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Partial distance estimate in a two-group SEIRD model

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We study the case of a two-group epidemic model, where the infectious individuals of each group transmit the disease to the susceptible individuals of both groups according to an asymmetric interaction and including an period of latency.

The system of equations for the two-group SEIRD model is the following

$$\begin{cases} S_1' = -(\beta_{11}I_1 + \beta_{12}I_2)S_1 & , \quad S_2' = -(\beta_{21}I_1 + \beta_{22}I_2)S_2 \\ E_1' = (\beta_{11}I_1 + \beta_{12}I_2)S_1 - \alpha_1 E_1 & , \quad E_2' = (\beta_{21}I_1 + \beta_{22}I_2)S_2 - \alpha_2 E_2 \\ I_1' = \alpha_1 E_1 - (\gamma_1 + \mu_1)I_1 & , \quad I_2' = \alpha_2 E_2 - (\gamma_2 + \mu_2)I_2 \\ R_1' = \gamma_1 I_1 & , \quad R_2' = \gamma_2 I_2 \\ D_1' = \mu_1 I_1 & , \quad D_2' = \mu_2 I_2 \end{cases} \tag{1}$$

where $\beta_{ij}, \alpha_i, \gamma_i, \mu_i$ are positive constants and $S_i(0), E_i(0), I_i(0), R_i(0), D_i(0) \in \mathbb{R}_+$ are nonnegative initial conditions, for all $i, j = 1, 2$.

We are interested in the following problem: if the solutions to the SEIRD model, corresponding to two different sets of paramenters and same initial consitions, are close to each other w.r.t. to the Group 1, how close the solutions w.r.t. Group 2 must be? The theorem below, which proof can be found in [1], presents an answer to this question under some hypotheses on the parameters:

Theorem 1. *Let $(\widetilde{S}, \widetilde{E}, \widetilde{I}, \widetilde{R}, \widetilde{D})$ and (S, E, I, R, D) be two solutions for (1) corresponding to two sets of parameters $\{\widetilde{\beta_{ij}}, \widetilde{\alpha}_i, \widetilde{\gamma}_i, \widetilde{\mu}_i\}_{ij=1,2}$ and $\{\beta_{ij}, \alpha_i, \gamma_i, \mu_i\}_{i,j=1,2}$ respectively, with the same initial conditions. Suppose that the two sets of parameters satisfy*

$$\frac{\widetilde{\beta}_{2i} - \widetilde{\beta}_{1i}}{\widetilde{\mu}_i} = \frac{\beta_{2i} - \beta_{1i}}{\mu_i}, \qquad \frac{\widetilde{\alpha}_1}{\widetilde{\alpha}_2} = \frac{\alpha_1}{\alpha_2}, \qquad \widetilde{\gamma}_1 - \widetilde{\gamma}_2 = \gamma_1 - \gamma_2, \qquad \widetilde{\mu}_1 - \widetilde{\mu}_2 = \mu_1 - \mu_2$$

If also $\|(\widetilde{S}_1, \widetilde{E}_1, \widetilde{I}_1, \widetilde{R}_1, \widetilde{D}_1) - (S_1, E_1, I_1, R_1, D_1)\|_{[0,T]} + \|D_2 - \widetilde{D}_2\|_{[0,T]} \leq \varepsilon$ then

$$\|(\widetilde{S}_2, \widetilde{E}_2, \widetilde{I}_2, \widetilde{R}_2) - (S_2, E_2, I_2, R_2)\|_{[0,T]} < K(\varepsilon)\varepsilon,$$

where $K(\varepsilon)$ is bounded as $\varepsilon \to 0$.

