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Phase diagrams of three-component attractive ultracold fermions in one dimension

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Abstract. We investigate trions, paired states and quantum phase transitions in one-dimensional SU(3) attractive fermions in external fields by means of the Bethe ansatz formalism. Analytical results for the ground state energy, critical fields and complete phase diagrams are obtained for the weak coupling regime. Higher-order corrections for these physical quantities are presented in the strong attractive regime. Numerical solutions of the dressed energy equations allow us to examine how the different phase boundaries are modified by varying the inter-component coupling throughout the whole attractive regime. The pure trionic phase existing in the strong coupling regime decreases smoothly with a decrease in this coupling, until the weak limit is reached. In this weak regime, a pure Bardeen–Cooper–Schrieffer (BCS)-like paired phase can be sustained under certain nonlinear Zeeman splittings.

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1. Introduction

Recent experiments on ultracold atomic systems confined to one dimension (1D) [1–4] have attracted renewed interest in Bethe ansatz integrable models of interacting bosons and multi-component fermions. The most recent experimental breakthrough is the realization of a one-dimensional (1D) spin-imbalanced Fermi gas of ${}^6\text{Li}$ atoms under the degenerate temperature [5]. This study demonstrates how ultracold atomic gases in 1D may be used to create nontrivial new phases of matter, and also paves the way for direct observation and further study of the Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) states [6, 7].

Three-component fermions exhibit a rich scenario, revealing more exotic phases [8–15]. Notably, strongly attractive three-component ultracold atomic fermions can form three-body bound states called *trions*. Consequently, a phase transition between Bardeen–Cooper–Schrieffer (BCS)-like pairing superfluid and trionic states is expected to occur in the strong attractive regime [10, 13, 16–22]. So far, most of the theoretical analyses have focused on the attractive strong coupling limit. A pertinent discussion in this context is what happens at the intermediate and weak attractive coupling regimes.

In this paper, we consider 1D three-component ultracold fermions with δ -function interaction in external magnetic fields. From a mathematical point of view, this model was solved long ago by Sutherland [23] and Takahashi [24] through Bethe ansatz techniques. Recently, integrable models of three-component interacting fermions [16, 17, 25] received renewed interest in connection with ultracold atomic gases. Advanced experimental techniques newly developed allow us to explore the three-component Fermi gas with different phases of trions, dimers and free atoms [26–29]. Remarkably, the direct observation of a trimer state consisting of fermionic ${}^6\text{Li}$ atoms in the three energetically lowest substates has recently been reported in [30]. This opens up a possibility to experimentally study such novel quantum phases of trions and pairs in 1D three-component Fermi gases, providing a physical ground and stimulus for the investigation of their theoretical aspects.

Our aim here is to expand on the theoretic knowledge of the 1D integrable model of three-component fermions by undertaking a detailed analysis of how the different phases are modified as the inter-component interaction decreases, ranging from a strong to a weak regime. We obtain analytical expressions for the critical fields and construct the full phase diagrams in the weak coupling regime by solving the Bethe ansatz equations (BAE). We extend previous work on this model [16, 19] to derive higher order corrections for these physical quantities in the strong coupling regime. Numerical solutions of the dressed energy equations show that the pure trionic phase existing in the strong coupling regime decreases as the coupling decreases and disappears in the presence of external fields when the weak regime is approached. In contrast to the two-component interacting fermions [31], nonlinear Zeeman splitting may sustain a BCS-like paired phase in the three-component attractive fermions for the weak coupling regime.

2. The model

We consider a δ -function (contact potential) interacting system of N fermions with equal mass m , which can occupy three possible hyperfine levels ($|1\rangle$, $|2\rangle$ and $|3\rangle$) with particle number N^1 , N^2 and N^3 , respectively. They are constrained to a line of length L with periodic boundary

conditions. The Hamiltonian reads [23]

$$H = -\frac{\hbar^2}{2m} \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2} + g_{1D} \sum_{1 \leq i < j \leq N} \delta(x_i - x_j) + E_Z. \quad (1)$$

