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Modelling Behaviour Diffusion with Dynamic Logic

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"the books that influence the world are those that it has not read"

— G.K. Chesterton

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ABSTRACT

This work is about agents and their adoption of behaviors in a given network. The work has focused on defining models and policies related to the "diffusion of information" not through agent reasoning but through a force exerted by the network. The model reasoning process checks whether each agent has received enough influence to surpass a threshold related to a given subject or behavior, deciding whether the agent enters into social conformity with its network of connections.
We consider models with multiple behaviors and different criteria for behavior adoption but with neighbors having the same level of social influence. We also define models and diffusion operations with directed influence with neighborhood connections with different weights on behavior adoption. We propose a minimal propositional dynamic logic language for all these variations and provide reduction axioms for each logic.
We also present naive algorithms for each model update operation.

Keywords: Diffusion of Behavior. Dynamic Logic. Reduction Axioms.

Modelando Difusão de Comportamento com Lógica Dinâmica

RESUMO

Este trabalho é sobre agentes e sua adoção de comportamentos em uma rede. O trabalho tem como foco a definição de modelos e políticas relacionadas à "difusão de informações" não por meio do raciocínio do agente, mas por meio de uma força exercida pela rede. O processo de difusão verifica, para cada agente, se ele recebeu influência suficiente para ultrapassar um limiar, relacionado a um determinado assunto ou comportamento, decidindo então, se o agente entra ou não em conformidade social com sua rede de conexões. Consideramos modelos com múltiplos comportamentos e diferentes critérios de adoção de comportamento, mas com vizinhos com o mesmo nível de influência social. Também são apresentados modelos com a influência pode ser em uma direção apenas e onde cada conexão pode ter um peso diferente na adoção de comportamentos. Para todas essas variações propomos uma lógica dinâmica proposicional mínima e, para cada lógica, fornecemos axiomas de redução. Também apresentamos algoritmos naïve para cada operação de atualização de modelos.

Palavras-chave: Difusão de Comportamento, Lógica Dinâmica, Axiomas de Redução..

LIST OF ABBREVIATIONS AND ACRONYMS

TM	Threshold Model
TMU	Threshold Model Update
TMIB	Threshold Model with Independent Behaviors
TMCB	Threshold Model with Conflicting Behaviors
TMCBPA	Threshold Model with Conflicting Behaviors and Priority for Adoption
TMDB	Threshold Model with Dependent Behaviors
TMDBPA	Threshold Model with Dependent Behaviors and Priority for Adoption
TMCDB	Threshold Model with Conflicting and Dependent Behaviors
TMCDBPA	Threshold Model with Conflicting and Dependent Behaviors and Priority for Adoption
TMWUN	Threshold Model with Weighted Undirected Networks
TMDN	Threshold Model with Directed Networks

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CONTENTS

1 INTRODUCTION

The importance of studying the dynamics of agents has long been recognized. It provides insights into how humans interact with each other and the environment. By studying agents, researchers can improve the understanding and prediction of human behavior and the effect of different behaviors on social networks and other related systems. It provides insights into designing better systems with artificial intelligence or robotics that can interact with humans meaningfully and naturally.

Our work has focused on social behavior, an area of study that originates from the field of Information Propagation and Epidemics. This study area received different approaches from researchers, dealing with cases ranging from energy distribution and blackouts to epidemiology and marketing. Although information propagation relates to transmitting information from one network node to another, in epidemics, the word "information" can be substituted by many others without causing a loss of sense of functioning. In epidemics, the propagation can be of infectious diseases (viral or bacterial) or meta-diseases, such as anxiety, worry, fear, and many others. In that case, these meta-diseases are spread by emotions, beliefs, and attitudes from person to person within a population. The field of study of epidemic diseases has always been a topic where biological issue mix with social ones (CHRISTAKIS; FOWLER, 2009; EASLEY; KLEINBERG, 2010).

Diseases and other spread contagions that create "epidemics" are studied in the works of Christakis and Fowler (2009), Watts (2004), Easley and Kleinberg (2010), with some examples being: obesity; depression; suicidal behavior; smoking and substance abuse; voting patterns; sexual and infectious diseases: influenza, measles, HIV/AIDS; technologies; ideas; products; political opinions; religious beliefs; cultural norms;

Rules applied to the spread of opinions in networks can be influenced by factors such as word of mouth, peer pressure, and the availability of information (WATTS, 2004; EASLEY; KLEINBERG, 2010; CHRISTAKIS; FOWLER, 2009).

Threshold Models (TM) of Collective Behaviors were introduced by Granovetter (1978). They are models describing the relationship between the number of people required to start a collective action and the probability that it will be successful. He argued that the number of people required to start a collective action and the probability that it will be successful are linked. A few examples of such collective actions are riot behavior, strikes, innovation and rumor diffusion, voting, and migration.

The "threshold" is a key concept in his approach and was later generalized in the work of Easley and Kleinberg (2010). Threshold Models have been an object of research in many areas, including in the logic community, that make use of Threshold Models to give semantics for a logic formalization for the diffusion of behaviors (CHRISTOFF, 2016; BALTAG et al., 2018; HANSEN, 2011; CHRISTOFF; HANSEN, 2015; ZHEN; SELIGMAN, 2011; LIBERMAN; RENDSVIG, 2022; HANSEN, 2015; GONZALEZ, 2022).

In this dissertation, we take as a departure point the non-epistemic part of the work of (BALTAG et al., 2018). We propose several extensions to their basic definitions of threshold model and threshold model update.

The element diffused in a network of agents is termed a *behavior*. However, it can express not only a behavior but also an opinion, a fashion, or product. Sometimes agents are refereed as *holders* or *not holders* of a behavior. Also, the verb *adopt* denotes that an agent became a holder of the behavior.

Along the dissertation, we adopt a minimal propositional dynamic logic to express the state of the network of agents, i.e, which agents are direct neighbors and which agents hold a behavior. We assume a set of propositional variables N_{ab} interpreted as *agents a and b are neighbors*, and a set of propositional variables B_a meaning that *agent a holds behavior B* For expressing the dynamic aspect, i.e., the diffusion of behavior, the logic has a dynamic modality for the adoption of a behavior.

This dissertation is organized as follows: in Chapter 2 we present the background that form the basis of our study. In Chapter 3, we consider threshold models with: Multiple Independent Behaviors; Conflicting Behaviors; Conflicting Behaviors and Priority for Adoption; Dependent Behaviors; Dependent Behaviors and Priority for Adoption; Conflicting and Dependent Behaviors; and lastly Conflicting and Dependent Behaviors and Priority for Adoption. In Chapter 4, we define two models with a single behavior where the adoption threshold takes into account how influential each neighbor of an agent is: a model with weighted undirected edges, and a model with directed unweighted edges. We conclude with a discussion on future work.

2 BACKGROUND AND RELATED WORKS

This preliminary chapter introduces the theory behind behavior diffusion in social networks. We start by evaluating ideas from sociology by discussing the possibilities of why this behavior diffusion effect occurs in networks. Subsequently, we will give the first steps into network information propagation, citing research areas and examples of use. We then discuss threshold models of collective behavior and cascade models, arriving at the work of Easley and Kleinberg (2010) that inspired the work of Baltag et al. (2018), which in turn inspired this dissertation.

We conclude this chapter with a brief introduction to aspects of Logic that will be used in the following chapters, and we introduce basic concepts about graph networks that may be necessary for the proper understanding of the operators for behavior diffusion to be proposed.

2.1 The Origins of Social Network Analysis

The book "The Development of Social Network Analysis" by Freeman (2004) states that some writers date the origin of social network analysis to beginning in the early 1930s with the work of Jacob Moreno introducing sociometry, which he published in his book "Who Shall Survive? A New Approach to the Problem of Human Interrelations.". In contrast, some argue that social network analysis began in the early 1970s when Harrison White started training graduate students at Harvard.

In the work of Moreno (1934), he developed sociograms to analyze choices of preferences within a group. A sociogram is a graphic representation of a person's social links, diagramming the structure and patterns of group interactions. In this sense, Moreno's work with sociograms was seen as an early application of graph theory to social network analysis.

A Columbia University sociologist named Allen Barton, in 1968, made the statement that mainstream social research was focused exclusively on the behavior of individuals, neglecting the *social* part of the behavior, neglecting the part concerned with the influential aspect that one has on another (FREEMAN, 2004).

The study of the *social relationships* linking individuals rather than the individuals themselves is not confined to the study of human social relationships; in fact, it is present in almost every field of science. Astrophysicists, for example, assign a lot of effort into the study of the relationship between planets, the gravitational attraction of each planet in the solar system on each of the others in order to account for planetary orbits; biologists study how each of the species interacts within an ecosystem; Electrical engineers, on how the interactions of various electronic components occur and affect the flow of current through a circuit; and many other examples can be found in other areas of study (FREEMAN, 2004).

An important fact to be noted is that the relationships that social network analysts study are usually those that link individual human beings. However, sometimes, the study may be more interesting, linking social individuals that are not human, like animals, agents, or robots, or that are not even individuals at all, like groups and organizations, once influence, for example, may be received from a brand, from an AI bot, an old subscript, and so on.

2.2 Information Propagation

In this area of study (social network analysis), the term *information propagation* is related to transmitting information from one network node to another. Depending on the field, the dynamics of these spreads can change. Over the years, approaches for propagation through networks have been developed by researchers working in areas such as energy distribution, marketing, politics, economics, social sciences, social networks, and epidemiology.

In social networks, the spread of opinions is often the result of a direct influence between individuals, such as family or friends, and it can be influenced by factors such as word of mouth, peer pressure, and the availability of information (WATTS, 2004; EASLEY; KLEINBERG, 2010; CHRISTAKIS; FOWLER, 2009). Individuals adopt the opinion of those they are connected with, creating an adoption cascade effect that quickly spreads, similar to the spread of virus contagion and epidemics. The network's structure also affects the spread, such as its size, density, and connectedness.

In epidemics research, "information" can be substituted by infectious diseases or meta-diseases, such as anxiety, worry, fear, and others. In that case, these meta-diseases are spread by emotions, beliefs, and attitudes from person to person within a population. Diseases and other spread contagions that create "epidemics" are vastly studied in the works of Christakis and Fowler (2009), Watts (2004), Easley and Kleinberg (2010). The study of epidemic diseases has always been a topic where biology and social ones influence each other (EASLEY; KLEINBERG, 2010).

2.3 Classification of Information Propagation

Classifying the propagation of information can help with understanding how information flows, the identification of potential risks and vulnerabilities, and also helps with the development of strategies for more effective propagation and control of the flow of information.

The classification we present in this section was given by the work of Liben-Nowell (2005), in which the author divides the field of information propagation into Information Propagation and Epidemics, Game-Theoretic Approaches, and Diffusion of Innovation. We briefly go through each one of these types situating this dissertation in the last one, namely Diffusion of Innovation.

2.3.1 Information Propagation and Epidemics

Most of the research investigating the flow of "information" through networks has been based upon the analogy between the spread of disease and the spread of information in networks. The study of epidemiology brings up essential aspects of networks and the diffusion effect.

Classical disease propagation in epidemiology is based on the disease cycle in a host, commonly called the SIR model. This model classifies individuals into three categories: susceptible, infected, and recovered. This model uses the contact rates between individuals and the transmission rate of the disease to simulate its spread into a population. The simulation results are used to identify potential interventions that could slow down the spread. This model can also be used to evaluate the effectiveness of interventions that have already been implemented (KEELING; ROHANI, 2008).

If someone is exposed to a disease and is susceptible, he becomes infected with some probability. The disease then runs its course in the host, who recovers. The recovered individual becomes immune for a while until it eventually may become susceptible again, and the cycle goes on. The SIR model is applied for diseases in which the host never becomes susceptible again, and the SIRS model is applied when the recovered host eventually becomes susceptible again. Other models are SI (susceptible-infected), SIS (susceptible-infected-susceptible), SEIR (susceptible-exposed-infectious-recovered), and others.

2.3.2 Game-Theoretic Approaches

Game Theory is the branch of mathematics used to analyze decision-making and strategic interactions between different agents, allowing the construction of rigorous models describing situations of conflict and cooperation between rational decision-makers. Game theory has been successfully applied to many situations, such as business competition, functioning of markets, political campaigning, jury voting, auctions, procurement contracts, and union negotiations. Game theory has also shed light in disciplines such as evolutionary biology and psychology (TADELIS, 2013).

The propagation of information through the perspective of Game Theory considers the utility for players to adopt an innovation or behavior. Players receive a positive payoff for each of their neighbors that have also adopted, in addition to an intrinsic benefit from adopting it. For example, if enough friends switched from videotape to DVDs, a person with friends who have made the same choice can benefit by borrowing movies (LIBEN-NOWELL, 2005).

2.3.3 Diffusion of Innovation

The Diffusion of Innovation is a technology diffusion and adoption theory proposed by Rogers (1983) that describes how, why, and at what rate new ideas and technologies spread across cultures. Many studies in other areas also use the model as a basis, including political science, public health, communications, history, economics, technology, and education (DOOLEY, 1999; STUART, 2001).

Adoption is a decision of "full use of an innovation," rejection is a decision of "not adoption of an innovation," and diffusion is "the process in which an innovation is communicated through certain channels over time among the members of a social system" (ROGERS, 1983).

The author mentions an interrelationship between the rate of knowledge about innovation in a system and its adoption rate. The diffusion effect means that, while an individual does not have a minimum level of information and peer influence from the environment, he is unlikely to adopt it. However, once the level of information increases past a certain threshold, adoption becomes more likely to occur. A threshold seems to occur when the leader's opinions favor innovation in the given system (ROGERS, 1983).

The literature considers two fundamental models for Diffusion of Innovation: Threshold Models by Granovetter (1978) and Cascade Models by Goldenberg, Libai and Muller (2001). Each of the models is discussed in the following subsections.

2.3.4 Threshold Models of Collective Behavior

The Threshold Models of Collective Behaviors, proposed by Granovetter (1978), describe the relationship between the number of people required to start a collective action and the probability of its success. He argued that the number of people required to start a collective action and the probability of success are linked. A few suggested applications are riot behavior, strikes, innovation and rumor diffusion, voting, migration, measurements, falsifications, and verification.

These models are developed for situations in which actors, who are assumed rational (given their goals, preferences, and perception of their situation, they act seeking max utility), are faced with two alternatives. The costs and benefits of the alternatives depend on the number of actors opting for each.

The key concept is given by the "threshold," which represents the proportion of others who have to decide before the actor decides (GRANOVETTER, 1978). Each node u in the network chooses a threshold $T_u \in [0, 1]$. Each edge between nodes v and u has a non-negative connection weight $w_{u,v}$, so that $\sum_{v \in \Gamma(u))} w_u, v \leq 1$, and u adopts if and only if $T_u \leq \sum_{v \in Adopters \cap \Gamma(u))} w_{u,v}$, where $\Gamma(u)$ denotes the graph-theoretic neighborhood of the node u (LIBEN-NOWELL, 2005).

The author gives a few descriptions of agents according to their threshold: "radical" being a low threshold owner, "instigators" being part of the radicals but with a threshold of 0%, and "conservatives" with thresholds varying from 80% to 100%.

2.3.5 Cascade Models

The work of Goldenberg, Libai and Muller (2001) examines word-of-mouth (w-o-m) communications, which is a phenomenon that occurs when people share information through informal conversation and other forms of personal interactions. It is a basic form of communication in many forms of marketing, as it is a powerful tool for spreading information about a product, service, or brand.

The Cascade Model of w-o-m has use of the theory of weak and strong ties given by the work of Granovetter (1973), which says that the strength of weak ties between individuals is, often, more beneficial than strong ties, providing access to new information and resources that may not be available through strong ties. Weak ties are typically formed between individuals with less in common or more distant. In contrast, strong ties are made between individuals who are closer to each other and have more in common.

Whenever a social contact v in the neighborhood of u adopts an innovation, then u adopts it with a given probability of $p_{v,u}$. In other words, every time someone adopts it, there is a chance of someone related to this person to adopt it as well (LIBEN-NOWELL, 2005).

The work of Kempe, Kleinberg and Tardos (2003) promotes the general cascade model, which generalizes the independent cascade model given by Goldenberg, Libai and Muller (2001), and also generalizes the threshold model.

