0×10^{-4}, for $s = \pm 1$, and three values of q. (a) q=0.1; (b) q=0.2; (c) q=0.3. z_i as a function of ε, for ε between 0.0 and 2.0×10^{-4}, for $s = \pm 1$, and three values of q. (d) q=0.1; (e) q=0.2; (f) q=0.3. As in Fig. 1, the curves identified as $s = +1, a$ and $s = -1, a$ pertain to the W branch in the limit of vanishing dust density, and the curves identified as $s = +1, b$ and $s = -1, b$ pertain to the branch of CP waves. Other parameters as in Fig. 1.

rate for increasing values of ε. Another qualitative feature to be remarked is that the damping of the two modes become progressively more separated even for small values of ε, for increasing values of q. These features continue to be seen for larger values of q, not shown in the figures.

The occurrence of mode coupling between waves in the W branch and waves in the CP branch, due to the presence in the plasma of dust particles with variable charge, is a phenomenum not yet reported in the literature. We are conducting further investigations about this novel feature, intending to report on more details in a forthcoming publication.

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