The Threshold Condition for Stochastic Diffusion of Energetic Ions due to Lower Hybrid Waves

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In the present paper we consider the interaction between energetic ions and lower hybrid waves in tokamak plasmas and study the parametric dependence of the threshold condition for stochastic ion diffusion. It is shown that the threshold condition as obtained in the work by Karney [C. F. F. Karney, Phys. Fluids **22**, 2188 (1979)] may be not easily satisfied in present day large tokamaks, but can attained in small tokamaks with relatively modest levels of wave power.

I Introduction

In a series of two papers published in 1978 and 1979, it has been shown that the movement of an ion in a uniform magnetic field becomes stochastic in the presence of a perpendicularly propagating coherent electrostatic wave, if the wave amplitude exceeds a threshold [1, 2]. Due to the stochasticity the ion diffuses in velocity space, and this diffusion may be described by a diffusion equation which also applies to the case of lower hybrid waves in the slow mode (non-vanishing $k_\parallel << k_\perp$), propagating in a weakly inhomogeneous magnetic field [2]. The possibility of this diffusion mechanism raised important questions related to the efficiency of proposed RF current drive schemes using lower hybrid (LH) waves [3, 4, 5]. In particular, it has been argued that in fusion reactors the presence of α particles originated from fusion reactions would contribute to decrease the efficiency of the lower hybrid current drive, since part of the energy of the LH waves would be absorbed by the α particles [1]. The potential importance of this interaction between LH waves and α particles has motivated subsequent investigations, and many examples may be found in the recent literature [6, 7, 8, 9, 10, 11, 12, 13]. Investigations have also been made by considering other kind of possible interactions, as the interaction between LH waves and a population of energetic ions generated by neutral beam injection [14, 15, 16, 17], and the interaction between LH waves and fast ions generated by IC waves utilized for ion heating. Some experimental results on this subject are available, as those originated from relatively recent experiments conducted in the JET tokamak (Joint European Torus) [18, 19]. The results of these experiments provide evidence of significant absorption of LH waves by fast ions, and therefore confirm the convenience of further investigations on this important issue concerning the dynamics of α particles or other energetic ions in the environment of a reactor, and the efficiency of the LH waves for radio frequency current drive.

In the present paper we study particular features of the subject, considering the threshold condition for stochastic diffusion of energetic ions, in a tokamak environment modelated by a plasma slab. We utilize the formalism proposed by Karney [2], discussing the dependence of the threshold condition on some wave and plasma parameters, for large present day tokamaks and for small scale tokamaks, and compare the value of the electric field predicted by the threshold condition with the wave electric field inside the tokamak.

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The plan of the paper is the following. In Sec. II it is presented a short summary about the equation describing stochastic ion diffusion, as proposed by Karney [1, 2], and it is introduced the threshold condition for the occurrence of this process. In Sec. III we describe the model to be utilized for the tokamak and for the lower hybrid wave packet. Using the model, in Sec. IV we discuss the dependence of the threshold condition on plasma and wave parameters, and present results of comparisons between the wave amplitude required by the threshold condition and the wave electric field which shall be present in situations typical of experiments conducted in large present day tokamaks and in small tokamaks. Finally, in Sec. V we summarize the main results of the paper, and comment on possible future developments of this investigation.

II The threshold condition for stochastic ion diffusion

According to the formalism developed by Karney [2], the diffusion in velocity space produced by a lower hybrid wave with angular frequency ω and components of the wave number $N_\perp$ and $N_\parallel$ may be described by the following equation, in normalized form,

$$(\partial_{\tau_i} f_i)_{lh} = \frac{1}{u_\perp}\partial_{u_\perp}\left(u_\perp D_{lh}\partial_{u_\perp} f_i\right), \qquad (1)$$

where the ion thermal velocity is used for velocity normalization ($u \equiv v/v_i$), f_i is the ion distribution function, and where

$$D_{lh}(u_\perp) = \begin{cases} \frac{\pi}{4}\frac{E_0^2}{c^2B_0^2}\frac{\mu_i^2}{N_\perp^2 u_\perp^2}\frac{\Omega_i}{\nu_i}\left|H_\nu^{(1)\prime}(\nu)\right|^2 g^2(\mathcal{A})\,\gamma, & \text{for } u_\perp \geq u_{ph}, \\ D_{lh}(u_{ph}), & \text{for } u_{ph} - u_{tr} \leq u_\perp < u_{ph}, \\ D_{lh}(u_{ph})\left[u_\perp - (u_{ph} - 2u_{tr})\right]^2, & \text{for } u_{ph} - 2u_{tr} \leq u_\perp < u_{ph} - u_{tr}, \\ 0, & \text{for } u_\perp < u_{ph} - 2u_{tr}. \end{cases} \qquad (2)$$