The first and second terms correspond to the kinetic energy and δ -interaction potential, respectively. The last term denotes the Zeeman energy $E_Z = \sum_{i=1}^3 N^i \epsilon_Z^i(\mu_B^i, B)$, with the Zeeman energy levels ϵ_Z^i determined by the magnetic moments μ_B^i and the magnetic field B . For later convenience, the Zeeman energy term can also be written as $E_Z = -H_1(N^1 - N^2) - H_2(N^2 - N^3) + N\bar{\epsilon}$, where the unequally spaced Zeeman splitting in three hyperfine levels can be specified by two independent parameters $H_1 = \bar{\epsilon} - \epsilon_Z^1(\mu_B^1, B)$ and $H_2 = \epsilon_Z^3(\mu_B^3, B) - \bar{\epsilon}$, with $\bar{\epsilon} = \sum_{\sigma=1}^3 \epsilon_Z^\sigma(\mu_B^\sigma, B)/3$ the average Zeeman energy.

The spin-independent contact interaction g_{1D} remains between fermions with different hyperfine states and preserves the spins in each hyperfine state, i.e. the number of fermions in each spin state is conserved. Although these conditions seem rather restrictive, it is possible to tune scattering lengths between atoms in different low sublevels to form nearly SU(3) degeneracy Fermi gases via broad Feshbach resonances [1, 26, 27, 32]. Consequently, the model still captures the essential physics relevant in the discussion of multiple phases in three-component ultracold Fermi gases. The inter-component coupling $g_{1D} = -\hbar^2 c/m$ with interaction strength $c = -2/a_{1D}$ is determined by the effective 1D scattering length a_{1D} [33]. It is attractive for $g_{1D} < 0$ and repulsive for $g_{1D} > 0$. Here we will focus on the attractive case. For simplicity, we choose the dimensionless units of $\hbar = 2m = 1$ and use the dimensionless coupling constant $\gamma = c/n$ with linear density $n = N/L$.

The Hamiltonian (1) exhibits spin SU(3) symmetry and was solved long ago by means of the nested Bethe ansatz [23, 24]. In this approach its spin content was incorporated via the symmetry of the wavefunction. The energy eigenspectrum is given in terms of the quasimomenta $\{k_i\}$ of the fermions through

$$E = \sum_{j=1}^N k_j^2, \quad (2)$$

satisfying the following set of coupled BAE [23, 24]

$$\begin{aligned} \exp(ik_j L) &= \prod_{\ell=1}^{M_1} \frac{k_j - \Lambda_\ell + ic/2}{k_j - \Lambda_\ell - ic/2}, \\ \prod_{\ell=1}^N \frac{\Lambda_\alpha - k_\ell + ic/2}{\Lambda_\alpha - k_\ell - ic/2} &= -\prod_{\beta=1}^{M_1} \frac{\Lambda_\alpha - \Lambda_\beta + ic}{\Lambda_\alpha - \Lambda_\beta - ic} \prod_{\ell=1}^{M_2} \frac{\Lambda_\alpha - \lambda_\ell - ic/2}{\Lambda_\alpha - \lambda_\ell + ic/2}, \\ \prod_{\ell=1}^{M_1} \frac{\lambda_\mu - \Lambda_\ell + ic/2}{\lambda_\mu - \Lambda_\ell - ic/2} &= -\prod_{\ell=1}^{M_2} \frac{\lambda_\mu - \lambda_\ell + ic}{\lambda_\mu - \lambda_\ell - ic}, \end{aligned} \quad (3)$$

written also in terms of the rapidities for the internal hyperfine spin degrees of freedom Λ_α and λ_μ . Above $j = 1, \dots, N$, $\alpha = 1, \dots, M_1$, $\mu = 1, \dots, M_2$, with quantum numbers $M_1 = N_2 + 2N_3$ and $M_2 = N_3$. For the irreducible representation $[3^{N_3} 2^{N_2} 1^{N_1}]$, a three-column Young tableau

encodes the numbers of unpaired fermions ($N_1 = N^1 - N^2$), bound pairs ($N_2 = N^2 - N^3$) and trions ($N_3 = N^3$).

3. Ground states

For attractive interaction, the BAE allow charge bound states and spin strings. In particular, the SU(3) symmetry carries two kinds of charge bound states: trions and pairs. In principle, different numbers of unpaired fermions, pairs and trions can be chosen to populate the ground state by carefully tuning H_1 and H_2 .