2.4 Social Conformation

As discussed in the previous sections, the importance of studying the dynamics of agents has long been recognized. It provides insights into how humans interact with each other and the environment. By studying agents, researchers can improve the understanding and prediction of human behavior and the effect of different behaviors on social networks and other related systems. The study of agents can yield insights into designing better systems in areas such as artificial intelligence or robotics that can interact with humans more meaningfully. A famous perspective about agents is given by Wooldridge (2009). The author state that agents should have the following abilities: the ability to operate without the direct intervention of humans and control over their internal state (autonomy), the ability to interact with humans and other agents (social behavior), the ability to perceive changes in the environment and respond to them in a timely fashion (reactivity), and the ability to exhibit goal-directed behavior (proactivity).

However, interactive agents often fail to adapt to real-world society's norms; the improperly developed agent can look like a dictator or, in other cases, the complete opposite, accepting everything and thus not contributing. For example, we can imagine a situation of earthquake rescue involving multiple robot agents and humans; both robots and humans have previous knowledge related to rescue missions; both are able to explore the terrain searching for track of survivors; and both are able to call for group actions, actions that a single unit can't perform. If the robot acts as a complete dictator, he will refuse to delay or abandon his plans to attend to calls, which might cost lives. At the same time, a one hundred percent passive robot may abandon his plans too early to participate in a group call, despite being very near to a rescue. In other words, the lack of mechanisms to balance agent decision-making can result in agents that are not "social" enough. Sometimes a priority list might fix it; sometimes, a group decision might be the best option. In the end, understanding the dynamics of group interactions is essential to developing more evolved agents.

Christakis and Fowler (2009), for instance, propose three different definitions for human social networks. They calls a *group of people* a network defined by an attribute (for example, woman, Democrats, lawyers, long-distance runners etc.); they define a *network* as a collection of people with a specific set of connections between them, where the pattern of these ties are often more important than the individual people themselves. Those groups defined by the network can do things that a disconnected collection of individuals cannot. Lastly, they define a *network community* as a group of people who are much more connected to one another than they are to other groups of connected people in other parts of the network.

An explanation for why individuals form groups is given by Peterson (2017). The author uses an insight from Stanford biologist Robert Sapolsky about zebras. Zebras (they are black with white stripes) are not camouflaged against their surrounding environment. Instead, they are camouflaged to blend in with the herd.

This blending effect makes it hard for researchers to study zebras in their natural environment. They make a note as soon as they notice something about a zebra. However, when they look back, they cannot tell if it is the same zebra. Researchers had the idea to red paint or ear tag zebras and found that once the zebra was market, it was quickly killed by the lions. Researchers realized that lions do not necessarily kill the weakest in the herd but typically go for the ones they can identify. Being a small zebra, a zebra with a limp, red painted, or those who stick their head up are all reasons to become a potential target for predators.

More proof of that can be seen in fish hordes, for example. The intelligent, healthy, large fishes stay in the center of the school. When the herd moves around, the healthier fishes keep moving to the middle of the herd, using the herd as protection. Another case is found in reindeer. When the herd is in danger, the reindeer begin to run in circles around the hunter, creating entire cyclones and making it nearly impossible for the hunter to target a single animal (WEISBERGER, 2019).

We didn't find proper Studies in the area relating this theory to humans, although we found that the explanation might be plausible, matching the dynamics of Diffusion of Innovation. We hope that the future brings more research being done in the area.

We point out that the book Christakis and Fowler (2009) seems to be a good candidate for analysis. It presents a collection of multiple cases of social epidemics related to humans that enforce the dynamic ideas behind the social conformation theory to humans. Although, despite our initial intentions, we decided that the cases were too harsh to be presented in our work without a deep and proper discussion. It's a sensible area, and it is possible that many of the cases reported in the book are not the standard but rather cases that, in the area of statistics, would be called outlier cases. Even so, we remember that the area of economy conducts many studies precisely to find characteristics of these outliers, discovering patterns and making, for instance, decisions about investments. The book Taleb (2008) illustrates that.

For now, what we found relating to humans is not complete work but a discussion done by Peterson (2017), where, according to him, people might act following the same principles of the social conformation theory: they try to place themselves at the center of the crowd so that there is a protective barrier between them and potential attacks. He says that humans tend to keep their opinions and behaviors not too distinctive from the crowd so they can feel protected by not "sticking out their heads."

Resuming, the theory itself says that: It is not the survival of the fittest; instead, it is the survival of the conformist.

2.5 Modelling Behavior Diffusion with Dynamic Logic

This dissertation is in the context of a logical treatment for the diffusion of behavior relying on the generalized model of *threshold limited influence* given by (EASLEY; KLEINBERG, 2010) that creates an effect that matches the definition of social conformity. According to this approach, agents adopt a behavior whenever a given proportion of their direct neighbors have already adopted it.

After the discussion in the previous sections of different ways of approaching diffusion of information in a network of individuals, in this section, we give some background on Dynamic Logic for reasoning about the effects of actions and, more specifically, about threshold models and the process of diffusion of behaviors.

Dynamic Logic was first proposed by Vaughan Pratt (PRATT, 1976) as a modal logic for reasoning about imperative programs. If P is a program and φ is a property about state involving values of variables manipulated by P, a formula $[P]\varphi$ of Dynamic Logic expresses that φ holds after the execution of a P. Dynamic Logic, as developed by Pratt, is a generalization of Floyd-Hoare (FLOYD, 1967; HOARE, 1969) logic for programs: a Hoare triple $\{\phi\}\ P\ \{\psi\}$ corresponds to $\phi \rightarrow [P]\psi$ in Dynamic Logic.

After the pioneering work of Pratt, Dynamic Logic attracted the attention of researchers working with Logic in areas other than program verification. Any logic for studying operators that can modify the proper structure in which their effect is being evaluated is based on Dynamic Logic. Examples are Dynamic Epistemic Logics (DITMARSCH; HOEK; KOOI, 2007) and Dynamic Preference Logics (VAN BENTHEM, 2009; LIU, 2011).

This dissertation follows the lines of (CHRISTOFF, 2016), (SELIGMAN; LIU; GIRARD, 2011), and, more specifically, (BALTAG et al., 2018), but instead of advancing in the epistemic part of their work, we took a step back: we provide non-epistemic logics for reasoning about different threshold models of behavior diffusion. Our point of departure as inspiration was the basic threshold model and model update operation given by Baltag et al. (2018). We also assume a minimal propositional logic for reasoning about behavior adoption and neighborhood relation between agents.

Our first extension to the basic model of Baltag et al. (2018) considers multiple independent behaviors instead of a single behavior, and it is given in Section 3.2. Since it is a straightforward extension to the basic model, that section can also be read as a background for the first, non-epistemic part of Baltag et al. (2018).

2.6 Graph Theory and Networks

At first, Euler's ideas for solving the famous Konigsberg Bridges puzzle and the "graph theory" he developed were used only for solving puzzles and in analyzing games and other recreations (HARRIS; HIRST; MOSSINGHOFF, 2009). In the mid-1800s, however, people began to realize that graphs could be used to model many things, e.g., people and friendships, computers and communication lines, chemicals and reactions

(HARRIS; HIRST; MOSSINGHOFF, 2009; NEWMAN; BARABASI; WATTS, 2006). Field applications continued to grow, and the *graph theory* saw applications in engineering, operations research, computer science, and sociology (NEWMAN; BARABASI; WATTS, 2006).

In mathematics *graph theory* studies graphs and mathematical structures used to model pairwise relations between objects (EASLEY; KLEINBERG, 2010; NEWMAN; BARABASI; WATTS, 2006).

This section provides basic graph theory concepts necessary for understanding the behavior diffusion operations presented in Chapters 3 and 4. We understand that most readers of our work are familiar with these concepts, and they can skip this section proceeding to chapters 3 and 4.

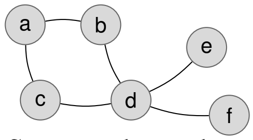
A *network graph*, or simple a *graph*, consists of two finite sets, V and E. Each element of V is called *vertex* (plural $vertices$) or *nodes*, and elements of E, called *edges* or *links*, are unordered pairs of vertices.

Some more specific examples of graphs to represent networks related to our theme of research are: *Communication networks* where nodes are agents, computers, or other devices that can relay messages, and edges represent direct links along which messages can be transmitted; *Social Networks* with nodes related to people, agents, or groups of those, and edges are some social interaction; and *Information networks* where nodes are some kind of information resources such as web pages and documents, and edges are logical connections such as hyperlinks, citations, or cross-references.

The vertex set of a graph G is denoted by $V(G)$, and the edge set is denoted by $E(G)$. The *order* of a graph G is the cardinality of its vertex set, and the *size* of a graph is the cardinality of its edge set (HARRIS; HIRST; MOSSINGHOFF, 2009).

Given two vertices u and v, if $(u, v) \in E$, then u and v are $adjacent$ to each other, if $(u, v) \notin E$, then u and v are $non\ adjacent$ (HARRIS; HIRST; MOSSINGHOFF, 2009). The $degree$ of v refers to the number of edges adjacents to v, the number of direct connections it has to other nodes.

Figure 2.1 – Visual representation of Graph G



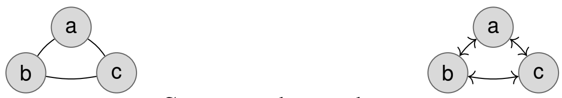
Source: the author.

The visual representation of the following graph G can be seen in Figure 2.1, the set $V = \{a, b, c, d, e, f\}$ and the set $E = \{(a, b), (a, c), (b, d), (c, d), (d, e), (d, f)\}$. Together, V and E are a graph G.

Figure 2.1 and Figure 2.2a are examples of *undirected graph*. The edges indicate a symmetric relationship between their vertices. Although less common, the graph in Figure 2.2b also represents a symmetric relation between edges.

Figure 2.2 – Two examples of graphs with symmetric relation between edges

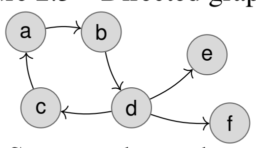
(a) Undirected graph (b) bi-directional directed graph



Source: the author.

Edges can represent an asymmetric relationship, creating a *directed graph* or *digraph*. In this case, the set E is a set of ordered pairs of vertices (a, b) drawn as an arrow departing from a and pointing to b. Figure 2.3 shows a *digraph*. *Digraphs* in the context of social networks allow the effect of directed influence among other cases.

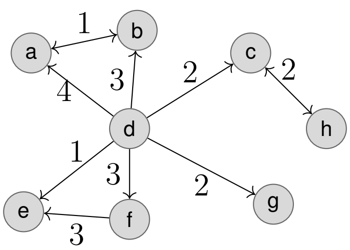
Figure 2.3 – Directed graph G



Source: the author.

Another typical case is related to how strong one agent's influence is on other agents. This relationship can be modeled using *weighted edges* as shown in Figure 2.4.

Figure 2.4 – Visual representation of Graph G with weighted edges

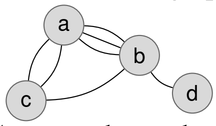


Source: the author.

A *Weighted graph* is a graph G with a *weight function* W that maps the edges of G to, commonly, nonnegative real numbers. Although in some cases, the weight function allows negative numbers, notes, tags, or anything that result in true, false, a number, or even a new tag after being evaluated by the quoted function. Graphically speaking, the edge usually is represented together with a label.

A graph that allows repeated pairs of edges, replacing the set E with a multiset, generates a *multigraph* (Figure 2.5). In the case of social networks, a possible example would be considering different edge types for information, money, friendship, antagonism, etc.

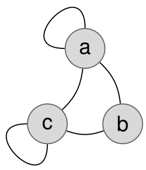
Figure 2.5 – Multigraph G



Source: the author.

When vertex self-connection is available, "loops," the graph is called *pseudograph* as in Figure 2.6.

Figure 2.6 – Pseudograph G



Source: the author.

3 DIFFUSION WITH MULTIPLE BEHAVIORS

Having a single subject of discussion isn't the norm in real societies. With that in mind, this chapter proposes the first variations to the basic model of diffusion presented in (BALTAG et al., 2018). We assume a finite set $\mathcal{B}$ of behaviors $B, B_i, \ldots B_1, \ldots$, and we also assume a function β that maps behaviors to the set of agents that hold them.

In this chapter, we consider different approaches for modeling behavior diffusion. They have in common that they are all formalized in dynamic logic and consider threshold models with multiple behaviors.

Each approach is presented in a separate section. All these sections have the same structure: they first present the definition of model and model update operation and provide an example of model update and a naive algorithm for it. After that, each section provides the syntax, semantics, and (except for the last approach) an axiomatization for a minimal propositional dynamic logic for reasoning about the neighborhood relation and behavior adoption.

3.1 Network of Agents

Throughout this dissertation, a network of agents is represented as a graph where nodes represent agents and edges represent a binary relationship between agents, which receive the general denomination of *neighborhood relation*.

We start with the definition of a *network of agents* following that of (BALTAG et al., 2018):

Definition 3.1.1 (Adapted from (BALTAG et al., 2018))**.** A **Network** is a pair ($\mathcal{A}$,N), where $\mathcal{A}$ is a non-empty finite set of *agents*, and $N : \mathcal{A} \rightarrow \mathcal{P}(\mathcal{A})$ is a function that assigns a set $N(a)$ of direct neighbors to each $a \in \mathcal{A}$, such that:

- $a \notin N(a)$ - irreflexivity
- $b \in N(a)$ iff $a \in N(b)$ - symmetry
- $N(a) \neq \emptyset$ - seriality

Although irreflexivity and seriality of the neighborhood relation will be kept throughout this work, symmetry is a requirement for the approaches of this chapter only, where we assume that all agents have the same capacity to influence other agents.

Symmetry and irreflexivity mean that the network graph is undirected and without self-loops, and with that, we can model relationships between agents such as neighborhood and friendship, for instance. Seriality means that agents have at least one neighbor, as isolated agents do not contribute to behavior diffusion.

3.2 Independent Behaviors

This first multi-behavior approach aims for behaviors that can be adopted by agents independently of any other behavior they might hold or not. We first describe the model and the model update operation, and then we introduce a logic language, its semantics, and its axiomatization.

Model and Model Update

Definition 3.2.1. A **Threshold Model with Independent Behaviors (TMIB)** is a tuple $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta, \theta)$ where ($\mathcal{A}$,N) is a network, $\mathcal{B}_{\mathrm{ID}}$ is a finite set of behaviors, $\beta : \mathcal{B} \to \mathcal{P}(\mathcal{A})$ is a total function mapping each behavior to the set of agents that hold it, and $\theta \in [0, 1]$ is the adoption threshold.

The behavior diffusion is essentially the same as that given in (BALTAG et al., 2018) except that it considers models that can have multiple behaviors. In what follows, the notation $\beta' = \beta[B \mapsto S]$ means that β' is equal to β, except that the behavior B is mapped to the set of agents S.

Definition 3.2.2. The **Threshold Model Update of a TMIB** $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta, \theta)$ by the diffusion of a behavior $B \in \mathrm{Dom}(\beta)$ results in the model $\mathcal{M}' = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta', \theta)$ where $\beta' = \beta[B \mapsto S]$ and S is a set of agents given by:

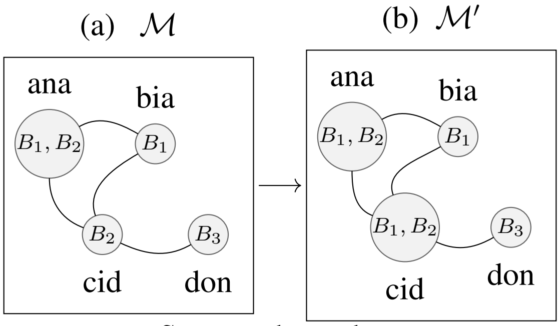
$$S = \beta(B) \cup \left\{ a \in \mathcal{A} : \frac{|N(a) \cap \beta(B)|}{|N(a)|} \geq \theta \right\}$$

We occasionally write $\mathsf{TMIB}(\mathcal{M}, B)$ for the model that results from the update of model $\mathcal{M}$ by the diffusion of B following the model update operation defined above.

Definition 3.2.2 provides quantitative criteria for the adoption of behavior as it expresses that the new set of agents adopting a behavior B are those agents whose fraction of their neighbors that have adopted B has reached or surpassed a given threshold θ.