We apply the theorem above to analyze a possible underreporting of the COVID-19 cases in the cities of Petrolina/PE [2] and Juazeiro/BA [3] for the period of February 13 to May 13 of 2021. For this, we rely on the data of cases from Petrolina and the data of deaths in both cities and we suppose that these data represent a model as (1) for some set of unknown parameters $\{\widetilde{\beta_{ij}}, \widetilde{\alpha}_i, \widetilde{\gamma}_i, \widetilde{\mu}_i\}_{ij=1,2}$ with solutions $(\widetilde{S}, \widetilde{E}, \widetilde{I}, \widetilde{R}, \widetilde{D})$. Then we fit the model and obtain an approximate set of parameters $\{\beta_{ij}, \alpha_i, \gamma_i, \mu_i\}_{i,j=1,2}$ with respective solutions (S, E, I, R, D). From the proof of the theorem,

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we obtain that the cumulative number of cases $\widetilde{C}_2(t)$ must be close to the function $C_2(t) := I_2(t) + R_2(t) + D_2(t)$ by

$$\left| C_2(t) - \widetilde{C}_2(t) \right| \leq K_C(t)$$

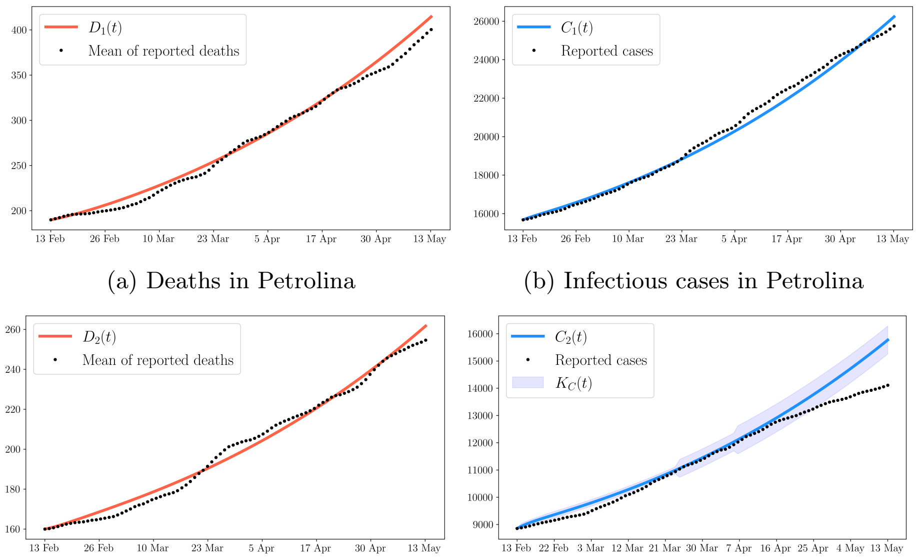
where

$$K_C(t) = \max_{s=1,\ldots,t} \left\{ S_2(s) \left(1 - \left(1 - \frac{\varepsilon_{S_1}(s)}{S_1(s)} \right) e^{-\left(|V_1|\varepsilon_{D_1}(s) + |V_2|\varepsilon_{D_2}(s)\right)} \right) \right\}$$

and

$$\varepsilon_{D_i}(t) := \max_{s=1,\ldots,t} \left| D_i(s) - \widetilde{D}_i(s) \right|, \quad \text{and} \quad \varepsilon_{S_1}(t) := \max_{s=1,\ldots,t} \left| S_1(s) - \widetilde{S}_1(s) \right|,$$

In (a) and (c) below we plot the reported deaths (black dots) and the solutions $D_1(t)$ and $D_2(t)$ (red solid line) for the fitted model. In (b) the cumulative number of reported infectious cases (black dots) and the function $C_1(t)$ (blue solid line). In (d), we plot the reported number of infectious cases (black square markers) for Juazeiro, the function cumulative number of infectious cases $C_2(t)$ obtained from the fitted model (dark blue solid line) and the estimated error range $K_C(t)$ (blue area) for this function.



(a) Deaths in Petrolina

(b) Infectious cases in Petrolina

(c) Deaths in Juazeiro

(d) Infectious cases in Juazeiro

Figure 1: Reported data for deaths and cases for Petrolina and Juazeiro, solutions to the fitted SEIRD system (1) and estimated error of cases in Juazeiro.

References

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[3] **Boletim Epidemiologico de Juazeiro: Prefeitura Municipal de Juazeiro**. Online. Acessado em 01/10/2021, https://www6.juazeiro.ba.gov.br/category/coronavirus.