In the strong coupling regime $L|c| \gg 1$, the imaginary parts of the bound states become equal spaced, i.e. a trionic state has the form $\{k_j = \Lambda_j \pm i|c|, \lambda_j\}$ and for the bound pair $\{k_r = \Lambda_r \pm i|c|/2\}$. Substituting these root patterns into the BAE (3), we find their real parts, from which the ground state energy in the strongly attractive regime can be obtained [16]:

$$\begin{aligned} \frac{E}{L} \approx & \frac{\pi^2 n_1^3}{3} \left(1 + \frac{8n_2 + 4n_3}{|c|} + \frac{12}{c^2} (2n_2 + n_3)^2 \right) - \frac{n_2 c^2}{2} \\ & + \frac{\pi^2 n_2^3}{6} \left(1 + \frac{12n_1 + 6n_2 + 16n_3}{3|c|} + \frac{1}{3c^2} (6n_1 + 3n_2 + 8n_3)^2 \right) - 2n_3 c^2 \\ & + \frac{\pi^2 n_3^3}{9} \left(1 + \frac{12n_1 + 32n_2 + 18n_3}{9|c|} + \frac{1}{27c^2} (6n_1 + 16n_2 + 9n_3)^2 \right). \end{aligned} \quad (4)$$

Here $n_a = N_a/L$ ($a = 1, 2, 3$) is the density for unpaired fermions, pairs and trions, respectively. This state can be considered as a mixture of unpaired fermions, pairs and trionic fermions, behaving basically like particles with different statistical signatures [34]. For strong attractive interaction, trions are stable compared to the BCS-like pairing and unpaired states. From (4) we can obtain the binding energy for a trion, given by $\varepsilon_t = \hbar^2 c^2/m$ and the pair binding energy, which is $\varepsilon_b = \hbar^2 c^2/4m$.

In the weak coupling regime $L|c| \ll 1$, the imaginary parts $i\gamma$ of the charge bound states are the roots of Hermite polynomials H_k of degree k . Specifically, $H_k(\sqrt{\frac{L}{2|c|}}y) = 0$, with $k = 2, 3$ for a bound pair and a trion, respectively [35]. The real parts of the quasimomenta deviate smoothly from the values evaluated for the $c = 0$ case. With this root configuration, the ground state energy in the weak attractive regime can be obtained:

$$\begin{aligned} \frac{E}{L} \approx & \frac{\pi^2}{3} (n_1^3 + 2n_2^3 + 3n_3^3) + \pi^2 (n_1(n_2 + n_3)(n_1 + n_2 + n_3) + 2n_2 n_3 (n_2 + n_3)) \\ & - 2|c|(n_1 n_2 + 2n_1 n_3 + 4n_2 n_3 + n_2^2 + 3n_3^2). \end{aligned} \quad (5)$$

The ground state energy (5) is dominated by the kinetic energy of composite particles and unpaired fermions and has an interaction energy consisting of density–density interaction between charge bound states and between charge bound states and unpaired fermions. From equation (5) we can obtain the binding energy for a trion, given by $\varepsilon_t = 3\hbar^2|c|/mL$, and the pair binding energy, which is $\varepsilon_b = \hbar^2|c|/mL$. For weak attractive interaction, the trionic state is unstable against thermal and spin fluctuations. This becomes apparent in the weak coupling phase diagrams presented in figures 1(d), 2(c) and 3(c).