Example 3.2.1. Figure 3.1 (a) has a graphical representation of a TMIB $\mathcal{M}$ with a set of agents $\mathcal{A} = \{\text{ana}, \text{bia}, \text{cid}, \text{don}\}$, a set of behaviors $\mathcal{B}_{\text{ID}} = \{B_1, B_2, B_3\}$ and a function β s.t. $\beta(B_1) = \{\text{ana}, \text{bia}\}$, $\beta(B_2) = \{\text{ana}, \text{cid}\}$, and $\beta(B_3) = \{\text{don}\}$. Considering a threshold $\theta = 0.5$, Figure 3.1 (b) illustrates the model $\mathcal{M}'$ that results from the update of $\mathcal{M}$ by one step of the diffusion of behavior B_1. Agents *cid* and *don* are the only agents that do not hold behavior B_1, but only *cid* adopts B_1 after the diffusion step from $\mathcal{M}$ to $\mathcal{M}'$ since two of its three neighbors already hold B_1, which is above the threshold θ.

Figure 3.1 – Evolution of model $\mathcal{M}$ after one step in the diffusion of B_1, considering $\theta = 0.5$ and model update of Definition 3.2.2.



Source: the author.

We present an algorithm for the model update of Definition 3.2.2. As explained in Chapter 1, the main goal of the algorithms we give in this dissertation is to provide a straightforward and clear operational view of the different model update operations presented. For that reason, the algorithms are intentionally left naïve.

We split the algorithm into two parts. Algorithm 1 defines a routine called $influence$ that, given as arguments an agent a, a network N, and a behavior B, returns the proportion of direct neighbors of agent a in the network N that hold behavior B. The Algorithm 2 receives as arguments a TMIB model $\mathcal{M}$ and behavior B and returns a new model $\mathcal{M}'$ with the set of agents holding behavior B updated. It initializes a variable $newHolders$ for storing the set of agents that will adopt the behavior (line 1). The algorithm goes through all agents that are not holders of B (lines 2-6), adding them to $newHolders$ (line 4) if the influence of their direct neighbors that hold behavior B is greater than or is equal to a

---

Algorithm 1: TMIB Update - influence.

Input: agent a, network N, behavior B

Output: a number in $[0..1]$

$\text{directN} \leftarrow \text{getNeighborsOf}(a, N)$

$\text{directNHolders} \leftarrow \text{directN} \cap B$

return $|\text{directNHolders}|/|\text{directN}|$

---

threshold θ (line 3). Finally, the algorithm adds newHolders to the set of agents holding behavior B and returns the updated model (lines 7-9).

Algorithm 2: TMIB Update.

Input: $\mathcal{M}$=($\mathcal{A}$,N,$\mathcal{B}_{\text{ID}}$, β,θ) & $B \in \text{Dom}(\beta)$

Output: $\mathcal{M}'$=($\mathcal{A}$,N,$\mathcal{B}_{\text{ID}}$, β',θ)

```
1 newHolders ← {}
2 foreach a ∈ 𝒜 s.t. a ∉ β(B_i) do
3     if influence(a, N, B) ≥ θ then
4         newHolders ← newHolders ∪ {a}
5     end if
6 end foreach
7 β' ← β
8 β'(B) ← β(B) ∪ newHolders
9 return (𝒜, N, ℬ_ID, β', θ)
```

The logic $\mathcal{LIB}_{[]}$

The syntax of the logic language for multiple independent behaviors is slightly different from that of (BALTAG et al., 2018) that considers a single behavior. Besides a finite set with propositional variables N_{ab}, one for each $a, b \in \mathcal{A}$, we also assume a finite set with propositional variables B_a, one for each $B \in \mathcal{B}_{\text{ID}}$ and each $a \in \mathcal{A}$. Also, instead of a single dynamic modality $[\text{adopt}]$ of (BALTAG et al., 2018) we have an *adopt* dynamic modality $[\text{B}]$ for each $B \in \mathcal{B}_{\text{ID}}$.

Syntactically, all the other logics presented in this dissertation have the same syntactic structure. They will differ in their semantics and axiomatizations.

Definition 3.2.3. The set of formulas of $\mathcal{LIB}_{[]}$ is defined by the following grammar:

$$\varphi \ ::= \ \text{N}_{ab} \mid \text{B}_a \mid \neg\varphi \mid \varphi \wedge \psi \mid [\text{B}]\,\varphi$$

Definition 3.2.4. The semantics of $\mathcal{LIB}_{[]}$, w.r.t. a TMIB model $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\text{ID}}, \beta, \theta)$ is given by:

$$
\begin{array}{lll}
\mathcal{M} \models \text{B}_a & \textit{iff} & a \in \beta(B) \\
\mathcal{M} \models \text{N}_{ab} & \textit{iff} & b \in N(a) \\
\mathcal{M} \models \neg\varphi & \textit{iff} & \mathcal{M} \not\models \varphi \\
\mathcal{M} \models \varphi \wedge \psi & \textit{iff} & \mathcal{M} \models \varphi \text{ and } \mathcal{M} \models \psi \\
\mathcal{M} \models [\text{B}]\,\varphi & \textit{iff} & \mathcal{M}' \models \varphi, \text{ where } \mathcal{M}' = \mathsf{TMIB}(\mathcal{M}, B) \text{ (Def. 3.2.2).}
\end{array}
$$

In order to reason about threshold models with independent behaviors, we give a Hilber-style axiomatization for the logic $\mathcal{LIB}_{[]}$.

Definition 3.2.5. The following are the axioms and the rules of inference for $\mathcal{LIB}_{[]}$:

Network axioms	
$\neg \mathrm{N}_{aa}$	Irreflexivity
$\mathrm{N}_{ab} \leftrightarrow \mathrm{N}_{ba}$	Symmetry
$\bigvee_{b \in \mathcal{A}} \mathrm{N}_{ab}$	Seriality
Reduction axioms	
$[\mathrm{B}]\, \neg\varphi \leftrightarrow \neg[\mathrm{B}]\, \varphi$	Red.Ax.$\neg$
$[\mathrm{B}]\, (\varphi \wedge \psi) \leftrightarrow [\mathrm{B}]\, \varphi \;\wedge\; [\mathrm{B}]\, \psi$	Red.Ax.$\wedge$
$[\mathrm{B}]\, \mathrm{N}_{ab} \leftrightarrow \mathrm{N}_{ab}$	Red.Ax.N
$[\mathrm{B}]\, \mathrm{B}'_a \leftrightarrow \mathrm{B}'_a$ B $\neq$ B'	Red.Ax.B1
$[\mathrm{B}]\, \mathrm{B}_a \leftrightarrow \mathrm{B}_a \;\vee\; \mathrm{B}_{\mathrm{N_a} \geq \theta}$	Red.Ax.B2
Inference rules	
If φ and $\varphi \to \psi$, then ψ	Modus Ponens
If φ, then $[\mathrm{B}]\, \varphi$	Necessitation$_{[B]}$
If φ and $\psi \leftrightarrow \chi$, then $\varphi[\psi/\chi]$	Replacement of Equivalents.

The first three axioms capture the properties of the network structure. The last three lines of the table have the inference rules for a minimal propositional modal logic with a box modality.

The *reduction axioms* are obtained following the standard method known as *reduction* developed in the context of Dynamic Epistemic Logic. Reduction axioms characterize the truth of a formula $[\mathrm{B}]\, \varphi$ in terms of the truth of another formula ψ obtained by *pushing* the modality $[\mathrm{B}]$ *inside* φ.

This reduction process proceeds recursively until a formula of propositional logic with no occurrences of modality $[\mathrm{B}]$ is reached, meaning that the truth value of a formula with modality $[\mathrm{B}]$ can be characterized by the truth value of a formula of propositional logic without the modality.

In the first two reduction axioms, Red.Ax.$\neg$ and Red.Ax.$\wedge$, we observe, in a left to right reading, the $[\mathrm{B}]$ modality being pushed inside the formula structure. This process proceeds until a propositional letter/symbol is reached, i.e., until we have either $[\mathrm{B}]\, \mathrm{N}_{ab}$, $[\mathrm{B}]\, \mathrm{B}'_a$ with $B \neq B'$, or $[\mathrm{B}]\, \mathrm{B}_a$.

For the case of $[\mathrm{B}]\ \mathrm{N}_{ab}$, reduction axiom $\mathrm{Red.Ax.N}$ expresses that the existence of a neighborhood relation between agents is not affected by the diffusion of a behavior. For the case of $[\mathrm{B}]\ \mathrm{B}'_a$ with $B \neq B'$, reduction axiom $\mathrm{Red.Ax.B1}$ expresses the independence among behaviors, i.e., the truth status of a formula B'_a is not affected by the diffusion of a different behavior B.

For this logic, the most interesting reduction axiom is for the case $[\mathrm{B}]\ \mathrm{B}_a$. The reduction axiom $\mathrm{Red.Ax.B2}$ expresses that the truth of formula $[\mathrm{B}]\ \mathrm{B}_a$. i.e., the truth of B_a after one step in the diffusion of B is characterized by the following disjunction: either B_a is true (agent a happens to hold behavior B), or, if that is not the case, the conditions for the adoption of behavior B by agent a are present.

The formula that captures the conditions for the adoption by agent a of behavior B was given in (BALTAG et al., 2018). In axiom $\mathrm{Red.Ax.B2}$, we use the notation $B_{N_a \geq \theta}$ as an abbreviation for it.

$$\mathrm{B}_{N_a \geq \theta} \quad \equiv \quad \bigvee_{\mathcal{G} \subseteq \mathcal{N} \subseteq \mathcal{A} : \frac{|\mathcal{G}|}{|\mathcal{N}|} \geq \theta} \left(\bigwedge_{b \in \mathcal{N}} \mathrm{N}_{ab} \wedge \bigwedge_{b \notin \mathcal{N}} \neg \mathrm{N}_{ab} \wedge \bigwedge_{b \in \mathcal{G}} \mathrm{B}_b \right)$$

The formula $B_{N_a \geq \theta}$ is true if there are sets of agents $\mathcal{N} \subseteq \mathcal{A}$ and $\mathcal{G} \subseteq \mathcal{N}$ with $|\mathcal{G}|/|\mathcal{N}| \geq \theta$ for which the following holds: all elements of $\mathcal{N}$ are neighbors of agent a, agents not in $\mathcal{N}$ are not neighbors of agent a, and agents in $\mathcal{G}$ hold behavior B.

3.3 Conflicting Behaviors

Certain behaviors are mutually exclusive. For instance, an agent cannot be at the same time vegan and carnivore. In this section, we consider a logical treatment that prevents agents from adopting behavious that conflict with behaviors they already hold, even if the threshold for adoption has been reached.

Model and Model Update

We start by introducing a new model component that maps a behavior to the set of its conflicting behaviors:

Definition 3.3.1. A function $C : \mathcal{B}_{\mathrm{ID}} \rightarrow \mathcal{P}(\mathcal{B}_{\mathrm{ID}})$ is a **Behavior Conflict** function if, for each $B \in \mathcal{B}_{\mathrm{ID}}$

- $B \notin C(B)$ - irreflexivity
- $B \in C(B_j)$ iff $B_j \in C(B)$ - symmetry

In models with possibly conflicting behaviors, the function β mapping behaviors to the set of agents that hold them, and the behavior conflict functions C should be consistent with each other, i.e. no agents can hold conflicting behaviors at the same time:

Definition 3.3.2. Let $\beta : \mathcal{B}_{\text{ID}} \rightarrow \mathcal{P}(\mathcal{A})$ be a function that maps behaviors to the set of agents that hold them, and let $C : \mathcal{B}_{\text{ID}} \rightarrow \mathcal{P}(\mathcal{B}_{\text{ID}})$ be a behavior conflict function. We say that β **and** C **are consistent with each other** if they are such that:

$$\forall B_i, B_j \in \text{Dom}(\beta).\ \ B_i \in C(B_j) \ \text{ implies that } \ \beta(B_i) \cap \beta(B_j) = \{\ \}$$

With the previous two definitions, we are ready to define threshold models with conflicting behaviors:

Definition 3.3.3. A **Threshold Model with Conflicting Behaviors (TMCB)** is a tuple $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\text{ID}}, \beta, C, \theta)$ where $(\mathcal{A}, N)$ is a network, $\mathcal{B}_{\text{ID}}$ is a finite set of behaviors, $\beta : \mathcal{B}_{\text{ID}} \rightarrow \mathcal{P}(\mathcal{A})$ is a mapping from behaviors to the set of agents that hold then, $C : \mathcal{B}_{\text{ID}} \rightarrow \mathcal{P}(\mathcal{B}_{\text{ID}})$ is a behavior conflict funcion, with β and C consistent with each other, and $\theta \in [0, 1]$ is the threshold.

Besides the satisfaction of the threshold criteria, the adoption of a behavior B by an agent only occurs if the agent does not hold a behavior that conflicts with B. We introduce an auxiliary notation to be used in the definition of model update considering conflicting behaviors. We write $\text{HasConflict}_{\mathcal{M}}(a, B)$ to express that, in model $\mathcal{M}$, the agent a holds a behavior that conflicts with behavior B.

$$\text{HasConflict}_{\mathcal{M}}(a, B) \ \ \equiv \ \ \exists B_i \in \text{Dom}(\beta_{\mathcal{M}})\,.\, B_i \in C_{\mathcal{M}}(B) \ \wedge \ a \in \beta_{\mathcal{M}}(B_i)$$

The subscript $\mathcal{M}$ can be omitted when the model is clear from the context.

Definition 3.3.4. The **Threshold Model Update of a TMCB** $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\text{ID}}, \beta, C, \theta)$, by the diffusion of a behavior $B \in \text{Dom}(\beta)$, produces the model $\mathcal{M}' = (\mathcal{A}, N, \mathcal{B}_{\text{ID}}, \beta', C, \theta)$ where $\beta' = \beta[B \mapsto S]$ and S is a set of agents given by:

$$S = \beta(B) \cup \{a \in \mathcal{A} \ \mid \ \neg\text{HasConflict}(a, B) \ \wedge \ \frac{|N(a) \cap \beta(B)|}{|N(a)|} \geq \theta\}$$

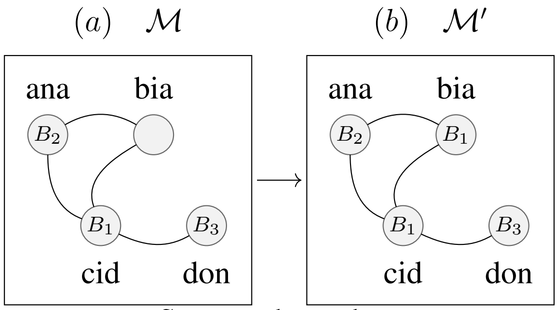
We will occasionally write $\mathsf{TMCB}(\mathcal{M}, B)$ for the model that results from the update of TMCB $\mathcal{M}$ by diffusion of behavior B.

Example 3.3.1. Assuming that the TMCB $\mathcal{M}$ of Figure 3.2 (a) has a threshold $\theta = 0.5$, and a behavior conflict function defined as

$$C(B_1) = \{B_2, B_3\} \qquad C(B_2) = \{B_1\} \qquad C(B_3) = \{B_1\}$$

we have that Figure 3.2 (b) illustrates the model $\mathcal{M}'$ that results from the update of model $\mathcal{M}$ of Figure 3.2 (a) by one step of the diffusion of B_1 according Definition 3.3.4. Observe that half of the direct neighbors of *ana* hold the behavior B_1, so judging only by the threshold criteria *ana* could adopt B_1, but that is not possible since *ana* holds B_2 which conflicts with B_1. A similar reason precludes *don* from adopting B_1.

Figure 3.2 – Evolution of model $\mathcal{M}$ after one step in the diffusion of B_1, considering $\theta = 0.5$ and model update of Def. 3.3.4.



Source: the author.

Algorithms 3 and 4 provide an operational description of the TMCB model update operation. Algorithm 3 describes a boolean function for the additional criteria to be used by the update algorithm 4. It goes through each behavior B_i that conflicts with behavior B (line 1) and stops with true when it finds that the agent holds that conflicting behavior B_i (lines 2-3).

Algorithm 3: TMCB - hasConflict.

Input: agent a, conflict funtion C, behavior mapping β, behavior B
Output: a boolean value

1 **foreach** $B_i \in C(B)$ **do**
2   **if** $a \in \beta(B_i)$ **then**
3   **return** true
4   **end if**
5 **end foreach**
6 **return** false

Algorithm 4 differs from Algorithm 2 only in line 3, where a test for conflict existence is added to the condition of the *if* command.

Algorithm 4: TMCB Update.