Moreover,

$$u_{ph} = \omega/(k_\perp v_i), \qquad u_{tr} = (q_i E_0/m_i k_\perp)^{1/2}/v_i,$$
$$\mathcal{A} = \frac{\alpha\nu}{r}|H_\nu^{(1)\prime}(\nu)|, \qquad (3)$$

with the introduction of the following definitions

$$\nu = \omega/\Omega_i, \qquad r = k_\perp v_\perp/\Omega_i,$$

and

$$\alpha = \frac{E_0}{B_0}\frac{k_\perp}{\Omega_i} = \frac{E_0}{B_0}\frac{N_\perp \nu}{c},$$

where $N_\perp = ck_\perp/\omega$. The quantity $k_\perp$ is the perpendicular component of the wave vector $\mathbf{k}$, Ω_i is the ion cyclotron frequency, E_0 is the amplitude of the wave electric field, and B_0 is the ambient magnetic field. We have also defined $\mu_i = m_i c^2/T_i$. The quantities $H_\nu^{(1)}$ and $H_\nu^{(1)\prime}$ are respectively the Hankel function of first kind and its derivative, given by

$$|H_\nu^{(1)}(r)| \simeq (2/\pi)^{1/2}(r^2-\nu^2)^{-1/4},$$

$$|H_\nu^{(1)\prime}(r)| \simeq (2/\pi)^{1/2}(r^2-\nu^2)^{1/4}/r,$$

for $r \geq \nu + (\nu/2)^{1/3}$ [2]; for $r < \nu + (\nu/2)^{1/3}$, it is used

$$|H_\nu^{(1)}(r)| \simeq |H_\nu^{(1)}(\nu + (\nu/2)^{1/3})|,$$

$$|H_\nu^{(1)\prime}(r)| \simeq |H_\nu^{(1)\prime}(\nu + (\nu/2)^{1/3})|.$$

In Eq. (2) the threshold condition is expressed by the quantity $g(\mathcal{A})$. This quantity is zero for $\mathcal{A}$ smaller than $\mathcal{A}_s$ and grows fast to $g(\mathcal{A}) = 1$ for $\mathcal{A}$ larger than $\mathcal{A}_s$ [2],

$$g(\mathcal{A}) = \max\left[1 - \mathcal{A}_s^2/\mathcal{A}^2, 0\right]. \qquad (4)$$

The stochasticity condition defines the situation in which a stochastic layer occupies a substantial fraction of the phase space, and therefore the diffusion becomes effective. This condition does not allow a precise definition of $\mathcal{A}_s$. Karney sets the requirement that for

$\mathcal{A} = \mathcal{A}_s$ the phase change suffered by a particle in the wave-particle resonance would be equal to $\pi/2$, and therefore assumes that $\mathcal{A}_s = 0.25$ [2]. The same condition is assumed for most of the present work.

Another quantity appearing in Eq. (2) is the fraction of time in which the particles remain under the effect of the lower hybrid waves, denoted by γ [2], which is a quantity dependent on the existence of models for the tokamak and the wave packet.

III Models of the wave packet and of the tokamak

In order to atribute to the quantities which have been defined a numerical value at a given position in the tokamak, it is necessary to have a picture about wave propagation and energy deposition of the wave. Accurate description of these processes is difficult to achieve, and therefore we assume a simplifying model which is intended to be useful providing an estimate about the feasibility of the stochastic diffusion, as a function of wave and plasma parameters.

For these purposes a model tokamak described by a slab of plasma shall be satisfactory. We assume the following simple profiles

$$n_\alpha(x) = n_\alpha(0)\left(1 - \frac{x^2}{a^2}\right)$$

$$T_\alpha(x) = T_\alpha(0)\left(1 - \frac{x^2}{a^2}\right)^2 \tag{5}$$

$$B_0(x) = B_0(0)\left(1 + \frac{x}{R}\right)^{-1},$$

where a is the minor radius, R is the major radius, and x is the radial coordinate in the equatorial plane, with the origin in the center of the plasma column. $n_\alpha(0)$, $T_\alpha(0)$, and $B_0(0)$ are respectively the density and electron temperature of species α and the ambient toroidal magnetic field, at the center of the plasma.