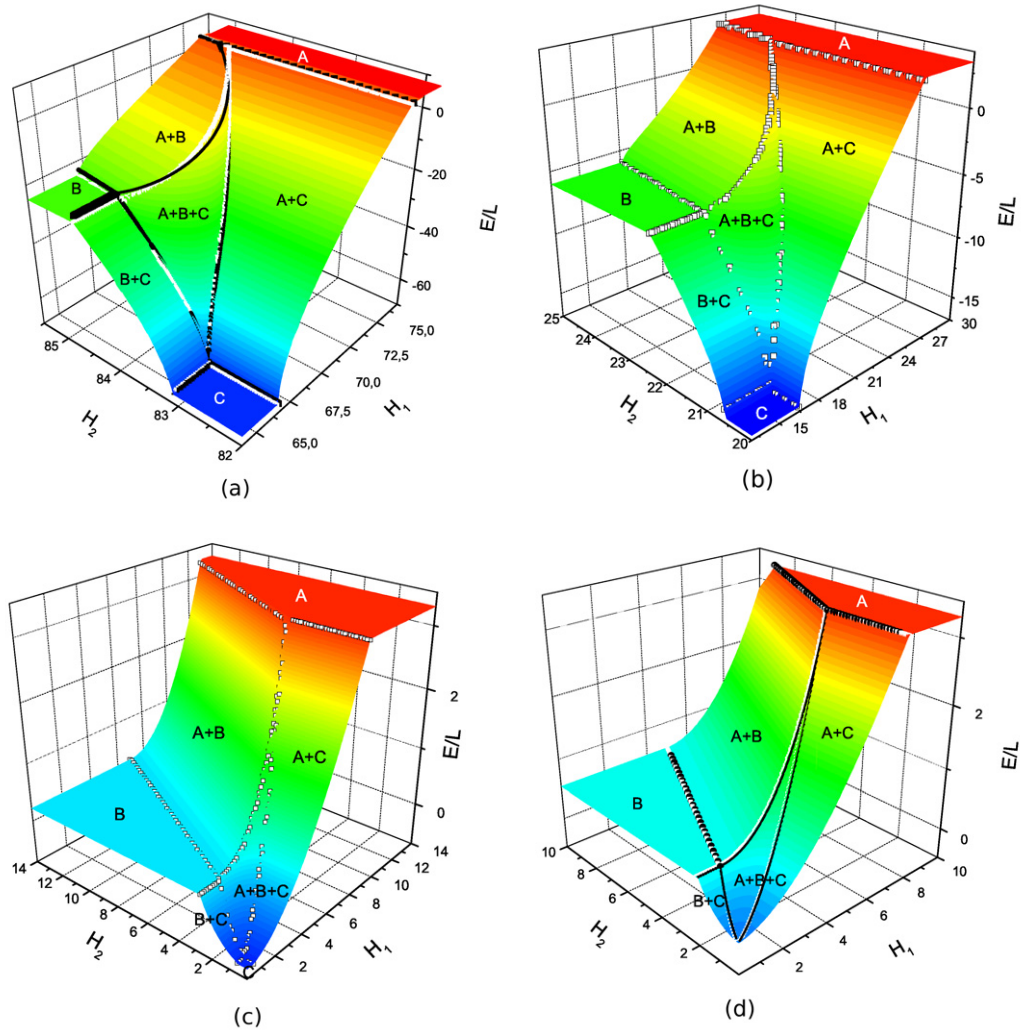


Figure 1. Ground state energy versus Zeeman splitting for different coupling values: (a) strong interaction $|\gamma| = 10$, (b) $|\gamma| = 5$, (c) $|\gamma| = 1$ and (d) weak interaction $|\gamma| = 0.5$. The white dots correspond to the numerical solutions of the dressed energy equations (6). The black lines in (a) and (d) are plotted from the analytical results (10) and (11), respectively. Good agreement was found between the analytical results and the numerical solutions in the strong and weak regimes. The pure trionic phase C , present in the strong coupling regime, decreases smoothly as $|\gamma|$ decreases and is suppressed in the weak limit.

4. Dressed energy formalism

In the thermodynamic limit, i.e. $L, N \rightarrow \infty$ with N/L finite, the grand partition function $Z = \text{tr}(e^{-\mathcal{H}/T}) = e^{-G/T}$ is given in terms of the Gibbs free energy $G = E + E_Z - \mu N - TS$, written in terms of the Zeeman energy E_Z , chemical potential μ , temperature T and entropy S [16, 36–38]. The Gibbs free energy can be expressed in terms of the densities of particles and holes for unpaired fermions, bound pairs and trions, as well as spin degrees of freedom, which