Input: $\mathcal{M}$=($\mathcal{A}$,N,$\mathcal{B}_{\mathrm{ID}}$, β,C,θ) & $B \in \mathrm{Dom}(\beta)$
Output: $\mathcal{M}'$=($\mathcal{A}$,N,$\mathcal{B}_{\mathrm{ID}}$, β',C,θ)
1 $\mathrm{newHolders} \leftarrow \{\}$
2 **foreach** $a \in \mathcal{A}$ *s.t.* $a \notin \beta(B)$ **do**
3   **if** $\neg$hasConflict(*a, C, β, B*) && $\mathrm{influence}(a, N, B) \geq \theta$ **then**
4     $\mathrm{newHolders} \leftarrow \mathrm{newHolders} \cup \{a\}$
5   **end if**
6 **end foreach**
7 $\beta' \leftarrow \beta$
8 $\beta'(B) \leftarrow \beta(B) \cup \mathrm{newHolders}$
9 **return** ($\mathcal{A}$,N,$\mathcal{B}_{\mathrm{ID}}$, β',C,θ)

The logic $\mathcal{LCB}_{[]}$

As mentioned before, the syntax of formulas of $\mathcal{LCB}_{[]}$ are exactly the same as in $\mathcal{LIB}_{[]}$ (Definition 3.2.3) so we do not repeat it here again. The definition of semantics differs in the clause for formulas [B] φ:

Definition 3.3.5. The semantics of $\mathcal{LCB}_{[]}$, w.r.t. a TMCB model $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta, C, \theta)$ is given by:

$\mathcal{M} \models \mathrm{B}_a$ *iff* $a \in \beta(B)$
$\mathcal{M} \models \mathrm{N}_{ab}$ *iff* $b \in N(a)$
$\mathcal{M} \models \neg\varphi$ *iff* $\mathcal{M} \not\models \varphi$
$\mathcal{M} \models \varphi \wedge \psi$ *iff* $\mathcal{M} \models \varphi$ and $\mathcal{M} \models \psi$
$\mathcal{M} \models [\mathrm{B}]\, \varphi$ *iff* $\mathcal{M}' \models \varphi$, where $\mathcal{M}' = \mathsf{TMCB}(\mathcal{M}, B)$ (Def. 3.3.4.)

The *Network Axioms* and the *Inference Rules* for $\mathcal{LCB}_{[]}$ are the same as those of $\mathcal{LIB}_{[]}$, given in Definition 3.2.5. The interesting reduction axioms, however, differ:

Definition 3.3.6. The axiomatization of $\mathcal{LCB}_{[]}$ is composed of the *Network Axioms* and the *Inference Rules* of Definition 3.2.5, and of the following *Reduction Axioms*:

Reduction axioms	
$[\mathrm{B}]\ \neg\varphi \ \leftrightarrow\ \neg[\mathrm{B}]\ \varphi$	Red.Ax.¬
$[\mathrm{B}]\ (\varphi \wedge \psi) \ \leftrightarrow\ [\mathrm{B}]\ \varphi \wedge [\mathrm{B}]\ \psi$	Red.Ax.∧
$[\mathrm{B}]\ \mathrm{N}_{ab} \ \leftrightarrow\ \mathrm{N}_{ab}$	Red.Ax.N
$[\mathrm{B}]\ \mathrm{B}'_a \ \leftrightarrow\ \mathrm{B}'_a \quad B \neq B'$	Red.Ax.B1
$[\mathrm{B}]\ \mathrm{B}_a \ \leftrightarrow\ \mathrm{B}_a \vee BC_{N_a \geq \theta}$	Red.Ax.B2

The reduction axiom *Red.Ax.B2* in Definition 3.3.6 uses the notation $\mathrm{BC}_{N_a \geq \theta}$ which is an abbreviation for the following formula:

$$\mathrm{BC}_{N_a \geq \theta} \ \equiv\ \Big(\bigwedge_{B' \,\in\, C(B)} \neg \mathrm{B}'_a \Big) \ \wedge\ \mathrm{B}_{N_a \geq \theta}$$

The subformula between parenthesis expresses that agent a does not hold any behavior that conflicts with B. The subformula $\mathrm{B}_{N_a \geq \theta}$ in the right side of the conjunction is the same used in the axiomatization of $\mathcal{LIB}_{[]}$ (axiom Red.Ax.B2 of Definition 3.2.5), and it expresses that the threshold for the adoption by agent a of behavior B has been reached.

3.4 Conflicting behaviors with Priority for Adoption.

We now consider a variant of the update operation of the previous section. By this variation, an agent is forced to *unadopt* behaviors when they conflict with a behavior that can be adopted by the threshold criteria. This approach models a scenario where, in case of conflicts, the most recent social influence has a higher priority over agents' current behaviors. This approach, and that of Section 3.8, are the only approaches presented in this dissertation where agents can drop behaviors.

Model and Model Update

The model we consider here is exactly the same as that of Definition 3.3.3. To hopefully, obtaining a clear definition of the new model update operation with behavior unadoption, we assign the name T_B to the set of agents that have reached or surpassed the threshold for the adoption of B in a given TMCB model, i.e, the agents have satisfied the quantitative criteria for behavior adoption. This notation will also be used in other parts of this work.

$$\mathrm{T}_B \;\equiv\; \{a \in \mathcal{A} : \frac{|N(a) \cap \beta(B)|}{|N(a)|} \geq \theta\}$$

Definition 3.4.1. The **Threshold Model Update of a TMCB model** $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta, C, \theta)$, that **prioritizes adoption** of a behavior $B \in \mathrm{Dom}(\beta)$ over conflicts, results in the model $\mathcal{M}' = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta', C, \theta)$ where the function β' is defined as

$$\beta'(B_i) = \begin{cases} \beta(B) \,\cup\, \mathrm{T}_B & \textit{if } B_i = B \\ \beta(B_i) -\, \mathrm{T}_B & \textit{if } B_i \in C(B) \\ \beta(B_i) & otherwise \end{cases}$$

We write $\mathsf{TMCBPA}(\mathcal{M}, B)$ for the TMCB model that results from the model update operation defined above with priority for adoption.

The first clause of the definition of β' above expresses that agents that pass the threshold criteria *do* adopt the behavior B. The second clause of the definition of β' above characterizes that agents that adopted B have to drop behaviors that conflict with it.

The operational aspect of Definition 3.4.1 is expressed in Algorithm 5. From lines 1-8, the algorithm is the same as the one for independent behaviors. The difference is in lines 9-11, which perform the *unadoption* by new holders of behavior B, of behaviors that conflict with B.

We do not consider the possible cascade side effects that this *unadoption* can have on other agents that might have adopted a behavior due to the social influence of neighbors who are now abandoning it.

The logic $\mathcal{LCBPA}_{[]}$

The syntax of formulas of $\mathcal{LCBPA}_{[]}$ are exactly the same as in $\mathcal{LIB}_{[]}$ and in $\mathcal{LCB}_{[]}$ (Definition 3.2.3). The definition of semantics differs in the clause for formulas $[\mathrm{B}]\ \varphi$:

Definition 3.4.2. The semantics of $\mathcal{LCBPA}_{[]}$, w.r.t. a TMCB model $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta, C, \theta)$

Algorithm 5: TMCB Update with Unadoption.

Input: $\mathcal{M}$=($\mathcal{A}$, N, $\mathcal{B}_{\text{ID}}$, β, C, θ) & $B \in \text{Dom}(\beta)$
Output: $\mathcal{M}'$=($\mathcal{A}$, N, $\mathcal{B}_{\text{ID}}$, β', C, θ)

newHolders $\leftarrow \{\}$
foreach $a \in \mathcal{A}$ *s.t.* $a \notin \beta(B_i)$ **do**
  if influence$(a, N, B) \geq \theta$ **then**
    newHolders $\leftarrow$ newHolders $\cup \{a\}$
  end if
end foreach
$\beta' \leftarrow \beta$
$\beta'(B) \leftarrow \beta(B) \cup$ newHolders
foreach $B_i \in C(B)$ **do**
  $\beta'(B_i) \leftarrow \beta(B_i) -$ newHolders
end foreach
return $(\mathcal{A}, N, \mathcal{B}_{\text{ID}}, \beta', C, \theta)$

is given by:

$\mathcal{M} \models \text{B}_a$ *iff* $a \in \beta(B)$

$\mathcal{M} \models \text{N}_{ab}$ *iff* $b \in N(a)$

$\mathcal{M} \models \neg\varphi$ *iff* $\mathcal{M} \not\models \varphi$

$\mathcal{M} \models \varphi \wedge \psi$ *iff* $\mathcal{M} \models \varphi$ and $\mathcal{M} \models \psi$

$\mathcal{M} \models [\text{B}]\, \varphi$ *iff* $\mathcal{M}' \models \varphi$, where $\mathcal{M}' = \mathsf{TMCBPA}(\mathcal{M}, B)$ (Def. 3.4.1.)

The *Network Axioms* and the *Inference Rules* for $\mathcal{LCBPA}_{[]}$ are the same as those of $\mathcal{LIB}_{[]}$, given in Definition 3.2.5. Next we give the reduction axioms only:

Definition 3.4.3. The axiomatization of $\mathcal{LCBPA}_{[]}$ is composed of the *Network Axioms* and the *Inference Rules* of Definition 3.2.5, and of the following *Reduction Axioms*:

Reduction axioms		
$[\text{B}]\, \neg\varphi \leftrightarrow \neg[\text{B}]\, \varphi$		Red.Ax.$\neg$
$[\text{B}]\, (\varphi \wedge \psi) \leftrightarrow [\text{B}]\, \varphi \wedge [\text{B}]\, \psi$		Red.Ax.$\wedge$
$[\text{B}]\, \text{N}_{ab} \leftrightarrow \text{N}_{ab}$		Red.Ax.N
$[\text{B}]\, \text{B}'_a \leftrightarrow \text{B}'_a$	B $\neq$ B' and B' $\notin C$(B)	Red.Ax.B1a
$[\text{B}]\, \text{B}'_a \leftrightarrow \text{B}'_a \wedge \neg\, B_{N_a \geq \theta}$	B $\neq$ B' and B' $\in C$(B)	Red.Ax.B1b
$[\text{B}]\, \text{B}_a \leftrightarrow \text{B}_a \vee B_{N_a \geq \theta}$		Red.Ax.B2

Observe that the reduction axiom Red.Ax.B2 is exactly the same as Red.Ax.B2 for the logic $\mathcal{LIB}_{[]}$ seen in Section 3.2 that considers multiple independent behaviors: the truth of a formula $[\mathrm{B}]\ \mathrm{B}_a$ is characterized by the truth of a formula saying that either agent a already holds behavior B, or that the conditions for the adoption of B by agent a are present. i.e., the threshold has been reached (formula $B_{N_a \geq \theta}$).

For a formula $[\mathrm{B}]\ \mathrm{B}'_a$, with $B \neq B'$, we have two axioms: the reduction axiom Red.Ax.B1a, for the case where B' *does not conflict* with B, and reduction axiom Red.Ax.B1b for the case where B' *does conflict* with B.

The interesting one is axiom Red.Ax.B1b that characterizes the truth of $[\mathrm{B}]\ \mathrm{B}'_a$, for $B \neq B'$ and B conflicting with B', by the formula saying that agent a holds behavior B' *and* it is *not the case* that the conditions for the adoption of B by agent a are present.

3.5 Dependent Behaviors

We now consider the case when the adoption of a behavior might be conditioned to the previous adoption of other behaviors.

Model and Model Update

We need to define a way to express the dependency relation between behaviors:

Definition 3.5.1. A function $D : \mathcal{B}_{\mathrm{ID}} \to \mathcal{P}(\mathcal{B}_{\mathrm{ID}})$ is a **behavior Dependency** function if, for each $B \in \mathcal{B}_{\mathrm{ID}}$

- $B \notin D(B)$ - irreflexivity
- if $B_i \in D(B_j)$ then $B_j \notin D(B_i)$ - asymmetry
- $B_i \in D(B_j)$ and $B_k \in D(B_i)$, implies $B_j \in D(B_k)$ - transitivity

The set $D(B)$ is the set of behaviors that are pre-requisites for the adoption of behavior B. Irreflexivity and asymmetry mean that the adoption cannot *deadlock* due to self or mutually dependent behaviors. Another reading for transitivity is: if B_j depends on B_i and B_i depends on B_k, then B_j depends on B_k.

We also must define a notion of consistency between the function β that maps behavious to set of agents and the behavior dependency function D, in the following sense: if an agent is in the set of agents that hold behavior B_j, then it must also be in the set of all behaviors B_i that B_j depends on:

Definition 3.5.2. Let $\beta : \mathcal{B}_{\text{ID}} \to \mathcal{P}(\mathcal{A})$ be a behavior mapping function, and let $D : \mathcal{B}_{\text{ID}} \to \mathcal{P}(\mathcal{B}_{\text{ID}})$ be a behavior dependency function. We say that β **and** D **are consistent with each other** if they are such that:

$$\forall B_i, B_j \in \mathcal{B}_{\text{ID}}, \forall a \in \mathcal{A}.\;\; a \in \beta(B_j) \;\text{ and }\; B_i \in D(B_j) \;\text{ implie that }\; a \in \beta(B_i)$$

With the previous two definitions, we are ready to define threshold models with behaviors depending on other behaviors:

Definition 3.5.3. A **Threshold Model with Dependent Behaviors (TMDB)** is a tuple $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\text{ID}}, \beta, D, \theta)$ where $(\mathcal{A}, N)$ is a network, $\mathcal{B}_{\text{ID}}$ is a finite set of behaviors, $\beta : \mathcal{B}_{\text{ID}} \to \mathcal{P}(\mathcal{A})$ maps each behavior to the set of agents that hold it, $D : \mathcal{B}_{\text{ID}} \to \mathcal{P}(\mathcal{B}_{\text{ID}})$ is a behavior Dependency function with β and D consistent with each other, and $\theta \in [0, 1]$ is the threshold.

For a more compact definition of model update, we write a HoldsReqOf B to express that (in a model that will be clear from context) the agent a holds all the behaviors that are pre-requisites for the adoption of B:

$$a \text{ HoldsReqOf}_{\mathcal{M}}\, B \;\;\equiv\;\; \forall B_j \in D_{\mathcal{M}}(B).\, a \in \beta_{\mathcal{M}}(B_j)$$

Definition 3.5.4. The **Threshold Model Update with Dependent behaviors** of a model $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\text{ID}}, \beta, D, \theta)$, by the diffusion of B, results in $\mathcal{M}' = (\mathcal{A}, N, \mathcal{B}_{\text{ID}}, \beta', D, \theta)$ where $\beta' = \beta[B \mapsto S]$ and S is a set of agents given by:

$$S = \beta(B) \,\cup\, \{a \in \mathcal{A} :\; a \;\textsf{HoldsReqOf}\; B \,\wedge\, \frac{|N(a) \cap \beta(B)|}{|N(a)|} \geq \theta\}$$

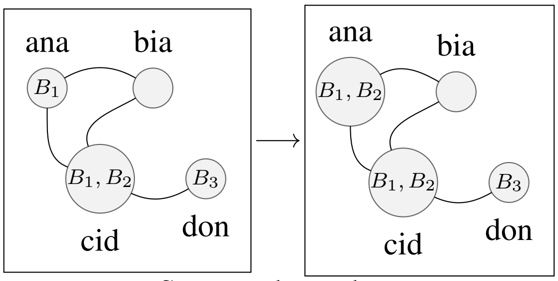
Example 3.5.1. An illustration of the diffusion process considering dependent behaviors can be seen in Figure 3.3. Assuming that the behavior dependency function of model $\mathcal{M}$ of Figure 3.3 (a) is defined as

$$D(B_1) = \{\} \qquad D(B_2) = \{B_1\} \qquad D(B_3) = \{\}$$

and that $\theta = 0.5$, observe that the behavior B_2 is adopted by the agent ana in $\mathcal{M}'$ since ana holds the behavior B_1 in $\mathcal{M}$ of Figure 3.3 (b), and the threshold for the adoption of B_2 has been reached by ana. But agents bia and don cannot adopt B_2 despite the fact that both agents satisfy the threshold criteria.

Figure 3.3 – Evolution of model $\mathcal{M}$ after one step in the diffusion of B_2, considering $\theta = 0.5$ and model update of Def. 3.5.4.

(a) $\mathcal{M}$ (b) $\mathcal{M}'$



Source: the author.

