Non-resonant Permittivity Measurement Methods

Sergio L. S. Severo, Álvaro A. A. de Salles,
Federal Institute of Science and Technology – IFSUL, Pelotas – RS, Brazil, sergiosevero@pelotas.ifsul.edu.br,
Electrical Engineering, Federal University of Rio Grande do Sul – UFRGS, Porto Alegre, RS, Brazil,
aasalles@ufrgs.br
Bruno Nervis, Braian K. Zanini
Electrical Engineering, Federal University of Rio Grande do Sul – UFRGS, Porto Alegre, RS,
bruno.nervis@ufrgs.br, braian.zanini@ufrgs.br

Abstract— **The measurement of the dielectric properties of materials has been applied in non-destructive tests, humidity measurement, soil analysis and even cancer detection. The methods have been developed for over 70 years based on the interaction of the electromagnetic waves with the material under test. This work presents a general model of scattering parameters for non-resonant methods of transmission/reflection and single-port reflection. Equations for determining permittivity are obtained. New equations for short-circuited load and coupled load in the double reflection method are presented.**

Index Terms— Microwave measurements, permittivity, short-circuit transmission line method, transmission / reflection method.

I. INTRODUCTION

The dielectric properties characterization is fundamental in engineering. This is employed in non-destructive test and evaluation [1], moisture measurements [2], soil analysis and tumor tissue detection. The physical concepts and technological aspects are related to determining the dielectric properties from the interaction of the electromagnetic fields with the material. These fields must be generated, guided or radiated over the sample (MUT – material under test) and detected after the interaction. Traditionally, these tasks were performed using microwave instrumentation techniques in laboratory [3]. Simultaneously, measurements methods [4] and mathematical methods for propagation, radiation and scattering of microwaves were developed [5]. These methods can be divided in resonant and non-resonant. The non-resonant methods are suitable for broadband measurement. Among these methods the most important ones are the SCTL (short-circuit transmission line) [3] and the NRW (Nicholson-Ross-Weir) [6][7]. The purpose of this work is to generate explicit equations for the permittivity using a straightforward scattering parameters model for load-terminated samples.

II. PERMITTIVITY MEASUREMENT METHODS

A. Historical Development

In 1946, Roberts and Von Hippel [8] presented a method for the measurement of permittivity using an air-filled rectangular waveguide with a sample of MUT in the end of the waveguide. By

comparing the standing wave pattern of the partially sample-filled waveguide and that of a short-terminated air-filled waveguide it is possible to determine the permittivity. Such method is known as SCTL reflection method. It obtains the line impedance from the peaks and valleys of the voltage standing wave pattern. This method was still widely used in 1961, when [9] reports uncertainties of 2% for the permittivity and 5% for the loss tangent. The use of charts for hyperbolic functions was avoided by having sample lengths of ¼ e ½ of the wavelength inside the material. In 1974, a computer program was developed aiming to increase the precision of the Roberts-von Hippel method [10].

In 1970, Nicolson and Ross [6], using a sampling oscilloscope, a sub nanosecond pulse generator and the Fourier transform, obtained the scattering parameters of a sample. With S_{11} and S_{21}, expressed as functions of the reflection coefficient in the material-air interface and the transmission coefficient between two faces of the sample, and measured by the aforementioned setup, they obtained the permittivity and permeability of the material. In 1974, Weir [7] obtained the scattering parameters directly from the frequency domain by using an automatic network analyzer, solving the phase ambiguity generated by larger than half wavelength sample length and measuring the group delays in different frequencies, assuming that the permittivity does not change significantly for small variations in frequency. In [11], the problems of the method in dispersive materials are discussed. Regardless of these problems, the method is widely accepted and known as Nicholson-Ross-Weir (NRW) algorithm.

An explicit equation for the permittivity as a function of the transmission and reflection parameters is presented in [12]. The authors show that it is possible to obtain the uncertainty of the permittivity as a function of the sample length, with the lowest uncertainty being obtained when the sample length is a quarter of the wavelength inside the material. The method becomes unstable when the sample length is a multiple of half wavelength.

The resonant methods are inadequate for characterization in the frequency domain. The reflection methods, also known as single-port methods, which measure the reflection coefficient of a guided wave or a wave in free-space [13], can be used for spectroscopy. In [14] such methods are reviewed and possible configurations for the measurements are presented. Among these, the method with two arbitrary terminations can be highlighted. In the reflection methods, the explicit equations for the permittivity are obtained through two or more measurements in two different configurations, as it is shown in [15].