Regarding the lower hybrid waves, let us assume a gaussian packet propagating in the tokamak, with spectral distribution centered at $N_\parallel = \overline{N}_\parallel$, and with half-width given by $\Delta N_\parallel$,

$$S(N_\parallel) = S_0 \frac{e^{-(N_\parallel - \overline{N}_\parallel)^2/(\Delta N_\parallel)^2}}{\sqrt{\pi}(\Delta N_\parallel)}, \tag{6}$$

where $\int dN_\parallel S(N_\parallel) = S_0$, the local wave intensity. The local amplitude of the electric field, which is necessary to evaluate the threshold condition, may be given by the following [20]

$$E_0(N_\parallel) = \left(\frac{16\pi}{(4\pi\epsilon_0)c}\frac{|B|}{|\partial D/\partial \mathbf{N}|}S(N_\parallel)\right)^{1/2}. \tag{7}$$

where

$$B = D_{11}D_{22} - |D_{12}|^2 + D_{11}D_{33} - |D_{13}|^2 + D_{22}D_{33} - |D_{23}|^2\,, \tag{8}$$

$$D = \epsilon_{11}N_\perp^4 + N_\perp^2(N_\parallel^2(\epsilon_{11} + \epsilon_{33}) - \epsilon_{11}^2 - \epsilon_{12}^2 - \epsilon_{11}\epsilon_{33}) + \epsilon_{33}[(\epsilon_{11} - N_\parallel^2)^2 + \epsilon_{12}^2] = 0\,, \tag{9}$$

and the D_{ij} are defined as $D_{ij} = N_iN_j - N^2\delta_{ij} + \epsilon_{ij}$, where the N_i are the components of $\vec{N} = c\vec{k}/\omega$ and the ϵ_{ij} are the components of the dielectric tensor obtained in the cold plasma approximation. The expression defined as D is the dispersion relation for LH waves.

Regarding the energy deposition, we consider the following: the wave packet propagating with half-width $\Delta N_\parallel$ has a spatial half-width $\Delta z = 2c/(\omega(\Delta N_\parallel))$. If we assume a poloidal width $\Delta\theta$ for the wave packet, the area affected by the wave in a magnetic surface of radius r will be given by $S_\omega = 2(\Delta z)(\Delta\theta)r$, while the area of the magnetic surface itself will be $S = 4\pi^2 rR$. If the wave power is given by W_0, the local intensity S_0 is given by $S_0 = W_0/S_\omega$.

This rather rough estimate of wave intensity tends to very large values near the center of the tokamak, and indeed diverges for $r \to 0$, since $S_\omega \to 0$. This divergence is not real, since the wave power will not really concentrate on a vanishing surface. In order to avoid this unphysical divergence of the model, we arbitrarily set a minimum value for S_ω and a corresponding maximum value for S_0, chosen as the values of S_ω and S_0 at $r = 2$ cm. Even with this cut-off value which avoids the divergence at $r \to 0$, it is possible that this simplified model is super-estimating the local magnitude of the wave electric field, for the central regions.

IV Parametric dependences of the threshold condition

The model introduced in the previous section may be used to evaluate the amplitude of the wave electric field, for each component of the wave packet, since the amplitude E_0 depends on $S(N_\parallel)$, according to Eq. (7). If the wave packet is sufficiently narrow, we may assume that only the central ray of te packet is significant, and that the diffusion coefficient as given by Eq. (2), derived for a single wave, is a good approximation to be employed in the analysis of ion diffusion in velocity space. Therefore, it is interesting to know the parametric dependence of the threshold condition for the occurence of the stochastic diffusion, and compare the threshold value with the wave electric field, for parameters typical of present day experiments.

Using the definition of $\mathcal{A}$ given by Eq. (3) and the condition $\mathcal{A} > \mathcal{A}_s$, the threshold value of the electric field amplitude at a given plasma position is given by the following condition

$$(E_0)_t = \frac{\mathcal{A}_s r}{N_\perp \nu^2 |H_\nu^{(1)\prime}(r)|} B_0 c. \qquad (10)$$

For some numerical estimates on this quantity, let us initially consider the case of a large tokamak, with parameters corresponding to those of the JET (Joint European Torus): $n_e(0) = 3.5 \times 10^{19}$ m^{-3}, $T_e(0) = 5.0$ keV, equal ion and electron densities and temperatures, and $B_0(0) = 3.0$ T (geometrical factors like the minor radius and the major radius will not be necessary for the determination of the threshold condition or the wave amplitude, according to the model which we have utilized). For the lower hybrid waves the relevant parameters are ω, $\overline{N}_\parallel$, $\Delta N_\parallel$, and W_0. Another necessary parameter is $\Delta\theta$, which will be assumed as $\Delta\theta = \pi/4$ [2]. For given values of ω and $N_\parallel$, the dispersion relation is solved and $N_\perp$ is determined.