Algorithm 7 provides an operational view of Definition 3.5.4. In line 3, besides checking if the threshold has been reached, it also checks if agents hold all the behaviors that behavior B depends on. Algorithm 6 performs this last checking.

Algorithm 6: TMDB - HoldReqsOf.

Input: agent a, dependency funtion D, behavior mapping β, behavior B
Output: a boolean value

foreach $B_i \in D(B)$ **do**
  if $a \notin \beta(B_i)$ **then**
    return false /* if a does not hold a behavior that B depends on
  end if
end foreach
return true

Algorithm 7: TMDB Update

Input: $\mathcal{M}$=($\mathcal{A}$,N,$\mathcal{B}_{\mathrm{ID}}$, β,D,θ) & $B \in \mathrm{Dom}(\beta)$
Output: $\mathcal{M}'$=($\mathcal{A}$,N,$\mathcal{B}_{\mathrm{ID}}$, β',D,θ)

1 $newHolders \leftarrow \{\}$
2 **foreach** $a \in \mathcal{A}$ *s.t.* $a \notin \beta(B)$ **do**
3   **if** $\mathrm{HoldsReqsOf}(a, D, \beta, B)$ && $\mathrm{influence}(a, N, B) \geq \theta$ **then**
4     $newHolders \leftarrow newHolders \cup \{a\}$
5   **end if**
6 **end foreach**
7 $\beta' \leftarrow \beta$
8 $\beta'(B) \leftarrow \beta'(B) \cup newHolders$
9 **return** ($\mathcal{A}$,N,$\mathcal{B}_{\mathrm{ID}}$, β',D,θ)

The logic $\mathcal{LDB}_{\square}$

The syntax of the logic language $\mathcal{LDB}_{\square}$ for dependent behaviors is the same as that of $\mathcal{LIB}_{[]}$. Its semantics is also analogous except that it is given with respect to the definition 3.5.4.

Definition 3.5.5. The semantics of $\mathcal{LDB}_{[]}$, w.r.t. a TMDB model $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\text{ID}}, \beta, C, \theta)$ is given by:

$\mathcal{M} \models \mathrm{B}_a$	*iff*	$a \in B$
$\mathcal{M} \models \mathrm{N}_{ab}$	*iff*	$b \in N(a)$
$\mathcal{M} \models \neg\varphi$	*iff*	$\mathcal{M} \not\models \varphi$
$\mathcal{M} \models \varphi \wedge \psi$	*iff*	$\mathcal{M} \models \varphi$ and $\mathcal{M} \models \psi$
$\mathcal{M} \models [\mathrm{B}]\, \varphi$	*iff*	$\mathcal{M}' \models \varphi$, where $\mathcal{M}' = \mathsf{TMDB}(\mathcal{M}, B)$ (Def. 3.5.4.)

The *Network Axioms* and the *Inference Rules* for $\mathcal{LDB}_{[]}$ are the same as those of $\mathcal{LIB}_{[]}$, given in Definition 3.2.5. Below we give the reduction axioms:

Definition 3.5.6. The axiomatization of $\mathcal{LDB}_{[]}$ is composed of the *Network Axioms* and the *Inference Rules* of Definition 3.2.5 and of the following *Reduction Axioms*:

Reduction axioms		
$[\mathrm{B}]\, \neg\varphi \;\leftrightarrow\; \neg\, [\mathrm{B}]\, \varphi$		Red.Ax.¬
$[\mathrm{B}]\, (\varphi \wedge \psi) \;\leftrightarrow\; [\mathrm{B}]\, \varphi \wedge [\mathrm{B}]\, \psi$		Red.Ax.∧
$[\mathrm{B}]\, \mathrm{N}_{ab} \;\leftrightarrow\; \mathrm{N}_{ab}$		Red.Ax.N
$[\mathrm{B}]\, \mathrm{B}'_a \;\leftrightarrow\; \mathrm{B}'_a$	B ≠ B'	Red.Ax.B1
$[\mathrm{B}]\, \mathrm{B}_a \;\leftrightarrow\; \mathrm{B}_a \;\vee\; BD_{N_a \geq \theta}$		Red.Ax.B2

The reduction axiom Red.Ax.B2 above uses the notation $\mathrm{BD}_{N_a \geq \theta}$ which is an abbreviation for the following formula:

$$\mathrm{BD}_{N_a \geq \theta} \quad \equiv \quad \Big(\bigwedge_{B' \,\in\, D(B)} \mathrm{B}'_a \Big) \;\wedge\; \mathrm{B}_{N_a \geq \theta}$$

The formula between parenthesis expresses that agent a holds all behaviors that B depends on.

3.6 Dependent Behaviors with Priority for Adoption

This section proposes a different treatment for dependent behaviors than that of Section 3.5. Instead of preventing the adoption of a behavior B by an agent a that does not hold all behaviors B depends on, this section proposes a multi-adoption, by a in the same diffusion step, of not only B but also of all behaviors that are requirements for B.

Model and Model Update

The model we consider here is exactly the same as that of Definition 3.5.3. In the following, we use again the notation T_B for the set of agents that satisfy the quantitative criteria for the adoption of behavior B.

Definition 3.6.1. The **Threshold Model Update of a TMDB model** $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta, D, \theta)$, with priority for the adoption of B, results in $\mathcal{M}' = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta', D, \theta)$ where the function β' is defined as

$$\beta'(B_i) = \begin{cases} \beta(B) \ \cup \ \mathrm{T}_B & \textit{if } B_i = B \\ \beta(B_i) \cup \ \{a \in \mathrm{T}_B : a \notin \beta(B_i)\} & \textit{if } B_i \in D(B) \\ \beta(B_i) & otherwise \end{cases}$$

We write $\mathsf{TMDBPA}(\mathcal{M}, B)$ for the model that results from the update of model $\mathcal{M}$ by the diffusion of behavior B following the update operation defined above that prioritizes adoption.

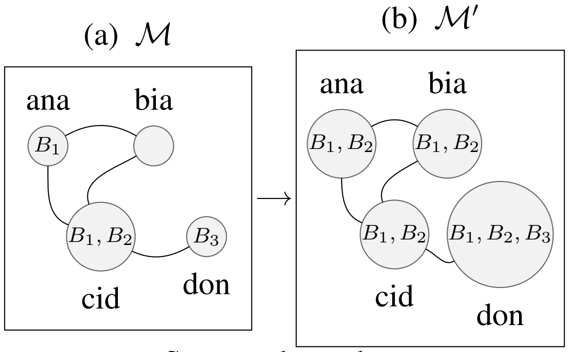
Example 3.6.1. Figure 3.4 illustrates the diffusion process considering dependent behaviors with priority for adoption. Assuming that the behavior dependency function of model $\mathcal{M}$ of Figure 3.4 (a) is defined as

$$D(B_1) = \{\} \qquad D(B_2) = \{B_1\} \qquad D(B_3) = \{B_4\}$$

and that $\theta = 0.5$, observe that the behavior B_2 is adopted by the agent ana in $\mathcal{M}'$ since ana holds the behavior B_1 in $\mathcal{M}$ of Figure 3.3 (b), and the threshold for the adoption of B_2 has been reached by ana. But agents bia and don cannot adopt B_2 despite the fact that both agents satisfy the threshold criteria.

The operational aspect of Definition 3.6.1 is expressed in Algorithm 8. From lines 1-8 the algorithm is the same as the one for independent behaviors. The difference is in lines 9-13 which perfom the *adoption*, by each new holder of behavior B, of behaviors B_i that B depends on.

Figure 3.4 – Evolution of model $\mathcal{M}$ after one step in the diffusion of B_2, considering $\theta = 0.5$ and model update of Definition 3.6.1.



Source: the author.

Algorithm 8: TMDB Update with Multiple Adoption.

Input: $\mathcal{M}$=($\mathcal{A}$, N, $\mathcal{B}_{\mathrm{ID}}$, β, C, θ) & $B \in \mathrm{Dom}(\beta)$
Output: $\mathcal{M}'$=($\mathcal{A}$, N, $\mathcal{B}_{\mathrm{ID}}$, β', C, θ)

newHolders $\leftarrow \{\}$
foreach $a \in \mathcal{A}$ *s.t.* $a \notin \beta(B_i)$ **do**
  if influence$(a, N, B) \geq \theta$ **then**
    newHolders $\leftarrow$ newHolders $\cup \{a\}$
  end if
end foreach
$\beta' \leftarrow \beta$
$\beta'(B) \leftarrow \beta(B) \cup$ newHolders
foreach $B_i \in D(B)$ **do**
  foreach $a \in$ newHolders, $s.t.\ a \notin \beta'(B_i)$ **do**
    $\beta'(B_i) \leftarrow \beta'(B_i) \cup \{a\}$
  end foreach
end foreach
return $(\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta', C, \theta)$

The logic $\mathcal{LDBPA}_{[]}$

The syntax of $\mathcal{LDBMA}_{[]}$ is the same as all the previous logics presented. The semantics is given by:

Definition 3.6.2. The semantics of $\mathcal{LCDBPA}_{[]}$, w.r.t. a TMDB model $\mathcal{M} = (\mathcal{A}, N, \beta, C, D, \theta)$ is given by:

$\mathcal{M} \models \mathrm{B}_a$	*iff*	$a \in B$
$\mathcal{M} \models \mathrm{N}_{ab}$	*iff*	$b \in N(a)$
$\mathcal{M} \models \neg\varphi$	*iff*	$\mathcal{M} \not\models \varphi$
$\mathcal{M} \models \varphi \wedge \psi$	*iff*	$\mathcal{M} \models \varphi$ and $\mathcal{M} \models \psi$
$\mathcal{M} \models [\mathrm{B}]\, \varphi$	*iff*	$\mathcal{M}' \models \varphi$, where $\mathcal{M}' = \mathsf{TMDB\_ma}(\mathcal{M}, B)$ (Def. 3.6.1)

Definition 3.6.3. The axiomatization of $\mathcal{LDBMA}$ is composed of the *Network Axioms* and the *Inference Rules* of Definition 3.2.5 and of the following *Reduction Axioms*:

Reduction axioms		
$[\mathrm{B}]\, \neg\varphi \;\leftrightarrow\; \neg\, [\mathrm{B}]\, \varphi$		Red.Ax.¬
$[\mathrm{B}]\, (\varphi \wedge \psi) \;\leftrightarrow\; [\mathrm{B}]\, \varphi \wedge [\mathrm{B}]\, \psi$		Red.Ax.∧
$[\mathrm{B}]\, \mathrm{N}_{ab} \;\leftrightarrow\; \mathrm{N}_{ab}$		Red.Ax.N
$[\mathrm{B}]\, \mathrm{B}'_a \;\leftrightarrow\; \mathrm{B}'_a$	B $\neq$ B' and B' $\notin D$(B)	Red.Ax.B1a
$[\mathrm{B}]\, \mathrm{B}'_a \;\leftrightarrow\; \mathrm{B}'_a \vee\; B_{N_a \geq \theta}$	B $\neq$ B' and B' $\in D$(B)	Red.Ax.B1b
$[\mathrm{B}]\, \mathrm{B}_a \;\leftrightarrow\; \mathrm{B}_a \vee B_{N_a \geq \theta}$		Red.Ax.B2

3.7 Conflicting and Dependent Behaviors

A set of behaviors can be composed of both conflicting and dependent behaviors. This section deals with this case by combining the approaches of the last two sections.

Model and Model Update

For combining conflicting and dependent behaviors, the Behavior Conflict function of Definition 3.3.1 and the Behavior Dependency function of Definition 3.5.1 must be consistent with each other in the following sense: if two behaviors, say B_i and B_j are conflicting, then no dependency relation should exist between then:

Definition 3.7.1. Let $C : \mathcal{B}_{\mathrm{ID}} \to \mathcal{P}(\mathcal{B}_{\mathrm{ID}})$ and $D : \mathcal{B}_{\mathrm{ID}} \to \mathcal{P}(\mathcal{B}_{\mathrm{ID}})$ be, respectively, a **Behavior Conflict** and a **behavior Dependency** function. We say that C and D are **conflict-dependency consistent** with each other if

$$B_i \in C(B_j) \quad \textit{implies} \quad B_i \notin D(B_j) \wedge B_j \notin D(B_i)$$

Definition 3.7.2. A **Threshold Model with Conflicting and Dependent Behaviors (TM-CDB)** is a tuple $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta, C, D, \theta)$ where $(\mathcal{A}, N)$ is a network, $\mathcal{B}_{\mathrm{ID}}$ is a finite set of behaviors, $\beta : \mathcal{B}_{\mathrm{ID}} \to \mathcal{P}(\mathcal{A})$ maps behaviors to agents holding them, $C : \mathcal{B}_{\mathrm{ID}} \to \mathcal{P}(\mathcal{B}_{\mathrm{ID}})$ and $D : \mathcal{B}_{\mathrm{ID}} \to \mathcal{P}(\mathcal{B}_{\mathrm{ID}})$ are, respectively a **Behavior Conflict** and a **behavior Dependency** function, such that β is consistent with both C and D, and C and D are conflict-dependency consistent, and $\theta \in [0, 1]$ is the threshold.

Definition 3.7.3. The **Threshold Model Update of a TMCDB** $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\text{ID}}, \beta, C, D, \theta)$ by the diffusion of a behavior $B \in \text{Dom}(\beta)$ results in $\mathcal{M}' = (\mathcal{A}, N, \mathcal{B}_{\text{ID}}, \beta', C, D, \theta)$ where $\beta' = \beta[B \mapsto S]$ and S is a set of agents given by

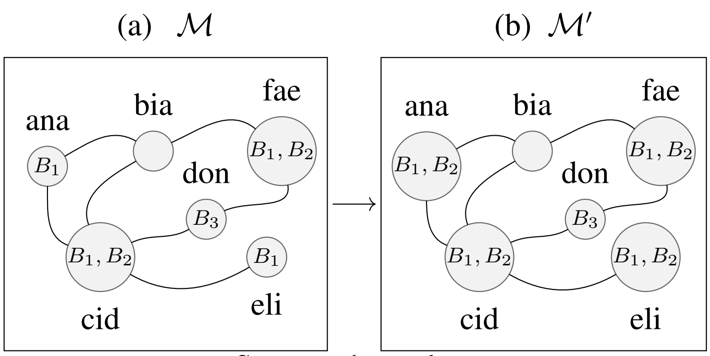
$$S = \beta(B) \cup \{a \in \mathcal{A} \mid a \text{ HoldsReqOf } B \wedge \neg\text{HasConflict}(a, B) \wedge \frac{|N(a) \cap \beta(B)|}{|N(a)|} \geq \theta\}$$

Example 3.7.1. Assuming that the TMCDB of Figure 3.5 (a) has a threshold $\theta = 0.5$ and conflict and dependency functions C and D defined as

$$\begin{array}{llll} C(B_1) = \{B_3, B_4\} & C(B_2) = \{\} & C(B_3) = \{B_1\} & C(B_4) = \{B_1\} \\ D(B_1) = \{\} & D(B_2) = \{B_1\} & D(B_3) = \{\} & D(B_4) = \{B_3\} \end{array}$$

Figure 3.5 (b) illustrates the model $\mathcal{M}'$ that results from $\mathcal{M}$ by one step diffusion of behavior B_2. Observe that the behavior B_2 is adopted by agents ana and eli, since they already have behavior B_1. Agents bia and don, however, cannot adopt B_2 because they do not hold B_1, despite both being able to pass the requirements for threshold θ. In fact, the agent don will never be able to adopt B_2 because the adoption of B_2 depends on B_1 which conflicts with B_3 held by don. In Section 3.8, we present another approach for diffusion combining conflicting with dependent behaviors that gives priority for behavior adoption. When that approach is applied to the model of this example, agents *bia* and *don* will be able to adopt B_2.

Figure 3.5 – Evolution of model $\mathcal{M}$ after one step in the diffusion of B_2, considering $\theta = 0.5$ and model update of Definition 3.7.3.



Source: the author.

Algorithm 9 has an operational view of the update of TMCDB models. For an agent a to be added to the set of new holders of behavior B (line 3), besides satisfying the threshold criteria, it cannot hold any behavior that conflicts with B, and it should hold all behaviors B depends on (line 2). The descriptions for subroutines *hasConflict* and *HoldsReqsOf* are given in Algorithm 3 and Algorithm 6, respectively.