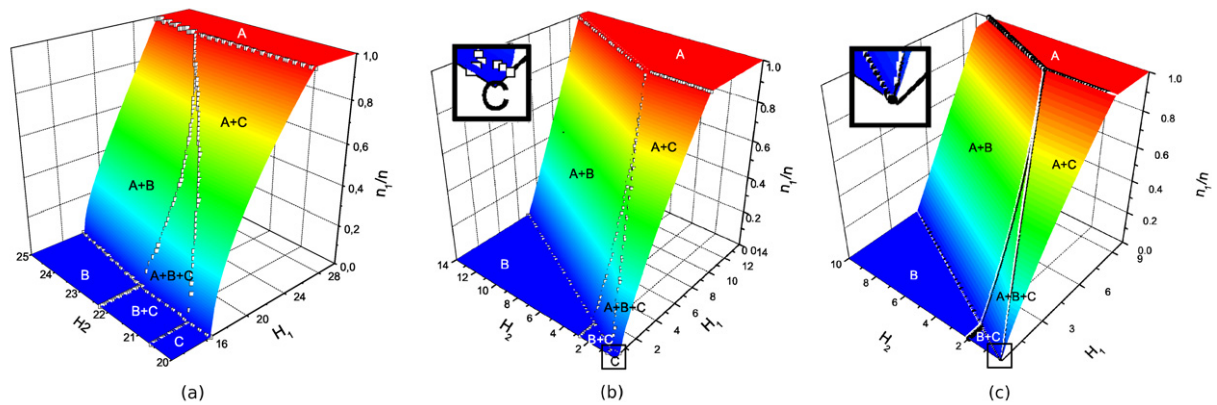


Figure 2. Phase diagram showing the polarization n_1/n versus the fields H_1 and H_2 for different coupling values (a) $|\gamma| = 5$, (b) $|\gamma| = 1$ and (c) weak interaction $|\gamma| = 0.5$. The white dots correspond to the numerical solutions of the dressed energy equations (6). Good agreement is found between the analytical results (11) (black lines) and the numerical solution in the weak regime. At intermediate coupling regimes, the pure trionic phase decreases by decreasing the coupling and it is not present in the weak regime. A zoom around the origin is presented in figure 2(b) (2(c)) to show the presence (absence) of the trionic phase.

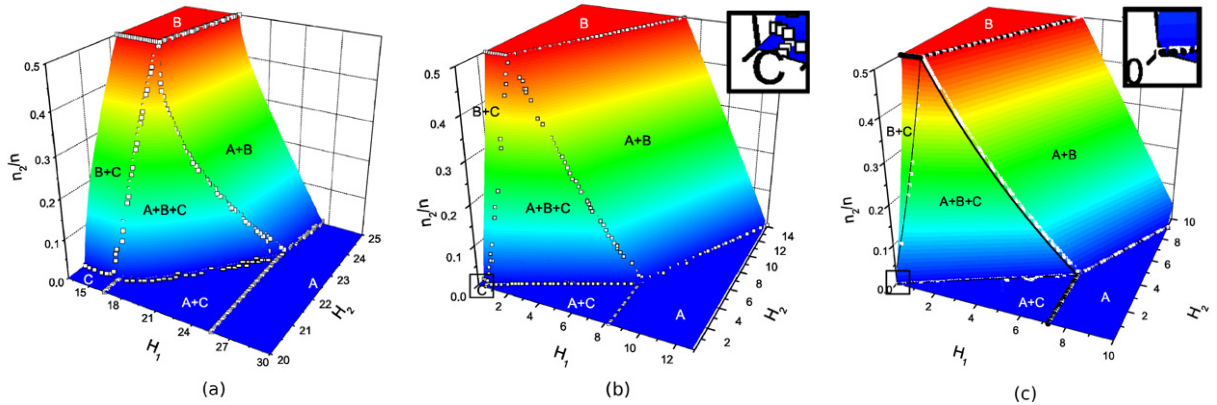


Figure 3. Phase diagram showing the polarization n_2/n versus the fields H_1 and H_2 for different coupling values (a) $|\gamma| = 5$, (b) $|\gamma| = 1$ and (c) weak interaction $|\gamma| = 0.5$. The white dots correspond to the numerical solutions of the dressed energy equations (6). The black lines plotted from the analytical results (11) are in good agreement with the numerical results in the weak regime. At intermediary coupling regimes, the pure trionic phase decreases by decreasing the coupling and it is not present in the weak regime. A zoom around the origin is presented in figure 3(b) (3(c)) to show the presence (absence) of the trionic phase.