B. Transmission-Reflection methods state-of-the-art

The work in simultaneous measurement of transmission and reflection coefficients of a sample to obtain permittivity is consolidated in [16], in which explicit equations independent of reference plane or sample length are shown. The half wavelength uncertainty is discussed and the measurement uncertainties are determined. Works aiming to solve the half wavelength problem are also referenced. In [17] a new method to solve problems related to dispersive materials is presented. A complete

review regarding the transmission-reflection methods is also done in [17].

C. Single-port reflection methods

  Reflection methods which employ the measurement of two reflection coefficients were already presented in [3]. These use which uses a short-terminated transmission line (as the SCTL method) and an open-circuit terminated transmission line. Although the equation for the permittivity is simple [14], the method only works at specific frequencies since, to create an open circuit, it is necessary to create a short-circuit at a quarter-wavelength distance. In [15] an explicit equation with the S_{11} parameter (measured with a coupled load or free-space and a short-circuit) is shown. Other approach is described in [18], using only the amplitude of the reflection coefficient. Two distant frequencies (in non-dispersive media) or three near frequencies (in dispersive media) can be used. The simplicity of the required instrumentation makes the method very attractive.

III. Dielectric slab scattering parameters model

A. Reflection coefficient Γ and propagation factor T

  Consider an uniform dielectric slab, with complex permittivity "ε_2" and thickness "d" immersed in a dielectric with permittivity ε_1 to the left and ε_3 to the right, splitting the space into regions 1 and 3, as shown in *Fig*. 1.
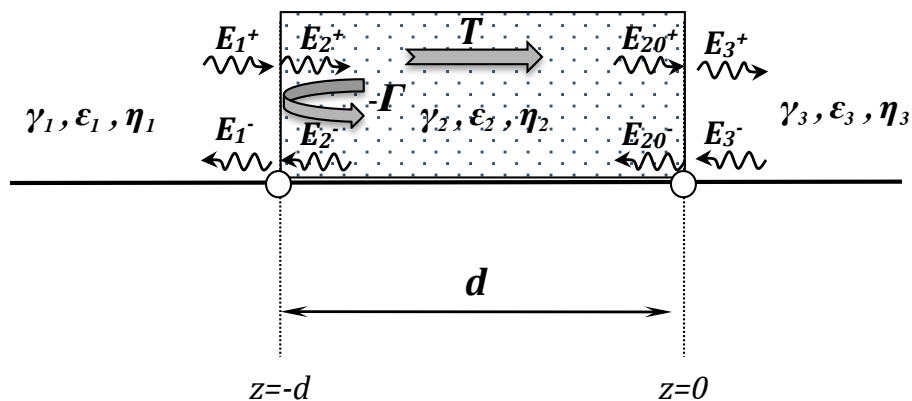

Fig. 1. Sample electromagnetic wave interaction

  Let us assume an electromagnetic wave, which is perpendicularly incident at the interface ($z=-d$). The incident electric and magnetic waves at the interface are E_1^+ and H_1^+, respectively. Both are parallel to the interface and are partially reflected to the medium 1 and partially transmitted to the interior of the slab. E_1^- and H_1^- are the reflected waves, which travel in the medium 1 in the negative z direction. From $z=-d$, the transmitted waves E_2^+ and H_2^+ travel in the positive z direction. On the interface between the slab and the medium 3 ($z=0$) there are the fields E_{20}^+ and H_{20}^+. These fields are partially transmitted to medium 3, indicated as waves E_3^+ and H_3^+ and partially reflected back to medium 2, the waves E_{20}^- and H_{20}^-. The propagation constants of the materials are γ_1, γ_2 e γ_3. The electric and magnetic fields are related in each medium by the intrinsic impedance of these media: η_1,

η_2 e η_3. If the medium 3 is infinite, there will be no propagation in the negative z direction in this medium (no reflection) and E_3^-=0. If medium 1 is vacuum and medium 3 has the same permittivity of medium 2, the value of **Γ**, *the reflection coefficient at the interface*, is given by:

$$\Gamma = \frac{E_1^-}{E_1^+} = \frac{\sqrt{\dfrac{\mu_r}{\varepsilon_r}} - 1}{\sqrt{\dfrac{\mu_r}{\varepsilon_r}} + 1} \tag{1}$$

where μ_r and ε_r are the relative permeability and permittivity of medium 2, respectively.