Algorithm 9: TMCDB Update

Input: $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta, C, D, \theta)$ & $B \in \mathrm{Dom}(\beta)$
Output: $\mathcal{M}' = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta', C, D, \theta)$

$newHolders \leftarrow \{\}$
if $\neg$hasConflict(a, C, β, B) && $\mathrm{HoldsReqsOf}(a, D, \beta, B)$ && influence$(a, N, B) \geq \theta$ **then**
  newHolders $\leftarrow$ newHolders $\cup \{a\}$
end if
$\beta' \leftarrow \beta$
$\beta'(B) \leftarrow \beta'(B) \cup$ newHolders
return $(\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta', C, D, \theta)$

The logic $\mathcal{LCDB}_{[]}$

The syntax of the logic language $\mathcal{LCDB}_{[]}$ for conflicting and dependent behaviors is the same as that of $\mathcal{LIB}_{[]}$. Its semantics is also analogous to that of $\mathcal{LIB}_{[]}$ except that it is given with respect to the definitions 3.7.2 and 3.7.3 above.

Definition 3.7.4. The semantics of $\mathcal{LCDB}_{[]}$, w.r.t. a TMCDB model $\mathcal{M} = (\mathcal{A}, N, \beta, C, D, \theta)$ is given by:

$\mathcal{M} \models \mathrm{B}_a$	*iff*	$a \in B$
$\mathcal{M} \models \mathrm{N}_{ab}$	*iff*	$b \in N(a)$
$\mathcal{M} \models \neg\varphi$	*iff*	$\mathcal{M} \not\models \varphi$
$\mathcal{M} \models \varphi \wedge \psi$	*iff*	$\mathcal{M} \models \varphi$ and $\mathcal{M} \models \psi$
$\mathcal{M} \models [\mathrm{B}]\, \varphi$	*iff*	$\mathcal{M}' \models \varphi$, where $\mathcal{M}' = \mathsf{TMCDB}(\mathcal{M}, B)$ (Def. 3.7.3)

The network axioms and the inference rules for $\mathcal{LCDB}_{[]}$ are the same as before.

Definition 3.7.5. The axiomatization of $\mathcal{LCDB}_{[]}$ is composed of the *Network Axioms* and the *Inference Rules* of Definition 3.2.5 and of the following *Reduction Axioms*:

Reduction axioms		
$[\mathrm{B}]\, \neg\varphi \leftrightarrow \neg\, [\mathrm{B}]\, \varphi$		Red.Ax.$\neg$
$[\mathrm{B}]\, (\varphi \wedge \psi) \leftrightarrow [\mathrm{B}]\, \varphi \wedge [\mathrm{B}]\, \psi$		Red.Ax.$\wedge$
$[\mathrm{B}]\, \mathrm{N}_{ab} \leftrightarrow \mathrm{N}_{ab}$		Red.Ax.N
$[\mathrm{B}]\, \mathrm{B}'_a \leftrightarrow \mathrm{B}'_a$	B $\neq$ B’	Red.Ax.B1
$[\mathrm{B}]\, \mathrm{B}_a \leftrightarrow \mathrm{B}_a \vee BCD_{N_a \geq \theta}$		Red.Ax.B2

The reduction axiom Red.Ax.B2 uses the notation $\mathrm{BCD}_{N_a \geq \theta}$ which is an abbreviation for the following formula:

$$\mathrm{BCD}_{N_a \geq \theta} \;\equiv\; \Big(\bigwedge_{B' \in C(B)} \neg \mathrm{B'}_a \;\wedge\; \bigwedge_{B' \in D(B)} \mathrm{B'}_a \,\Big) \;\wedge\; \mathrm{B}_{N_a \geq \theta}$$

The formula between parentheses expresses that agent a doe not hold any behavior conflicting with B and holds all behaviors that B depends on.

3.8 Conflicting and Dependent Behaviors with Priority for Adoption

In Section 3.7, we defined an approach for model update that combines conflicting and dependent behaviors and that follows the following principle: besides satisfying the quantitative threshold, an agent a can only adopt a behavior B if it does not hold a behavior that conflicts with B and if it holds all behaviors that B depends on.

In this section, we present a different version of this combination of conflicting and dependent behaviors that prioritizes adoption of a behavior B by agent a even if that leads a to drop behaviors conflicting with B and to adopt all behaviors that are required for the adoption of B.

Model and model update

The model is defined exactly as in Definition 3.7.2 of Section 3.7. The difference lies in the model update operation. As before, T_B names the set of agents that satisfy the quantitative criteria for adoption of a behavior:

Definition 3.8.1. The **Threshold Model Update of a TMCDB** $\mathcal{M} = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta, C, D, \theta)$ by the diffusion of a behavior $B \in \mathrm{Dom}(\beta)$ and **with priority for adoption** results in a model $\mathcal{M}' = (\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta', C, D, \theta)$ where β' is defined as

$$\beta'(B_i) = \begin{cases} \beta(B) \,\cup\, \mathrm{T} & \text{if } B_i = B \qquad (1) \\ \beta(B_i) \cup \,\{a \in \mathrm{T} : a \notin \beta(B_i)\} & \text{if } B_i \in D(B) \qquad (2) \\ \beta(B_i) - \mathrm{T} & \text{if } B_i \in C(B) \text{ or } B_i \in C(B_j), \text{ with } B_j \in D(B) \qquad (3) \\ \beta(B_i) & \mathit{otherwise} \end{cases}$$

By clause (1) above, all agents that satisfy the quantitative criteria adopt the behavior B. By clause (2), these agents, new holders of B, also adopt all behaviors that B depends on, and, by clause (3), they drop any behavior B_i that conflicts with B and also drop any behavior B_i that conflicts with behaviors that B depends on.

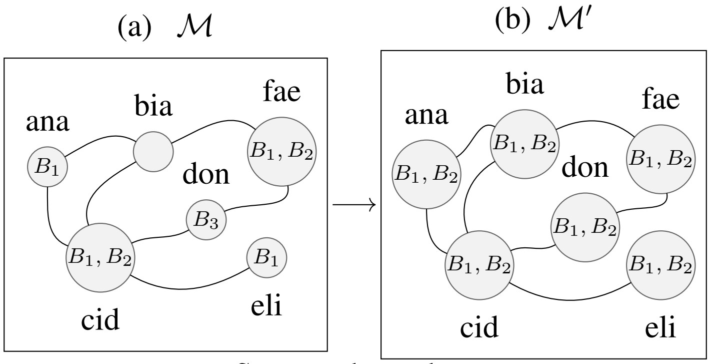
Example 3.8.1. Assuming that the TMCDB of Figure 3.6 (a) has a threshold $\theta = 0.5$, and has conflict and dependency functions C and D defined as

$$\begin{array}{llll} C(B_1) = \{B_3, B_4\} & C(B_2) = \{\} & C(B_3) = \{B_1\} & C(B_4) = \{B_1\} \\ D(B_1) = \{\} & D(B_2) = \{B_1\} & D(B_3) = \{\} & D(B_4) = \{B_3\} \end{array}$$

Figure 3.6 (b) illustrates the model $\mathcal{M}'$ that results from $\mathcal{M}$ of Figure 3.6 (a) by one step diffusion of behavior B_2. Observe that the behavior B_2 is adopted by agents ana and eli, and they don't have to adopt or drop any other behavior for that since they already have the behavior B_1 that B_2 depends on.

The agent *bia* satisfies the quantitative criteria for adoption of B_2, but *bia* needs to adopt B_1 too. Agent *don* also satisfies the quantitative criteria for adopting B_2 and since this adoption has priority for him, he must adopt B_1. behavior B_1, however, conflicts with B_3 he holds, so *don* drops B_3.

Figure 3.6 – Evolution of model $\mathcal{M}$ after one step in the diffusion of B_2, considering $\theta = 0.5$ and model update of Definition 3.8.1.



Source: the author.

Algorithm 10 has an operational view of the update of TMCDB models with priority for adoption. From lines 2 to 7 the algorithm obtains the set *thlds* of agents whose proportion of neighbors holding behavior B has reached the threshold. These agents adopt the behavior B (lines 8-9). From line 11 to 15 these agents also adopt all behaviors B_i that B depends on. Lines 17 to 20 collect the behaviors that have to be dropped: the behaviors that conflict with B (line 17), and all the behaviors that conflict with behaviors

B_j that B depends on (lines 18 to 20). From lines 22 to 24 the algorithm removes agents *thlds* from the behaviors these agents must drop.

Algorithm 10: TMDB Update with Multiple Adoption.

Input: $\mathcal{M}$=($\mathcal{A}$, N, $\mathcal{B}_{\mathrm{ID}}$, β, C, θ) & $B \in \mathrm{Dom}(\beta)$
Output: $\mathcal{M}'$=($\mathcal{A}$, N, $\mathcal{B}_{\mathrm{ID}}$, β', C, θ)

1
2 $\mathrm{thlds} \leftarrow \{\}$
3 **foreach** $a \in \mathcal{A}$ *s.t.* $a \notin \beta(B_i)$ **do**
4   **if** $\mathrm{influence}(a, N, B) \geq \theta$ **then**
5         $\mathrm{thlds} \leftarrow \mathrm{thlds} \cup \{a\}$
6   **end if**
7 **end foreach**
8 $\beta' \leftarrow \beta$
9 $\beta'(B) \leftarrow \beta'(B) \cup \mathrm{thlds}$
10
11 **foreach** $B_i \in D(B)$ **do**
12   **foreach** $a \in \mathrm{thlds},\ s.t.\ a \notin \beta'(B_i)$ **do**
13         $\beta'(B_i) \leftarrow \beta'(B_i) \cup \{a\}$
14   **end foreach**
15 **end foreach**
16
17 $\mathrm{toDrop} \leftarrow C(B)$
18 **foreach** $B_j \in D(B)$ **do**
19     $\mathrm{toDrop} \leftarrow \mathrm{toDrop} \cup C(B_j)$
20 **end foreach**
21
22 **foreach** $B_i \in \mathrm{toDrop}$ **do**
23     $\beta'(B_i) \leftarrow \beta'(B_i) - \mathrm{thlds}$
24 **end foreach**
25 **return** $(\mathcal{A}, N, \mathcal{B}_{\mathrm{ID}}, \beta', C, \theta)$

The logic $\mathcal{LCDBPA}_{\Box}$

The syntax of the logic language $\mathcal{LCDBPA}_{\Box}$ for conflicting and dependent behaviors with priority for adoption is the same as that of $\mathcal{LIB}_{\Box}$. Its semantics is also analogous to that of $\mathcal{LIB}_{\Box}$ except that it is given with respect to the definitions 3.7.2 and 3.8.1 above.

Definition 3.8.2. The semantics of $\mathcal{LCDBPA}_{\Box}$, w.r.t. a TMCDB model $\mathcal{M} = (\mathcal{A}, N, \beta, C, D, \theta)$

is given by:

$$\begin{array}{lll}
\mathcal{M} \models \mathrm{B}_a & \textit{iff} & a \in B \\
\mathcal{M} \models \mathrm{N}_{ab} & \textit{iff} & b \in N(a) \\
\mathcal{M} \models \neg\varphi & \textit{iff} & \mathcal{M} \not\models \varphi \\
\mathcal{M} \models \varphi \land \psi & \textit{iff} & \mathcal{M} \models \varphi \text{ and } \mathcal{M} \models \psi \\
\mathcal{M} \models [\mathrm{B}]\, \varphi & \textit{iff} & \mathcal{M}' \models \varphi, \text{ where } \mathcal{M}' = \mathsf{TMCDBPA}(\mathcal{M}, B) \text{ (Def. 3.8.1)}
\end{array}$$

4 DIFFUSION WITH SINGLE BEHAVIOR AND DIFFERENT NETWORKS

This chapter proposes another kind of variation to the basic behavior diffusion model based on different notions of networks. Each notion leads to different ways to check if a given threshold has been reached. In order to focus on the network aspects, we assume models with a single behavior only. With a single behavior, the syntax of the corresponding logic languages are slightly changed: they all have a single dynamic modality $[\mathrm{adopt}]$ instead of one modality $[\mathrm{B}]$ for each behavior B.

We present two approaches. In Section 4.1, we keep the undirected (symmetric) neighborhood relation between agents but add weights to each connection between agents. In Section 4.2, we present the only approach in this work that considers directed connections between agents.

4.1 Weighted and undirected connections between agents

In more complex multiagent systems, the influence received by different sources is far from equal. Different neighbors may cause a different level of social influence in one agent. This level of influence, when it is reciprocal, is modeled in the network as an undirected weighted edge connecting two agents. We observe that all the approaches presented in the previous chapter assumed undirected edges connecting agents. In this section, we keep undirectdness, but we add weights to each edge.

Models and model uddate

We start by defining weighted undirected networks, where the weight refers to the reciprocal influence between the pair of agents connected by that edge.

Definition 4.1.1. A **Weighted Undirected Network** is a tuple $(\mathcal{A}, N, W)$, where $\mathcal{A}$ and N form a network and $W : \mathcal{A} \times \mathcal{A} \to \mathbb{N}$ is a function that assigns a weight to each edge connecting a pair of agents in the network.

Observe that the neighborhood relation between agents keeps the same properties defined in the previous chapter, i.e., it is irreflexive, symmetric, and serial.

Definition 4.1.2. A **Threshold Model with Weighted Undirected Network (TMWUN)** is a tuple $\mathcal{M} = (\mathcal{A}, N, W, B, \theta)$ where $\mathcal{A}$, N, and W form a weighted undirected network,

$B \subseteq \mathcal{A}$ is a *behavior*, and $\theta \in [0..1]$ is a uniform *adoption threshold*.

The model update is not anymore based on the proportion of neighbors of an agent that hold a behavior, but instead, it takes into account the weight of the social influence of each neighbor:

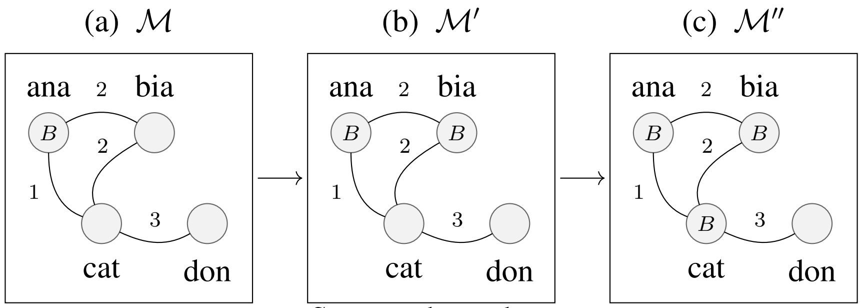
Definition 4.1.3. The **Threshold Model Update of a TMWUN** $\mathcal{M} = (\mathcal{A}, N, W, B, \theta)$ results in $\mathcal{M}' = (\mathcal{A}, N, W, B', \theta)$ where

$$B' = B \cup \left\{ a \in \mathcal{A} : \frac{\sum_{b \in N(a) \cap B} W(a,b)}{\sum_{b \in N(a)} W(a,b)} \geq \theta \right\}$$

We write $\mathsf{TMWUN}(\mathcal{M})$ for the model that results from one step update of $\mathcal{M}$ following the update operation defined above.

Example 4.1.1. An illustration of the diffusion process considering the update of Definition 4.1.3 can be seen in Figure 4.1. Considering, for instance, a threshold $\theta = 0.5$, observe that after the update of $\mathcal{M}$ only agent *bia* adopts behavior B (Fig. 4.1 (b)) since the sum of weights of *bia*'s connections with her neighbors holding B divided by the total weight of all connections with her neighbors is equal to the threshold. Figure 4.1 (c) shows model $\mathcal{M}''$ which results from model $\mathcal{M}'$ after one step diffusion of B. We can see that agent *cat* can only adopt B after the adoption by *bia*.

Figure 4.1 – Evolution of model $\mathcal{M}$ after one step in the diffusion, considering $\theta = 0.5$ and model update of Def. 4.1.3



Source: the author.

The Algorithm 11 is straightforward: it goes through the network, adding the weights of all neighbors of each agent (line 3) and adding the weight of all neighbors that hold the behavior (line 4). The agent is added to the set of new holders (line 6) if the fraction of these two values is greater than or equal to the threshold (line 5).