are determined from the BAE (3). Thus the equilibrium state is established by minimizing the Gibbs free energy with respect to these densities. This procedure leads to a set of coupled nonlinear integral equations, from which the dressed energy equations are obtained in the limit $T \rightarrow 0$ (see [16, 38] for details)

$$\begin{aligned}\epsilon^{(3)}(\lambda) &= 3\lambda^2 - 2c^2 - 3\mu - a_2 * \epsilon^{(1)}(\lambda) - [a_1 + a_3] * \epsilon^{(2)}(\lambda) - [a_2 + a_4] * \epsilon^{(3)}(\lambda), \\ \epsilon^{(2)}(\Lambda) &= 2\Lambda^2 - 2\mu - \frac{c^2}{2} - H_2 - a_1 * \epsilon^{(1)}(\Lambda) - a_2 * \epsilon^{(2)}(\Lambda) - [a_1 + a_3] * \epsilon^{(3)}(\Lambda), \\ \epsilon^{(1)}(k) &= k^2 - \mu - H_1 - a_1 * \epsilon^{(2)}(k) - a_2 * \epsilon^{(3)}(k).\end{aligned}\quad (6)$$

Here $\epsilon^{(a)}$, $a = 1, 2, 3$ are the dressed energies for unpaired fermions, bound pairs and trions, respectively, and $a_j(x) = \frac{1}{2\pi} \frac{j|c|}{(jc/2)^2 + x^2}$. The symbol ‘*’ denotes the convolution $a_j * \epsilon^{(a)}(x) = \int_{-Q_a}^{+Q_a} a_j(x-y)\epsilon^{(a)}(y) dy$ with the integration boundaries Q_a given by $\epsilon^{(a)}(\pm Q_a) = 0$. The Gibbs free energy per unit length at zero temperature can be written in terms of the dressed energies as $G = \sum_{a=1}^3 \frac{a}{2\pi} \int_{-Q_a}^{+Q_a} \epsilon^{(a)}(x) dx$.

The dressed energy equations (6) can be analytically solved just in some special limits. In particular, they were solved in [16] for strongly attractive interaction through a lengthy iteration method. Here we numerically solve these equations to determine the full phase diagram of the model for any value of the coupling. This allows us to examine how the different phase boundaries deform by varying the coupling from the strong to weak regime. The numerical solution is also employed to confirm the analytical expressions for the physical quantities and the resulting phase diagrams of the model in the weak coupling limit (for a similar discussion in the strong regime see [19]).

5. Full phase diagrams

Basically, there are two possible Bethe ansatz schemes to construct the phase diagram of the system. One possibility is to handle it with the dressed energy equations (6). This approach was discussed for the strong attractive regime in [16], where expressions for the fields in terms of the densities were obtained up to the order of $1/|c|$. Alternatively, one can handle it directly with its discrete version (equations (2) and (3)) by solving the BAE. We adopt this second strategy here. In order to obtain the explicit forms for the fields in terms of the polarizations, we consider the energy for arbitrary population imbalances

$$E/L = \mu n + G/L + n_1 H_1 + n_2 H_2, \quad (7)$$

which coincides with the ground state energy (2) obtained by solving the BAE (3). Then the fields H_1 and H_2 are determined through the relations

$$H_1 = \frac{\partial E/L}{\partial n_1}, \quad H_2 = \frac{\partial E/L}{\partial n_2} \quad (8)$$

together with the constraint

$$n = n_1 + 2n_2 + 3n_3. \quad (9)$$

In the strong coupling regime, using the ground state energy (4) we find

$$\begin{aligned}
H_1 = & \pi^2 n_1^2 \left(1 - \frac{4n_1}{9|c|} + \frac{8n_2}{|c|} + \frac{4n_3}{|c|} + \frac{12}{c^2} (2n_2 + n_3)^2 - \frac{8n_1}{3c^2} (2n_2 + n_3) \right) \\
& - \frac{\pi^2 n_3^2}{9} \left(1 + \frac{4n_1}{3|c|} + \frac{32n_2}{9|c|} + \frac{4n_3}{3|c|} + \frac{(6n_1 + 16n_2 + 9n_3)^2}{27c^2} - \frac{2n_3(6n_1 + 16n_2 + 9n_3)}{9c^2} \right) \\
& + \frac{10\pi^2 n_2^3}{27|c|} \left(1 + \frac{6n_1 + 3n_2 + 8n_3}{|c|} \right) + \frac{2c^2}{3}, \\
H_2 = & \frac{\pi^2 n_2^2}{2} \left(1 + \frac{4n_1}{|c|} + \frac{40n_2}{27|c|} + \frac{16n_3}{3|c|} + \frac{(6n_1 + 3n_2 + 8n_3)^2}{3c^2} - \frac{14n_2(6n_1 + 3n_2 + 8n_3)}{27c^2} \right) \\
& - \frac{2\pi^2 n_3^2}{9} \left(1 + \frac{4n_1}{3|c|} + \frac{32n_2}{9|c|} + \frac{8n_3}{9|c|} + \frac{(6n_1 + 16n_2 + 9n_3)^2}{27c^2} \right. \\
& \left. - \frac{10n_3(6n_1 + 16n_2 + 9n_3)}{27c^2} \right) + \frac{16\pi^2 n_1^3}{9|c|} \left(1 + \frac{6(2n_2 + n_3)}{|c|} \right) + \frac{5c^2}{6}. \tag{10}
\end{aligned}$$