Initially, let us choose the position $x = 0$ cm and $\omega = 1.5\,\omega_{lh}$, where ω_{lh} is the local lower hybrid angular frequency, $\Delta N_\parallel = 0.2$, and four values of $\overline{N}_\parallel$, chosen in order to be resonant with electrons of 3, 4, 5 and 6 local electron thermal velocities. For JET-like parameters, these correspond respectively to $\overline{N}_\parallel = 3.37, 2.53, 2.02$, and 1.68. These values of $\overline{N}_\parallel$ all satisfy the acessibility condition to the center of the tokamak [18]. For these parameters, in Fig. 1 we show the normalized threshold amplitude $(E_0)_t/(cB_0)$ at $x = 0$ cm, evaluated according to Eq. (10). The threshold amplitude is shown as a function of $u_\perp$, the perpendicular ion velocity normalized to local ion thermal velocity, for $u_\perp$ above perpendicular phase velocity for the central ray of the wave packet, $\omega/k_\perp(\overline{N}_\parallel, \omega)$. The figure also shows the normalized amplitude of the central ray of the wave packet, $E_0(\overline{N}_\parallel)/(cB_0)$, as a function of $u_\perp$, for $W_0 = 2.0$ MW and the same values of $\overline{N}_\parallel$, for the whole range where the diffusion coefficient may be non-vanishing. This amplitude, of course, has been evaluated with the use of Eq. (7) applied to the case of $N_\parallel = \overline{N}_\parallel$, the value of $N_\parallel$ at the center of the wave packet.

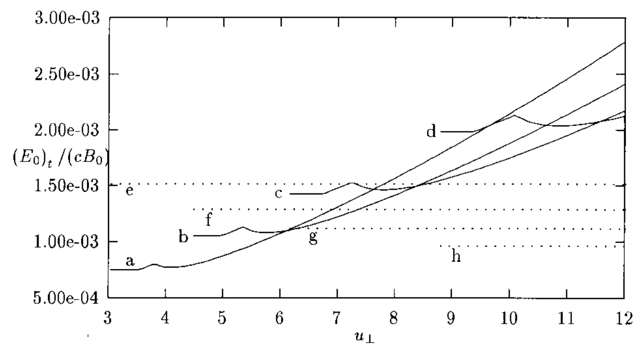

Figure 1. Normalized threshold amplitude $(E_0)_t/(cB_0)$ as a function of $u_\perp$, for $\omega = 1.5\ \omega_{lh}$, $\Delta N_\parallel = 0.2$, for $x = 0$ cm and parameters similar to those of the JET tokamak. (a) $\overline{N}_\parallel$=3.37; (b) $\overline{N}_\parallel$=2.53; (c) $\overline{N}_\parallel$=2.02; (d) $\overline{N}_\parallel$= 1.68 ($v_{res} = 3, 4, 5$, and $6v_e$, respectively)). The dotted straight lines (e), (f), (g), and (h) represent the normalized amplitude of the electric field for the central ray, $E_0(\overline{N}_\parallel)/(cB_0)$, as a function of $u_\perp$, for $W_0 = 2.0$ MW and the same values of $\overline{N}_\parallel$, respectively.

It is seen that for the parameters utilized the threshold condition is satisfied by the wave at the center of the packet only if $\overline{N}_\parallel$ is such that the resonance is in the near-tail of the electron distribution, nearly up to 5 thermal velocities.