Algorithm 11: TMWUN update

Input: $\mathcal{M}$=($\mathcal{A}$,N, W,B,θ)
Output: $\mathcal{M}'$=($\mathcal{A}$,N, W, B',θ)
$newHolders \leftarrow \{ \}$
foreach $a \in \mathcal{A}$ *s.t.* $a \notin B$ **do**
    $totalNWeight \leftarrow addNeighborsWeight(a, N, W)$
    $holdersWeight \leftarrow addNeighborsHoldersWeight(a, N, B)$
    if $holdersWeight$ / $totalNWeight$ $\geq \theta$ **then**
        $newHolders \leftarrow newHolders \cup \{a\}$
    end if
end foreach
$B' \leftarrow B \cup newHolders$
return ($\mathcal{A}$,N, W, B',θ)

The logic $\mathcal{LWUN}_{[]}$

The logic language $\mathcal{LWUN}_{[]}$ for threshold models with a weighted undirected network is syntactically the same as the previous languages, except that, as we now have a single behaviour, we have a single modality $[adopt]$. The semantics of the language is also similar, except that it is given w.r.t. definitions 4.1.2 and 4.1.3.

Definition 4.1.4. The semantics of $\mathcal{LWUN}_{[]}$, w.r.t. a TMWUN $\mathcal{M} = (\mathcal{A}, N, W, B, \theta)$ is given by:

$\mathcal{M} \models \mathrm{B}_a$	*iff*	$a \in B$
$\mathcal{M} \models \mathrm{N}_{ab}$	*iff*	$b \in N(a)$
$\mathcal{M} \models \neg\varphi$	*iff*	$\mathcal{M} \not\models \varphi$
$\mathcal{M} \models \varphi \wedge \psi$	*iff*	$\mathcal{M} \models \varphi$ and $\mathcal{M} \models \psi$
$\mathcal{M} \models [\mathrm{adopt}]\,\varphi$	*iff*	$\mathcal{M}' \models \varphi$, where $\mathcal{M}' = \mathsf{TMWUN}(\mathcal{M})$ (Def. 4.1.3)

Definition 4.1.5. The axiomatization of $\mathcal{LWUN}_{[]}$ is composed of the *Network Axioms* and the *Inference Rules* of Definition 3.2.5 and of the following *Reduction Axioms*:

Reduction axioms	
$[\mathrm{adopt}]\,\neg\varphi \;\leftrightarrow\; \neg\,[\mathrm{adopt}]\,\varphi$	Red.Ax.¬
$[\mathrm{adopt}]\,(\varphi \wedge \psi) \;\leftrightarrow\; [\mathrm{adopt}]\,\varphi \wedge [\mathrm{adopt}]\,\psi$	Red.Ax.∧
$[\mathrm{adopt}]\,\mathrm{N}_{ab} \;\leftrightarrow\; \mathrm{N}_{ab}$	Red.Ax.N
$[\mathrm{adopt}]\,\mathrm{B}_a \;\leftrightarrow\; \mathrm{B}_a \;\vee\; BWUN_{N_a \geq \theta}$	Red.Ax.B

The interesting axiom is Red.Ax.B which uses the notation $BWN_{N_a \geq \theta}$ as an abbreviation for the following formula:

$$\mathrm{BWN}_{N_a \geq \theta} \equiv \bigvee_{\mathcal{G} \subseteq \mathcal{N} \subseteq \mathcal{A}\colon \mathcal{W} \geq \theta} \Big(\bigwedge_{b \in \mathcal{N}} \mathrm{N}_{ab} \wedge \bigwedge_{b \notin \mathcal{N}} \neg \mathrm{N}_{ab} \wedge \bigwedge_{b \in \mathcal{G}} \mathrm{B}_b \Big)$$

where

$$\mathcal{W} \equiv \frac{\sum_{b \in \mathcal{G}} \mathrm{W}(a,b)}{\sum_{b \in \mathcal{N}} \mathrm{W}(a,b)}$$

The formula $\mathrm{BWN}_{N_a \geq \theta}$ is true if there are sets of agents $\mathcal{N} \subseteq \mathcal{A}$ and $\mathcal{G} \subseteq \mathcal{N}$ with $\mathcal{W} \geq \theta$ for which the following holds: all elements of $\mathcal{N}$ are neighbors of agent a, agents not in $\mathcal{N}$ are not neighbors of agent a and all agents in $\mathcal{G}$ hold the behaviour B.

4.2 Directed (and unweighted) connections between agents

We now consider networks where agents are connected with directed edges. Hence, the neighborhood relation is non-symmetric. The word *neighborhood*, however, suggests a symmetric relation (a is neighbor of b iff b is neighbor of a, or, as formalized in the previous chapter, N_{ab} iff N_{ba}). Because of non-symmetry, we now might have that N_{ab}, but not necessarily N_{ba}. We will keep the letter N for this relationship but we will call it *influence* relation. If N_{ab} we say that agent a **is influenced by** agent b and, graphically, that will be represented as a directed arrow pointing to a and departing from b[1]

Differently from the other sections of this dissertation, we start with an example of behaviour diffusion of a model with directed edges.

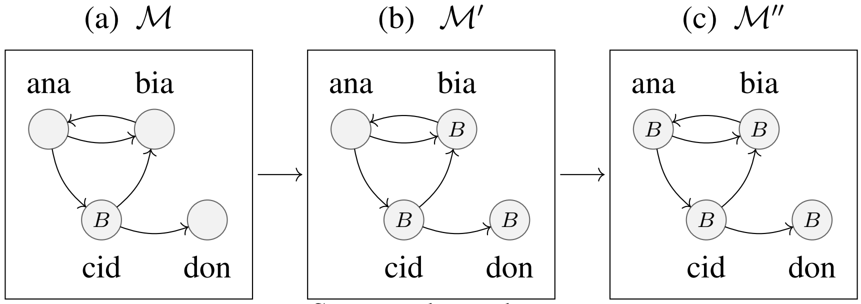
Example 4.2.1. The Figure 4.2 shows an illustration of the following model $\mathcal{M}$ with a non-symmetrical relation N:

$$\mathcal{M} = \begin{cases} \mathcal{A} = \{ana, bia, cid, don\}, \\ B = \{cid\}, \\ N = \{(bia, ana), (bia, cid), (don, cid), (cid, ana), (ana, bia))\}, \\ \theta = 0.5 \end{cases}$$

We can read N above as: *bia is influenced by ana and cid*, *don is influenced by cid*,

[1]Usually in a directed graph the direction of the arrow departs from a, the first component in the pair, and has b, the second component of the pair, as its destination. We opted for doing the other way around for reusing, in this section, the notation $N(a)$ in the update operation with a different interpretation.

Figure 4.2 – Evolution of model $\mathcal{M}$ in two steps of diffusion of behaviour B.



Source: the author.

cid is influenced by ana, and *ana is influenced by bia*. Although half of *ana*'s connections hold the behaviour B (the threshold is 0.5) in $\mathcal{M}$, none of the connections that influence her hold B, so *ana* does not adopt B in $\mathcal{M}$'. The fraction of agents that influence *bia* and *don* and that hold B in $\mathcal{M}$ has reached the threshold, so these two agents adopt B in $\mathcal{M}$'. After *bia*'s adoption of B, *ana* will finally be influenced by her and also adopt B in $\mathcal{M}$".

Model and model update

The influence relationship is irreflexive since an agent cannot be influenced by itself. Moreover, it is serial, meaning that any agent influences other agents or is influenced by other agents.

Definition 4.2.1. A **Network of Influence** is a pair ($\mathcal{A}$,N), where $\mathcal{A}$ is a non-empty finite set of *agents*, and $N : \mathcal{A} \to \mathcal{P}(\mathcal{A})$ is a function that assigns, to each $a \in \mathcal{A}$, a set $N(a)$ of agents that influence a such that:

- $a \notin N(a)$ - irreflexivity
- for all a, there is b, s.t. $b \in N(a)$ or $a \in N(b)$

Definition 4.2.2. A **Threshold Model with Directed Network (TMDN)** is a tuple $\mathcal{M} = (\mathcal{A}, N, B, \theta)$ where $\mathcal{A}$ and N form a network of influence, $B \subseteq \mathcal{A}$ is a *behavior*, and $\theta \in [0..1]$ is a uniform *adoption threshold*.

Definition 4.2.3. The **Threshold Model Update of a TMDN** $\mathcal{M} = (\mathcal{A}, N, B, \theta)$ results in $\mathcal{M}' = (\mathcal{A}, N, B', \theta)$ where

$$B' = B \cup \left\{ a \in \mathcal{A} : \frac{|N(a) \cap B|}{|N(a)|} \geq \theta \right\}$$

The algorithm for this update operation is essentially the same as that of Section 3.2 for independent behaviours

Algorithm 12: TMDN - fracOfInfluencersHolders

Input: agent a, N, behaviour B
Output: a number in $[0..1]$
$\text{influencers} \leftarrow \text{getInfluencersOf}(a, N)$
$\text{influencersHolders} \leftarrow \text{influencers} \cap B$
return $|\text{influenceHolders}|/|\text{influencers}|$

Algorithm 13: TMDN Update.

Input: $\mathcal{M}$=($\mathcal{A}$,N,B,θ)
Output: $\mathcal{M}'$=($\mathcal{A}$,N,B',θ)
$\text{newHolders} \leftarrow \{\}$
foreach $a \in \mathcal{A}$ *s.t.* $a \notin \beta(B_i)$ **do**
  if $\text{fracOfInfluencersHolders}(a, N, B) \ \geq \ \theta$ **then**
    $\text{newHolders} \leftarrow \text{newHolders} \cup \{a\}$
  end if
end foreach
$B' \leftarrow B \ \cup \ \text{newHolders}$
return $(\mathcal{A}, N, B', \theta)$

Logic $\mathcal{LDN}_{[]}$

The syntax of $\mathcal{LDN}_{[]}$ is the same as the previous section The semantics is given w.r.t definitions 4.2.2 and 4.2.3.

Definition 4.2.4. The semantics of $\mathcal{LDN}_{[]}$, w.r.t. a TMDN model $\mathcal{M} = (\mathcal{A}, N, B, \theta)$ is given by:

$$
\begin{array}{lll}
\mathcal{M} \models \mathrm{B}_a & \textit{iff} & a \in \beta(B) \\
\mathcal{M} \models \mathrm{N}_{ab} & \textit{iff} & b \in N(a) \\
\mathcal{M} \models \neg\varphi & \textit{iff} & \mathcal{M} \not\models \varphi \\
\mathcal{M} \models \varphi \wedge \psi & \textit{iff} & \mathcal{M} \models \varphi \text{ and } \mathcal{M} \models \psi \\
\mathcal{M} \models [\text{adopt}]\, \varphi & \textit{iff} & \mathcal{M}' \models \varphi, \text{where } \mathcal{M}' = \mathsf{TMDN}(\mathcal{M}) \text{ (Def. 4.2.3).}
\end{array}
$$

As for the axiomatization: the inference rules remain the same, symmetry is removed from the network axioms, and seriality is reformulated.

Definition 4.2.5. The following are the axioms and the rules of inference for $\mathcal{LDN}_{[]}$:

Network axioms	
$\neg \mathrm{N}_{aa}$	Irreflexivity
$\bigvee_{b \in \mathcal{A}} (\mathrm{N}_{ab} \vee \mathrm{N}_{ba})$	Seriality
Reduction axioms	
$[\text{adopt}]\ \neg\varphi \ \leftrightarrow\ \neg[\text{adopt}]\ \varphi$	Red.Ax.$\neg$
$[\text{adopt}]\ (\varphi \wedge \psi) \ \leftrightarrow\ [\text{adopt}]\ \varphi \ \wedge\ [\text{adopt}]\ \psi$	Red.Ax.$\wedge$
$[\text{adopt}]\ \mathrm{N}_{ab} \ \leftrightarrow\ \mathrm{N}_{ab}$	Red.Ax.N
$[\text{adopt}]\ \mathrm{B}_a \ \leftrightarrow\ \mathrm{B}_a \ \vee\ \mathrm{B}_{\mathrm{N_a} \geq \theta}$	Red.Ax.B

The formula that captures the conditions for the adoption by agent a of behaviour B in axiom $\mathrm{Red.Ax.B}$ is exactly the same as that used in the axiomatizations of $\mathcal{LIB}_{[]}$ in Definition 3.2.5. The semantics of N_{ab} in the formula $\mathrm{B}_{N_a \geq \theta}$, however, is different.

5 CONCLUSION

In this dissertation, we developed behavior diffusion models and policies. We define a model update operation for each model, a logic with its semantics, and axiomatization.

We separate the different model policies into two chapters: Chapter 3 deals with seven different model policies with multiple behaviors that can be independent, conflicting, or dependent on other behaviors. Chapter 4 deals with model policies with only one behavior and two versions for the network of agents: weighted undirected and directed edges. We believe that these different policies make it possible to experiment with models closer to the real world than those previously proposed by other works.

We aimed to work with models, one at a time, to facilitate understanding of their main features. However, combining more than one model in a single model is possible. Experiments of this were illustrated with the Multiple Conflicting Dependent Behaviors and Multiple Conflicting Dependent Behaviors with Priority for Adoption given in Chapter 3. In what follows, we mention future work we believe is worth pursuing:

Proofs of Soundness and Completeness and **Reduction Axioms for** $\mathcal{LCDBPA}_{\square}$. Soundness and Completeness proofs are important for establishing a proper relationship between the syntactic and semantic aspects of a logical system. Together they imply that all and only validities (truths) are provable. As we mentioned at the beginning of Chapter 3, the reduction axioms for $\mathcal{LCDBPA}_{\square}$ were not defined. These two tasks have a higher priority on our list of future work.

Epistemic Operators. Epistemic logic uses the notion of possible worlds, made well-known by Saul Kripke, where possible worlds are interpreted as epistemic alternatives. In this approach, each possible world represents a complete description of a given reality, and the set of all possible worlds represents all the ways the world could be. Agents have knowledge or beliefs about the world. Epistemic models also have accessibility relationships between the worlds for each agent (DITMARSCH; HOEK; KOOI, 2007; RENDSVIG; SYMONS, 2021; HINTIKKA, 1962). In all the approaches proposed in this dissertation, behavior diffusion occurs in a way that the environment imposes on the agents the adoption of a behavior.

With epistemic threshold models for behavior diffusion, agents adopt a behavior based on what they *know* about the environment. More specifically, the adoption only happens when it is true, in all possible worlds accessible to the agent, that a given threshold has been reached.

We believe that the combination of the extensions we defined in this dissertation and epistemic models is worth pursuing to obtain a better understanding of realistic scenarios. The work of Christoff (2016), Baltag et al. (2018) presents a basic model with epistemic operators for diffusion and can be used as the basis of new expansions.

Logical framework. The works of EIJCK (2008), Gattinger (2018b), Gattinger (2018a), Hansen (2011), Said (2010), Schwarzentruber (2019) are examples of logical frameworks for different logics, including modal logic, dynamic epistemic logic, hybrid logic, and others. Logical frameworks are a powerful tool for learning, illustrating model and world changes, and allowing experimentation with different concrete models. Our first intention for this dissertation was the implementation of a logical framework, but due to technical difficulties, the focus of the work changed. We do have a preliminary implementation for the basic model with multiple agents and we plan to resume activities with that project. More information about the project and for access to the source code of the preliminary version of the tool can be found at gsamorim.github.io/SocialDiffusionProj/

Topology analysis and network modification. The book of Watts (2004) classifies network topology into three different types: Regular, Small-World, and Random Networks. Analyzing the effect of different operators applied to each type of topology, given by Watts (2004) and others, would allow a better understanding of network dynamics that could be used to understand better and prevent the spread of diseases, fake news, and hate speech. The work of Newman, Barabasi and Watts (2006) recognizes that networks evolve as the product of dynamical processes that add or remove vertices or edges. For example, a social network of friendships changes as individuals make and break ties with others. Behavior diffusion considering network modifications also deserves further investigation in conjunction with the different diffusion models we presented.

REFERENCES

BALTAG, A. et al. Dynamic epistemic logics of diffusion and prediction in social networks. **Studia Logica**, v. 107, 07 2018.

CHRISTAKIS, N.; FOWLER, J. **Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives**. Little, Brown, 2009. ISBN 9780316071345. Available from Internet: <https://books.google.com.br/books?id=LXHi4wgIkzEC>.

CHRISTOFF, Z. **Dynamic Logics of Networks: Information Flow and the Spread of Opinion**. Thesis (PhD) — University of Amsterdam, 2016.

CHRISTOFF, Z.; HANSEN, J. U. A logic for diffusion in social networks. **Journal of Applied Logic**, v. 13, n. 1, p. 48–77, 2015. ISSN 1570-8683.

DITMARSCH, H. v.; HOEK, W. van der; KOOI, B. **Dynamic Epistemic Logic**. 1st. ed. [S.l.]: Springer Publishing Company, Incorporated, 2007. ISBN 1402058381.