These equations provide higher order corrections to those derived in [16] using the dressed energy equations. To determine the full phase boundaries, we also need the energy–field transfer relation between the paired and unpaired phases $H_1 - H_2/2$, which can be extracted from the underlying two-component system with SU(2) symmetry [31, 34]. These equations determine the full phase diagram and the critical fields activated by the fields H_1 and H_2 .

Figure 1(a) shows the ground state energy versus Zeeman splitting parameters H_1 and H_2 determined from equation (4) with the densities n_1 and n_2 obtained from (10). There are three pure phases: an unpaired phase A , a pairing phase B and a trion phase C and four different mixtures of these states. For small H_1 , a transition from a trionic state into a mixture of trions and pairs occurs as H_2 exceeds the lower critical value H_2^{c1} . When H_2 is greater than the upper critical value H_2^{c2} , a pure pairing phase takes place. Trions and BCS-like pairs coexist when $H_2^{c1} < H_2 < H_2^{c2}$. These critical fields, derived from equation (10), are given by $H_2^{c1} \approx n^2 \left(\frac{5\gamma^2}{6} - \frac{2\pi^2}{81} \left(1 + \frac{8}{27|\gamma|} - \frac{1}{27\gamma^2} \right) \right)$ and $H_2^{c2} \approx n^2 \left(\frac{5\gamma^2}{6} + \frac{\pi^2}{8} \left(1 + \frac{20}{27|\gamma|} - \frac{1}{36\gamma^2} \right) \right)$. The phase transitions from $B \rightarrow A + B \rightarrow A$ induced by increasing H_1 are reminiscent of those in the two-component systems [34, 41]. Basically, in this region the highest level is far away from the other two levels, so the system reduces to the spin-1/2 fermion case. The mixed phase containing BCS-like pairs and unpaired fermions can be called an FFLO phase. We mention that a discussion on the pairing nature of 1D many-body systems can be found in, for instance, [39, 40]. For small H_2 , a phase transition from a trionic to a mixture of trions and unpaired fermions occurs. Using equation (10), we find that the trionic state with zero polarization $n_1/n = 0$ forms the ground state when the field $H < H_1^{c1}$, where $H_1^{c1} \approx n^2 \left(\frac{2\gamma^2}{3} - \frac{\pi^2}{81} \left(1 + \frac{4}{9|\gamma|} + \frac{1}{9\gamma^2} \right) \right)$. When H_1 is greater than the upper critical value $H_1^{c2} \approx n^2 \left(\frac{2\gamma^2}{3} + \pi^2 \left(1 - \frac{4}{9|\gamma|} \right) \right)$, all trions are broken and the state becomes a normal Fermi liquid.

At intermediate coupling regimes, it is not possible to construct the full phase diagrams analytically. However, they can be determined by numerically solving the dressed energy equations (6), as illustrated in figures 1(b) and (c), figures 2(a) and (b) and figures 3(a) and (b),

for the intermediate values of the coupling $|c| = 5$ and $|c| = 1$, respectively. The different phase boundaries are modified slightly by varying the inter-component coupling through the whole attractive regime. In particular, the pure trionic phase existing in the strong coupling regime decreases smoothly by decreasing this coupling until it is completely suppressed. A careful numerical analysis of the phase diagrams for $n = 1$ and different values of $|c|$ between $|c| = 1$ and $|c| = 0.5$ indicates that the critical coupling value at which the trionic phase disappears is around $c_c \approx 0.6$. Other mixed phases involving trions, especially the phase $(B + C)$, also decrease by decreasing $|c|$.