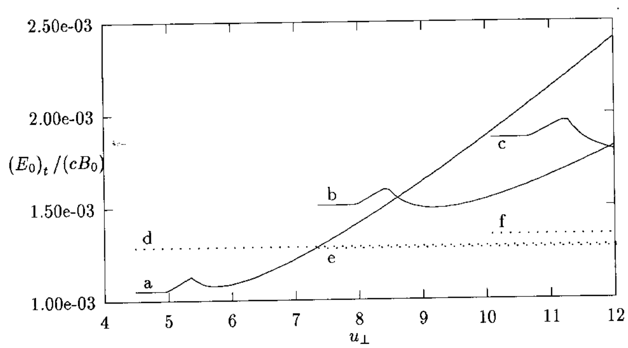

Figure 2. Normalized threshold amplitude $(E_0)_t/(cB_0)$ as a function of $u_\perp$, for $\Delta N_\parallel = 0.2$, $\overline{N}_\parallel = 2.53$ ($v_{res} = 4\ v_e$), for $x = 0$ cm, for parameters as those of the JET tokamak. (a) $\omega/\omega_{lh} = 1.5$; (b) $\omega/\omega_{lh} = 2.0$; (c) $\omega/\omega_{lh} = 2.5$. The dotted straight lines (d), (e), and (f) represent the normalized amplitude of the electric field for the central ray, $E_0(\overline{N}_\parallel)/(cB_0)$, as a function of $u_\perp$, for $W_0 = 2.0$ MW and the same values of ω/ω_{lh}, respectively.

Fig. 2 displays the normalized threshold amplitude $(E_0)_t/(cB_0)$ at $x = 0$ cm, for $\overline{N}_\parallel = 2.53$, for several values of the ratio ω/ω_{lh}, and other parameters as in Fig. 1. It is also seen the quantity $E_0(\overline{N}_\parallel)/(cB_0)$, as a function of $u_\perp$, for $W_0 = 2.0$ MW and the same values of ω/ω_{lh}. The comparison between these curves shows that for this wave power the stochastic diffusion vanishes for frequencies somewhat above the value of $\omega/\omega_{lh} = 1.5$.

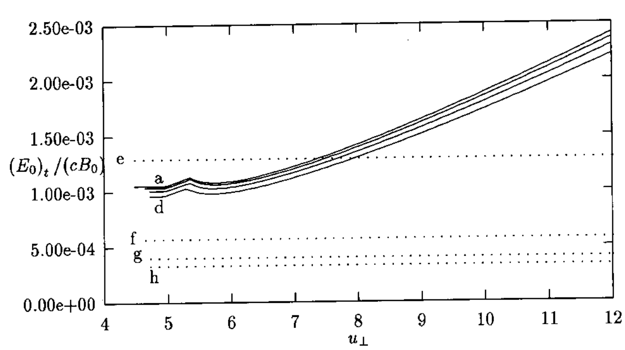

Figure 3. Normalized threshold amplitude $(E_0)_t/(cB_0)$ as a function of $u_\perp$, for $\Delta N_\parallel = 0.2$, $\overline{N}_\parallel = 2.53$ ($v_{res} = 4\ v_e$), $\omega/\omega_{lh} = 1.5$, for parameters as those of the JET tokamak. (a) $x = 0$ m; (d) $x = 0.30$ m; between these two curves, there are two corresponding to $x = 0.10$ and 0.20 m. The dotted straight lines (e), (f), (g), and (h) represent the normalized amplitude of the electric field for the central ray, $E_0(\overline{N}_\parallel)/(cB_0)$, as a function of $u_\perp$, for $W_0 = 2.0$ MW and the same values of x, respectively.

In Fig. 3 the quantity $(E_0)_t/(cB_0)$ appears for $\overline{N}_\parallel = 2.53$ and $\omega/\omega_{lh} = 1.5$, and other parameters also as in Fig. 1, for several values of x ($x = 0.00$, 0.10, 0.20, and 0.30 m). The curves depict the weak dependence of the threshold amplitude on the position, according to the model utilized. On the other hand, the amplitude of the central ray of the wave packet is more strongly dependent on position, as seen from the curves for $E_0(\overline{N}_\parallel)/(cB_0)$, as a function of $u_\perp$, for $W_0 = 2.0$ MW and the same values of x, also appearing in Fig. 3. For this wave power, it is seem that the stochastic diffusion is effective only in the central region of the tokamak, for the model utilized.