If the media have the magnetic permeability of vacuum ($\mu_r = 1$), the coefficient is simplified to:

$$\Gamma = \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} \tag{2}$$

The propagation constant γ in a dielectric with negligible conductivity and with magnetic permeability equal to the vacuum can be approximated to

$$\gamma \cong j\frac{\omega}{c}\sqrt{\varepsilon_r} \tag{3}$$

where c is the velocity of light in the vacuum. Therefore, the propagation of a TEM (transversal electromagnetic) wave through the distance d in a material with the propagation constant γ can be expressed by the propagation factor T [7]. Using (3) is possible to define:

$$T \triangleq e^{-\gamma d} = e^{-j\frac{\omega}{c}\sqrt{\varepsilon_r}\, d} \tag{4}$$

Some authors call T the "transmission coefficient" [4]. To avoid confusion with the transmission coefficient through a slab, the original term "propagation factor" will be kept.

B. MUT (Material Under Test) scattering parameters.

The intrinsic impedance variation between the two different media will result in that part of the incident wave to be transmitted and part of it to be reflected. In a dielectric slab, as shown in *Fig.* 2, there are two interfaces and then multiple reflections will happen inside the slab. Using harmonic analysis, this is simplified in the case of a high loss sample, because the multiple reflections add up to an attenuated standing wave pattern.
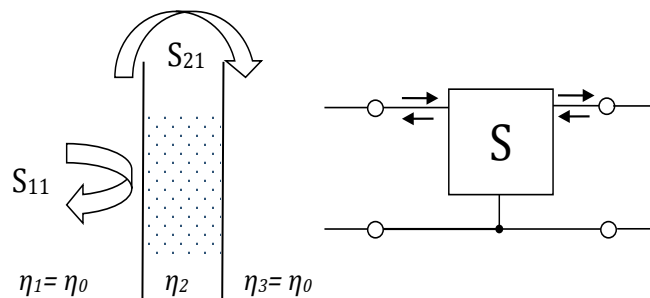

Fig. 2. MUT scattering parameters

The total fields can be obtained through the complete solution of the wave equation inside the slab

when the dielectric characteristics of the material and the sample dimensions are known. The total field, for 0 > z > -d, is:

$$E_2^t(z) = E_{20}^+ e^{-\gamma_2 z} + E_{20}^- e^{\gamma_2 z} \tag{5}$$

The reflection coefficient at the interface between the two media, when the sample is infinite, allows for the substitution of E_{20-} through Γ since:

$$\Gamma = \frac{E_1^-}{E_1^+} = -\frac{E_{20}^-}{E_{20}^+} \tag{6}$$

$$E_2^t(z) = E_{20}^+ \left(e^{-\gamma_2 z} - \Gamma e^{\gamma_2 z} \right) \tag{7}$$

The same procedure is applicable to the magnetic field. The total magnetic field can be written as:

$$H_2^t(z) = H_{20}^+ e^{-\gamma_2 z} + H_{20}^- e^{\gamma_2 z} \tag{8}$$

Since the magnetic fields are related to the electric fields through the intrinsic impedance of the medium, (8) can be rewritten as:

$$H_2^t(z) = \frac{E_{20}^+}{\eta_2} e^{-\gamma_2 z} - \frac{E_{20}^-}{\eta_2} e^{\gamma_2 z} \tag{9}$$

Applying (6) in (9):

$$H_2^t(z) = \frac{E_{20}^+}{\eta_2} \left(e^{-\gamma_2 z} + \Gamma e^{\gamma_2 z} \right) \tag{10}$$

Since the electrical and magnetic fields are tangential to the interface, it is possible to write, for z=-d:

$$E_2^t = E_1^+ + E_1^- \tag{11}$$

$$H_2^t = H_1^+ + H_1^- = \frac{E_1^+}{\eta_1} - \frac{E_1^-}{\eta_1} \tag{12}$$

Considering the propagation factor T along the slab, the total fields at z = -d, obtained from (7) and (10), are:

$$E_2^t(z = -d) = E_{20}^+ \left(T^{-1} - \Gamma T \right) \tag{13}$$

$$H_2^t(z = -d) = \frac{E_{20}^+}{\eta_2} \left(T^{-1} + \Gamma T \right) \tag{14}$$

From (11) (13) and (12) (14), the boundary conditions allow to write:

$$E_1^+ + E_1^- = E_{20}^+ \left(T^{-1} - \Gamma T \right) \tag{15}$$

$$E_1^+ - E_1^- = \frac{\eta_1}{\eta_2} E_{20}^+ \left(T^{-1} + \Gamma T \right) \tag{16}$$

Assume that the incident electric field in the slab at $z=-d$ is E_1^+. The reflected electric field is E_1^-.