In the weak coupling regime, we obtain the expressions between the fields and the polarizations using equations (5), (8) and (9):

$$\begin{aligned} H_1 &= \frac{\pi^2}{3}(2n_1^2 + n_2^2 + 4n_1n_2 + 4n_1n_3 + 2n_2n_3) + \frac{2|c|}{3}(2n_1 + n_2), \\ H_2 &= \frac{\pi^2}{3}(n_1^2 + 2n_2^2 + 2n_1n_2 + 2n_1n_3 + 4n_2n_3) + \frac{2|c|}{3}(2n_2 + n_1). \end{aligned} \quad (11)$$

These equations together with the energy–field transfer relation $H_1 - H_2/2$ determine the full phase diagram and the critical fields activated by the Zeeman splitting H_1 and H_2 . We observe that the density of trions n_3 does not appear independently in equations (11), in contrast to the corresponding equations in the strong regime (10). Figure 1(d) presents the ground state energy versus the fields H_1 , H_2 , while figures 2(c) and 3(c) show the polarizations n_1/n and n_2/n in terms of Zeeman splitting, respectively. Now in the weak coupling regime there are just six different phases in the H_1 – H_2 plane: we observe the disappearance of the pure trionic phase C in the presence of the fields, i.e. the trionic state is unstable against thermal and spin fluctuations. This behavior is in contrast to the strong coupling regime, where the phase C is robust and trion states populate the ground state for a considerable interval of the fields. In addition, the phase where trions and pairs coexist $(B + C)$ decreases significantly compared to the strong coupling regime. Interestingly, in contrast to the weak attractive spin-1/2 fermion system, a pure paired phase can be sustained under certain Zeeman splittings. For certain tuning H_1 and H_2 , the two lowest levels are almost degenerate. Therefore, the paired phase naturally occurs and is stable. The persistence of this phase is relevant for the investigation of phase transition between BCS-like pairs and FFLO states. All these boundary modifications occur smoothly, as shown by a numerical analysis of the phase diagrams for different values of the coupling across all regimes. This indicates that all phase transitions in the vicinity of critical points are second order. This conclusion is consistent with previous analytical results [34, 41]. We also mention that quantum phase transitions between different superfluid phases have been discussed in [42].

We perform a similar analysis as in the previous strong case to extract the critical fields. Since the trionic phase C disappears for non-vanishing fields, less critical fields are found compared to the strong coupling case. For small H_1 a transition from a mixture of trions and pairs to a pure paired phase occurs as H_2 exceeds the critical value $H_2^c \approx n^2(\frac{2\gamma}{3} + \frac{\pi^2}{6})$. The transition from a mixture of trions and unpaired fermions to a normal Fermi liquid phase occurs as H_1 exceeds the critical value $H_1^c \approx n^2(\frac{4\gamma}{3} + \frac{2\pi^2}{3})$. The phase transitions $B \rightarrow A + B \rightarrow A$ are reminiscent of those in spin-1/2 fermion systems. However, in the weak attractive two-component case the pure BCS-like paired phase is suppressed [31] and consequently it is not possible to investigate the phase separation between a BCS-like paired phase and an FFLO state, in contrast to the three-component case, where this study is still possible in a weak regime.

6. Conclusion

We have studied the three-component attractive 1D Fermi gas in external fields through the Bethe ansatz formalism. New results for the critical fields and complete zero-temperature phase diagrams have been presented for the weak coupling regime. Previous work on this model has been extended to derive higher order corrections to these physical quantities in the strong regime. We have further confirmed that the system exhibits exotic phases of trions, bound pairs, a normal Fermi liquid and a mixture of these phases in the strongly attractive limit. We have also shown how the different phase boundaries deform by varying the inter-component coupling across the whole attractive regime. In particular, the trionic phase that may occur in the strong coupling regime for certain values of the Zeeman splittings decreases smoothly by decreasing the coupling, until the weak limit is approached, when the trionic phase is suppressed. Interestingly, in the weak regime, a pure paired phase can be maintained under certain nonlinear Zeeman splittings, in contrast to the two-component attractive 1D Fermi gas. Our high precision of critical phase boundaries paves the way for further investigation of quantum criticality in the three-component interacting Fermi gas through the finite-temperature Bethe ansatz.

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