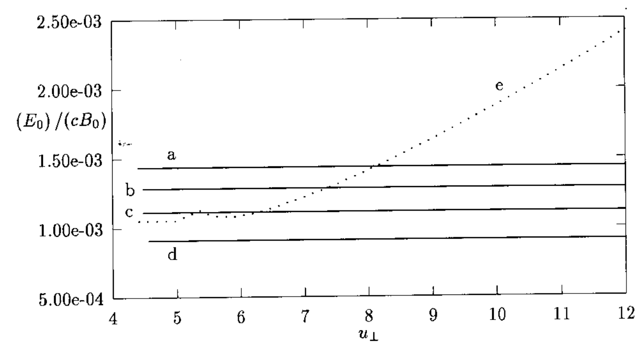

Figure 4. Normalized amplitude of the electric field for the central ray, $E_0(\overline{N}_\parallel)/(cB_0)$ as a function of $u_\perp$, for $\overline{N}_\parallel = 2.53$ ($v_{res} = 4v_e$) and the other parameters as in Fig. IV, and several values of W_0 ((d) $W_0 = 1.0$ MW; (c) $W_0 = 1.5$ MW; (b) $W_0 = 2.0$ MW; (a) $W_0 = 2.5$ MW). The dotted curve (e) represents the normalized threshold amplitude $(E_0)_t/(cB_0)$, for $\overline{N}_\parallel = 2.53$.

In Fig. 4 we show $E_0(\overline{N}_\parallel)/(cB_0)$ as a function of $u_\perp$, for $\overline{N}_\parallel = 2.53$ and the other parameters as in Fig. 1, and several values of W_0 (1.0, 1.5, 2.0, and 2.5 MW). For comparison it is also shown in this figure the curve for $(E_0)_t/(cB_0)$ for $\overline{N}_\parallel = 2.53$. These results show that the threshold condition is only satisfied for wave powers somewhat close to 1.5 MW and above.

In order to illustrate the dependence of threshold amplitude and wave amplitude on different plasma conditions, we also consider a small tokamak, with the following parameters: $n_e(0) = 1.0 \times 10^{19}$ m^{-3}, $T_e(0) = 0.5$ keV, equal ion and electron densities and temperatures, $B_0(0) = 1.5$ T, $a = 0.2$ m, and $R = 1.0$ m. As in the case of the JET-like tokamak, we choose the position $x = 0$ cm and $\omega = 1.5\ \omega_{lh}$, $\Delta N_\parallel = 0.2$, and four values of $\overline{N}_\parallel$, in order that the center of the wave packet is resonant with electrons of 3, 4, 5 and 6 electron thermal velocities. For the parameters assumed for the small tokamak, these correspond respectively to $\overline{N}_\parallel = 10.6, 7.99, 6.39$, and 5.33. For these parameters, in Fig. 5 we show the normalized threshold amplitude $(E_0)_t/(cB_0)$ at $x = 0$ cm, as a function of $u_\perp$. The comparison with the wave amplitudes which appear in the same figure for the same values of $\overline{N}_{\|}$ shows that for these parameters of a small tokamak the threshold condition is satisfied for the wave at the center of the packet up to resonant velocities near 6 v_e.

In Fig. 6 we show $E_0(\overline{N}_{\|})/(cB_0)$ as a function of $u_\perp$, for $\overline{N}_{\|} = 7.99$ and the other parameters as in Fig. 5, and several values of W_0 (25, 50, 75, and 100 kW). It is also seen in this figure the curve for $(E_0)_t/(cB_0)$ for $\overline{N}_{\|} = 7.99$. The figure shows that for these parameters even the wave power of 25 kW is sufficient to satisfy the threshold condition and cause stochastic diffusion.

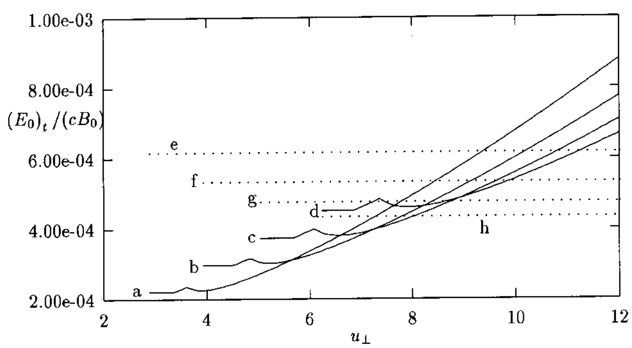

Figure 5. Normalized threshold amplitude $(E_0)_t/(cB_0)$ as a function of $u_\perp$, for $\omega = 1.5\ \omega_{lh}$, $\Delta N_{\|} = 0.2$, $x = 0$ cm and parameters typical of a small tokamak ($n_e(0) = 1.0 \times 10^{19}$ m^{-3}, $T_e(0) = 0.5$ keV, equal ion and electron densities and temperatures, $B_0(0) = 1.5$ T, $a = 0.2$ m, and $R = 1.0$ m). (a) $\overline{N}_{\|} = 3.37$; (b) $\overline{N}_{\|} = 2.53$; (c) $\overline{N}_{\|} = 2.02$; (d) $\overline{N}_{\|} = 1.68$ ($v_{res} = 3, 4, 5$, and $6v_e$, respecively). The dotted straight lines (e), (f), (g), and (h) represent the normalized amplitude of the electric field for the central ray, $E_0(\overline{N}_{\|})/(cB_0)$, as a function of $u_\perp$, for $W_0 = 50$ kW and the same values of $\overline{N}_{\|}$, respectively.