DOOLEY, K. E. Towards a holistic model for the diffusion of educationaltechnologies: An integrative review of educational innovationstudies. **J. Educ. Technol. Soc.**, v. 2, 1999.

EASLEY, D.; KLEINBERG, J. **Networks, Crowds, and Markets: Reasoning About a Highly Connected World**. USA: Cambridge University Press, 2010. ISBN 0521195330.

EIJCK, J. V. Demo — a demo of epistemic modelling. 01 2008.

FLOYD, R. W. Assigning meanings to programs. In: **Proc. Sympos. Appl. Math., Vol. XIX**. [S.l.]: ,, 1967. p. 19–32.

FREEMAN, L. The development of social network analysis. 2004.

GATTINGER, M. **New Directions in Model Checking Dynamic Epistemic Logic**. Thesis (PhD) — University of Amsterdam, 2018. Available from Internet: <https://malv.in/phdthesis>.

GATTINGER, M. Smcdel–an implementation of symbolic model checking for dynamic epistemic logic with binary decision diagrams. **Version**, v. 1, n. 0, p. 7–92, 2018.

GOLDENBERG, J.; LIBAI, B.; MULLER, E. Talk of the network: A complex systems look at the underlying process of word-of-mouth. **Marketing Letters**, v. 12, p. 211–223, 08 2001.

GONZALEZ, S. F. Change in social networks: Some dynamic extensions of Social Epistemic Logic. **Journal of Logic and Computation**, v. 32, n. 6, p. 1212–1233, 03 2022. ISSN 0955-792X.

GRANOVETTER, M. Threshold models of collective behavior. **American Journal of Sociology**, v. 83, n. 6, p. 1420–1443, 1978.

GRANOVETTER, M. S. The Strength of Weak Ties. **American Journal of Sociology**, v. 78, n. 6, p. 1360–1380, 1973.

HANSEN, J. U. A logic toolbox for modeling knowledge and information in multi-agent systems and social epistemology. **Computer Science Research Report**, n. 133, p. 1–236, 2011.

HANSEN, J. U. Reasoning about opinion dynamics in social networks. **Journal of Logic and Computation**, v. 29, n. 7, p. 1121–1137, 12 2015. ISSN 0955-792X.

HARRIS, J.; HIRST, J.; MOSSINGHOFF, M. **Combinatorics and Graph Theory**. Springer New York, 2009. (Undergraduate Texts in Mathematics). ISBN 9780387797113. Available from Internet: <https://books.google.com.br/books?id=DfcQaZKUVLwC>.

HINTIKKA, J. Knowledge and belief: An introduction to the logic of the two notions. **Studia Logica**, Springer, v. 16, p. 119–122, 1962.

HOARE, C. A. R. An axiomatic basis for computer programming. **Commun. ACM**, Association for Computing Machinery, New York, NY, USA, v. 12, n. 10, p. 576–580, oct 1969. ISSN 0001-0782. Available from Internet: <https://doi.org/10.1145/363235.363259>.

KEELING, M. J.; ROHANI, P. **Modeling Infectious Diseases in Humans and Animals**. Princeton: Princeton University Press, 2008. ISBN 9781400841035. Available from Internet: <https://doi.org/10.1515/9781400841035>.

KEMPE, D.; KLEINBERG, J.; TARDOS, E. Maximizing the spread of influence through a social network. In: **Proceedings of the Ninth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining**. New York, NY, USA: Association for Computing Machinery, 2003. (KDD '03), p. 137–146. ISBN 1581137370. Available from Internet: <https://doi.org/10.1145/956750.956769>.

LIBEN-NOWELL, D. **An Algorithmic Approach to Social Networks**. Thesis (PhD) — Massachusetts Institute of Technology, USA, 2005. AAI0808764.

LIBERMAN, A. O.; RENDSVIG, R. K. Reasoning about epistemic social network dynamics using dynamic term-modal logic. **Journal of Logic and Computation**, v. 32, n. 6, p. 1067–1087, 05 2022. ISSN 0955-792X.

LIU, F. **Reasoning about preference dynamics**. New York, US: Springer, 2011.

MORENO, J. L. **Who shall survive?: A new approach to the problem of human interrelations**. New York, US: Nervous and Mental Disease Publishing Co, 1934.

NEWMAN, M.; BARABASI, A.-L.; WATTS, D. J. (Ed.). **The Structure and Dynamics of Networks**. Princeton, NJ, USA: Princeton University Press, 2006. Available from Internet: <http://www.amazon.com/Structure-Dynamics-Networks-Princeton-Complexity/dp/0691113572/sr=8-1/qid=1163938454/ref=pd_bbs_1/002-9391936-8318412?ie=UTF8&s=books>.

PETERSON, J. **Lectures: Exploring the Psychology of Creativity**. 2017. <https://youtu.be/KxGPe1jD-qY?t=2801>. In comment section, accessed 2023-15-05.

PRATT, V. R. Semantical consideration on floyd-hoare logic. In: **Proceedings of the 17th Annual Symposium on Foundations of Computer Science**. USA: IEEE Computer Society, 1976. (SFCS '76), p. 109–121. Available from Internet: <https://doi.org/10.1109/SFCS.1976.27>.

RENDSVIG, R.; SYMONS, J. Epistemic Logic. In: ZALTA, E. N. (Ed.). **The Stanford Encyclopedia of Philosophy**. Summer 2021. [S.l.]: Metaphysics Research Lab, Stanford University, 2021.

ROGERS, E. **DIFFUSION OF INNOVATIONS 3RD E REV**. Free Press, 1983. (Diffusion of Innovations). ISBN 9780029266502. Available from Internet: <https://books.google.com.br/books?id=pXRkAAAAIAAJ>.

SAID, B. **Graph rewriting for model construction in modal logic. (Réécriture de graphes pour la construction de modèles en logique modale)**. Thesis (PhD) — Paul Sabatier University, Toulouse, France, 2010. Available from Internet: <https://tel.archives-ouvertes.fr/tel-00466115>.

SCHWARZENTRUBER, F. **Epistemic reasoning in artificial intelligence**. Thesis (PhD) — University of Rennes, Habilitation Thesis, 2019.

SELIGMAN, J.; LIU, F.; GIRARD, P. Logic in the community. In: **Indian Conference on Logic and Its Applications**. [S.l.: s.n.], 2011.

STUART, W. D. Influence of sources of communication, user characteristics and innovation characteristics on adoption of a communication technology. In: . [S.l.: s.n.], 2001.

TADELIS, S. **Game Theory: An Introduction**. Princeton University Press, 2013. ISBN 9780691129082. Available from Internet: <https://books.google.com.br/books?id=eLkOJPwAdu8C>.

TALEB, N. N. **The Black Swan: The Impact of the Highly Improbable**. 1. ed. London: Random House, 2008. ISBN 1400063515.

VAN BENTHEM, J. For better or for worse: Dynamic logics of preference. In: **Preference Change**. Dordrecht, NL: Springer Netherlands, 2009, (Theory and Decision Library A:, v. 42). p. 57–84.

WATTS, D. J. **Six degrees: The science of a connected age**. [S.l.]: WW Norton & Company, 2004.

WEISBERGER, M. **Reindeer cyclones are real, and you definitely don't want to get caught in one**. Purch, 2019. Available from Internet: <https://www.livescience.com/64778-vikings-reindeer-cyclone.html>.

WOOLDRIDGE, M. **An Introduction to MultiAgent Systems**. 2nd. ed. [S.l.]: Wiley Publishing, 2009. ISBN 0470519460.

ZHEN, L.; SELIGMAN, J. A logical model of the dynamics of peer pressure. **Electronic Notes in Theoretical Computer Science**, v. 278, p. 275–288, 2011. ISSN 1571-0661. Proceedings of the 7th Workshop on Methods for Modalities (M4M'2011) and the 4th Workshop on Logical Aspects of Multi-Agent Systems (LAMAS'2011).

APPENDIX A — RESUMO EXPANDIDO

Este trabalho é sobre agentes e sua adoção de comportamentos em uma rede. O trabalho tem como foco a definição de modelos e políticas relacionadas à "difusão de informações" não por meio do raciocínio do agente, mas por meio de uma força exercida pela rede. O processo de difusão verifica, para cada agente, se ele recebeu influência suficiente para ultrapassar um limiar, relacionado a um determinado assunto ou comportamento, decidindo então, se o agente entra ou não em conformidade social com sua rede de conexões.

Consideramos modelos com múltiplos comportamentos e diferentes critérios de adoção de comportamento, onde os agentes vizinhos se situam com o mesmo nível de influência social. Também são apresentados modelos onde a influência pode ser direcionada entre agentes dadas suas conexões, também definimos modelos onde há definição de peso para cada influência no processo de adoção de comportamentos. Para todas essas variações propomos uma lógica dinâmica proposicional mínima e, para cada lógica, fornecemos axiomas de redução. Também apresentamos algoritmos naïve para cada operação de atualização d modelos.

O estudo de influência social teve sua origem na área hoje chamada de Information Propagation and Epidemics (propagação de informação e epidemias), e que trata de diversos tipos de propagações dada uma rede, dentre os quais podemos citar: casos de redes de distribuição de energia e apagões, casos de epidemias bacterianas e virais, casos de propagação de comportamento de consumidores em economia, entre outros. Nesta área, onde muitas vezes questões biológicas se misturam com questões sociais (CHRISTAKIS; FOWLER, 2009; EASLEY; KLEINBERG, 2010), o conceito de grafos é essencial. O primeiro uso de grafos, chamados então de *sociograms* (*sociogramas*), servia representar escolhas de preferências dado um grupo de agentes..

Conforme sugerido por Liben-Nowell (2005), a área pode ser subdividida em tres vertentes : Information Propagation and Epidemics (propagação de informação e epidemias), Game-Theoretic Approaches (abordagem da teoria dos jogos), e Diffusion of Innovation (difusão de inovações). Dada esta divisão, denotamos que nosso trabalho se situa na sub-área de Diffusion of Innovation, que teve origem no estudo de adoção de comportamento de novas tecnologias dada uma rede de influência. Está sub-área faz uso de dois conceitos essenciais, o *threshold* (melhor traduzido em *limiar*) que precisa ser alcançado, dado o cálculo de influência, para que o agente possa então adotar o comportamento. E o conceito de *Cascade Models* (modelos de efeito em cascata), que diz que cada vez que um contato social v na vizinhança de u adota uma inovação, então u a adota também considerando uma probabilidade $p_{v,u}$. Ou seja, cada vez que alguém adota, existe a chance de alguém relacionado a está pessoa também adotar. (LIBEN-NOWELL, 2005; GOLDENBERG; LIBAI; MULLER, 2001; GRANOVETTER, 1973; ROGERS, 1983).

Ademais, chamamos de *behavior* (*comportamento*) o elemento a ser propagado na rede e de *holders* (adotantes) os agentes que são adotantes do comportamento trabalhado.

Nosso trabalho prático segue as linhas de Christoff (2016), Seligman, Liu and Girard (2011), e mais especificamente de Baltag et al. (2018), mas ao invés de avançarmos em questões epistêmicas do trabalho deles, resolvemos nos concentrar em incorporar novos modelos lógicos de raciocínio não-epistêmicos para difusão de comportamento. Com os modelos propostos e possível representar situações envolvendo mais riqueza de detalhes, que não eram consideradas no modelo original de Baltag et al. (2018).

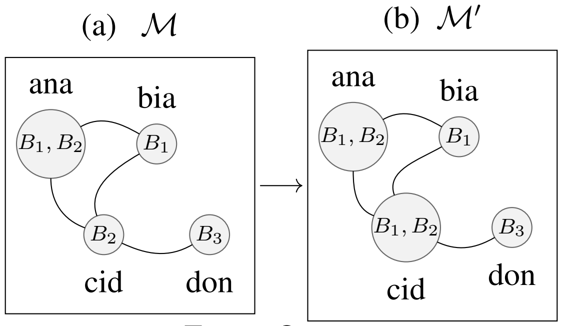
Fizemos uma divisão de capítulos onde o 3 e o 4 apresentam os modelos propostos. O capítulo 3 apresenta sete modelos considerando uma rede de agentes respeitando irreflexibilidade ($a \notin N(a)$), simetria ($b \in N(a)$ iff $a \in N(b)$) e também serialidade $N(a) \neq \emptyset$, uma vez que agentes isolados na rede não nos interessam. E o capítulo 4 volta a considerar somente um comportamento e apresenta dois modelos com variações na estrutura da rede.

No capítulo 3 desenvolvemos modelos que permitem simular múltiplos comportamentos sendo espalhados em diferentes etapas da simulação; a possibilidade de modelar comportamentos conflitantes, como por exemplo, determinado agente a ser vegano e comer carne; a possibilidade de que um agente deixe de ser vegano e passar a comer carne, e a possibilidade de dependência entre comportamentos, por exemplo, para o agente ser portador de uma carta de motorista precisa antes saber dirigir.

Nosso primeiro modelo propõe uma extensão do modelo básico de (BALTAG et al., 2018) mas considerando múltiplos comportamentos independentes, ou seja um agente pode adotar um comportamento independentemente de outros comportamentos que ele tenha ou não adotado.

A Figura A.1 ilustra um modelo $\mathcal{M}$ contendo quatro diferentes agentes (ana, bia, cid, don) representados por nodos nos grafos, três comportamentos independentes B_1, B_2, B_3 e um limiar $\theta = 0.5$. Após uma etapa da difusão do comportamento B_1 tem-se o modelo resultante $\mathcal{M}'$. O agente cid adotou B_1 em $\mathcal{M}'$ por sofrer influência de dois terços de seus vizinhos, o que é superior ao limiar de $0,5$.

Figure A.1 – Evolução do modelo $\mathcal{M}$ depois de uma etapa da difusão do comportamento B_1, considerando $\theta = 0.5$ e o modelo de update dado por 3.2.2.



Fonte: O autor.

Nosso segundo modelo proposto contempla a possibilidade de comportamentos conflitantes. Nste caso, a adoção de um novo comportamento B_i por um agente é impedida caso esse agente já seja adotante de um comportamento B_j conflitante com B_i, mesmo que o limiar de influência ja tenha sido atingido.

O terceiro modelo proposto chamado contempla comportamentos conflitantes com prioridade para adoção. Esse modelo também considera a possibilidade de comportamentos conflitantes entre si, mas caso haja conflito, o agente influenciado a adotar, que apresente um comportamento B_j conflitante com um novo comportamento B_i deve fazer a "desadoção" de B_j e assumir B_i.

O quarto modelo considera a possibilidade de haver uma reloação de dependência entre comportamentos. Dessa forma, um agente é impedido de adotar um comportamento B_j caso ele não possua comportamentos dos quais o comportamento B_j dependa. O quinto modelo é semelhante ao anterior, mas, caso um agente a não possua o comportamento B_i do qual B_j é dependente, então, em caso de difusão de B_j, o agente a adotaria tanto B_j quanto B_i no mesmo turno.

O sexto e o sétimo modelo proposto apresentam combinações de modelos anteriores. O sexto é uma combinação dos modelos conflitantes e dependentes , e o sétimos dos modelos conflitantes e dependentes, ambos com prioridade para adoção.

No capítulo 4, o primeiro modelo proposto adiciona peso às influências exercidas pela vizinhança, e mantém a simetria de influências entre agentes. O segundo modela influência direcionada entre agentes, ou seja, a remoção da simetria na relaçao de vizinhança. Se determinado comportamento sendo espalhado for relacionado a área de saúde por exemplo, é natural que um agente especialista naquela área exerça mais influência sobre outros agentes do que um agente leigo no assunto. Outro aspecto que pode ser modelado é o que se observa em redes sociais, como Instragram por exemplo: um agente pode seguir e ser influencidado por um agente famoso e não ser seguido por esse agente *influcencer*.

Ao final da dissertação, mencionamos possíveis trabalhos futuros relacionados, dentre os quais destacamos os seguintes: (i) a adição da noção de operadores epistêmicos; (ii) estudos sobre a influência da topologia de rede com a utilização dos operadores propostos, afim de para avaliar impactos dos diferentes tipos de redes sobre as influências espalhadas; e (iii) dar continuidade a implementação de um framework lógico para realizar simulações com os modelos propostos (fonte e executável disponíveis respectivamente em github.com/gsamorim/SocialDiffusionProj e gsamorim.github.io/SocialDiffusionProj/).