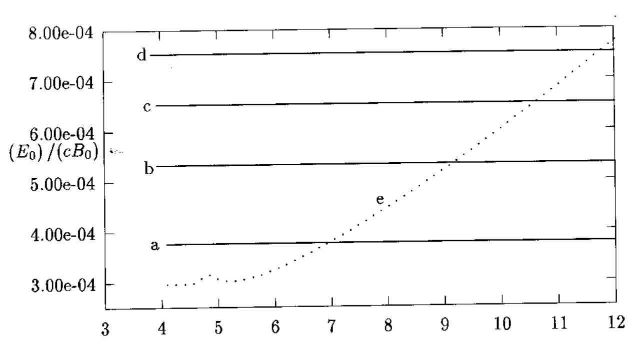

Figure 6. Normalized amplitude of the electric field for the central ray, $E_0(\overline{N}_{\|})/(cB_0)$ as a function of $u_\perp$, for $\overline{N}_{\|} = 7.99$ ($v_{res} = 4v_e$) and the other parameters as in Fig. IV, and several values of W_0 ((a) 25 kW; (b) 50 kW; (c) 75 kW; (d) 100 kW). The dotted curve (e) represents the normalized threshold amplitude $(E_0)_t/(cB_0)$, for $\overline{N}_{\|} = 7.99$.

We now consider the effect of the parameter $\mathcal{A}_s$ on the threshold condition. The motivation for this is the reasoning that the simultaneous presence of more than one LH wave should contribute to the increase of the stochastic layer in phase space, as compared to the case where only one LH wave is interacting with the ions, as considered in the formulation employed in the present work. Therefore diffusion in velocity space should be made easier by the presence of a group or spectrum of LH waves, which could appear as a reduced threshold requirement, or reduced value of $\mathcal{A}_s$. An interesting and more rigorous line of investigation would be to follow similar procedures as those employed by Karney, but considering the existence of more than one LH wave, and verify the modification in the threshold condition, as a function of the number of waves or spectral width. Such an investigation is beyond of the objectives of the present work, and is one of the possibilities offered to continue our research on the subject of interaction between LH waves and energetic ions. The present interest rests only on the observation of the effects of the value of $\mathcal{A}_s$ on the threshold condition, as a function of some important parameters. Therefore, in Fig. 7 we show $E_0(\overline{N}_{\|})/(cB_0)$ as a function of $u_\perp$, for $\overline{N}_{\|} = 2.53$ and $W_0 = 1.0$ MW, other parameters as in Fig. 1, and several values of $\mathcal{A}_s$, starting from the value used in the work by Karney, $\mathcal{A}_s$=0.25 [2]. It is seen that, while the original threshold condition would not be satisfied for $W_0 = 1.0$ MW, it is already satisfied over a limited range of $u_\perp$ for $\mathcal{A}_s = 0.2$, and is well satisfied over a large range of $u_\perp$ for $\mathcal{A}_s = 0.1$.

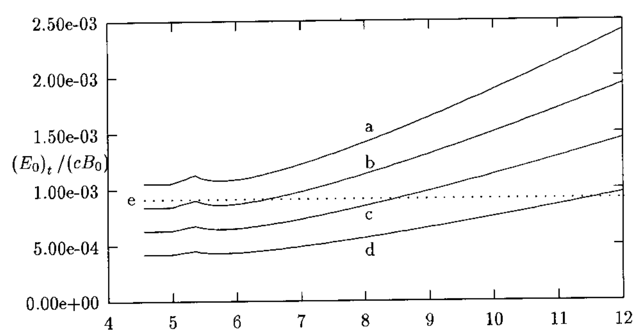

Figure 7. Normalized threshold amplitude $(E_0)_t/(cB_0)$ as a function of $u_\perp$, for $\omega = 1.5\ \omega_{lh}$, $\overline{N}_{\|} = 2.53$ ($v_{res} = 4v_e$), $\Delta N_{\|} = 0.2$, $x = 0$ cm, and parameters as those of the JET tokamak. (a) $\mathcal{A}_s$=0.25; (b) $\mathcal{A}_s$=0.20; (c) $\mathcal{A}_s$=0.15; (d) $\mathcal{A}_s$= 0.10. The dotted straight line (e) represents the normalized amplitude of the electric field for the central ray, $E_0(\overline{N}_{\|})/(cB_0)$, as a function of $u_\perp$, for $W_0 = 1.0$ MW.

Finally, in Fig. 8 we show the normalized threshold amplitude $(E_0)_t/(cB_0)$ at $x = 0$ cm, for $\overline{N}_\| = 2.53$ and $\mathcal{A}_s = 0.125$, for several values of the ratio ω/ω_{lh}, and other parameters as in Fig. 1. It is also seen the quantity $E_0(\overline{N}_\|)/(cB_0)$, as a function of $u_\perp$, for $W_0 = 1.0$ MW and the same values of ω/ω_{lh}. We see the threshold condition still satisfied over a small range of values of $u_\perp$, even for $\omega/\omega_{lh} = 2.5$, while Fig. (2) shows that for this frequency the threshold was not attained even by considering $W_0 = 2.0$ MW.

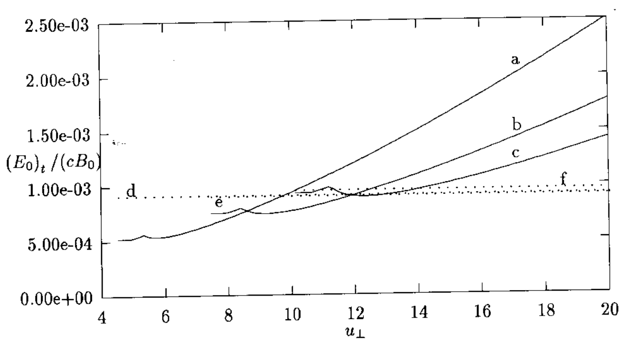

Figure 8. Normalized threshold amplitude $(E_0)_t/(cB_0)$ as a function of $u_\perp$, at $x = 0$ cm, for $\overline{N}_\| = 2.53$ ($v_{res} = 4\ v_e$), $\Delta N_\| = 0.2$, and $\mathcal{A}_s$=0.125, for parameters as those of the JET tokamak. (a) $\omega/\omega_{lh} = 1.5$; (b) $\omega/\omega_{lh} = 2.0$; (c) $\omega/\omega_{lh} = 2.5$. The dotted straight lines (d), (e), and (f) represent the normalized amplitude of the electric field for the central ray, $E_0(\overline{N}_\|)/(cB_0)$, as a function of $u_\perp$, for $W_0 = 1.0$ MW and the same values of ω/ω_{lh}, respectively.

V Summary and conclusions

In the present paper we have carried out an analysis of the threshold condition for stochastic diffusion of energetic ions in a tokamak, due to the interaction with lower hybrid waves of sufficiently high intensity. The tokamak plasma and the lower hybrid wave have been described by a simple model which is intended to contribute for the understanding of basic features related to the stochastic diffusion and its theoretical description.

In the description of the lower hybrid waves we have considered the presence of a spectrum of waves, which has allowed us to evaluate at each position the amplitude of the wave electric field, for a given wave power. However, we have assumed that for a narrow spectra it is possible for certain purposes to describe it approximately as a single wave, and based on this assumption we have analyzed the dependence of the threshold condition on some plasma and wave parameters. We have concluded that, although the stochastic diffusion is possible for present day large tokamaks and for reasonable LH wave power, the threshold condition is not always satisfied in the range of frequency and wave power where it has been observed in some recent experiments. The fact that the lower hybrid induced diffusion has been observed under conditions which do not satisfy the stochastic condition for the individual waves of the wave packet, may be related to the presence of a spectrum of waves instead of a single wave as assumed in the theoretical description which has been utilized, based on the work by Karney [2]. It is indeed conceivable that the effect of the simultaneous presence of waves with different wave numbers would increase the efficiency of the diffusion process. We intend to verify the validity of this conjecture and a possible modification of the threshold condition in our forthcoming investigations on the subject, but we have nevertheless evaluated the threshold value of the wave electric field, for different values of the parameter which controls the threshold condition, as a function of different parameters.

The model developed for the spectrum of LH waves propagating in a tokamak can be utilized to evaluate a diffusion coefficient for stochastic diffusion of energetic ions, to be employed in a quasilinear description of the time evolution of the ion distribution function. We are investigating the subject and intend to publish our findings in a forthcoming publication.

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