Since the media 1 and 3 have the same intrinsic impedance, a scattering parameters model can be applied.

Therefore, the reflection coefficient of the slab, as seen by the incident wave (input), will be the S_{11} parameter itself, or:

$$S_{11} = \frac{E_1^-}{E_1^+}$$ (17)

By expressing E_1^- as $S_{11}E_1^+$ and dividing (16) by (15):

$$\frac{(1-S_{11})}{(1+S_{11})} = \frac{\eta_1}{\eta_2}\frac{(T^{-1}+\Gamma T)}{(T^{-1}-\Gamma T)}$$ (18)

If $\eta_1 = \eta_3 = \eta_0$ and the medium 2 is non-magnetic, the ratio η_1 / η_2 is equal to the square root of the relative dielectric permittivity of the medium 2. From (2), is possible to isolate this square root as a function of Γ and then obtain the reflection coefficient at the input of the slab:

$$S_{11} = \frac{\Gamma(1-T^2)}{(1-\Gamma^2 T^2)}$$ (19)

The relation between the incident electromagnetic wave at the interface at z=-d and the emerging wave at the interface at z=0, when the medium 3 is equal to the medium 1 is the parameter S_{21} itself:

$$S_{21} = \frac{E_3^+}{E_1^+}$$ (20)

At the interface z = 0, the total tangential fields must be equal in both sides. For the electric fields, assuming that there are no fields traveling in the negative *z* direction in the medium 3:

$$E_3^+ = E_{20}^+ + E_{20}^-$$ (21)

Γ relates the fields E_{20}^+ e E_{20}^-, therefore:

$$E_3^+ = E_{20}^+(1-\Gamma)$$ (22)

Substituting E_{20}^+, from (22), and E_3^+ by $S_{21}E_1^+$ in (20), into (15) and (16) and replacing the ratio between the characteristic impedances by the reflection coefficient Γ:

$$E_1^+ + E_1^- = \frac{S_{21}E_1^+}{(1-\Gamma)}(T^{-1}-\Gamma T)$$ (23)

$$E_1^+ - E_1^- = \frac{(1-\Gamma)}{(1+\Gamma)}\frac{S_{21}E_1^+}{(1-\Gamma)}(T^{-1}+\Gamma T)$$ (24)

Adding (23) and (24), eliminating E_1^- e E_1^+ it is possible to isolate the transmission coefficient through the slab:

$$S_{21} = \frac{T(1-\Gamma^2)}{1-\Gamma^2 T^2}$$ (25)

Given that the dielectric slab is symmetrical and the material is isotropic and homogeneous, the scattering parameters matrix is completely specified by making $S_{22} = S_{11}$ and $S_{12} = S_{21}$.

IV. SIMPLE MODEL FOR NON-RESONANT METHODS

A. NRW algorithm

Consider a sample, as shown in Fig. 3, inside a coaxial cable with a termination impedance connected immediately after the sample ($d_L = 0$) or the medium 3 with an intrinsic impedance different of that of the medium 1 in free-space. In both cases, it is possible to model the system as a slab represented by its scattering parameters and loaded by impedance Z_L or an infinite medium of intrinsic impedance η_L.
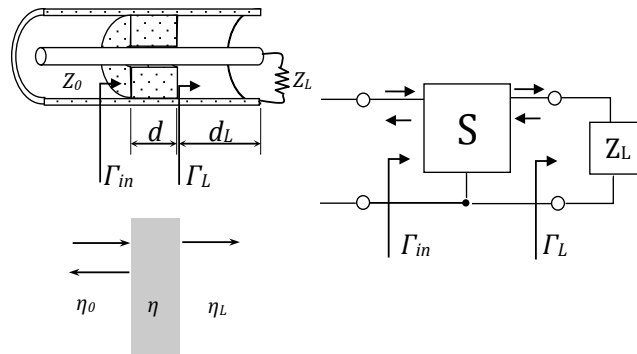

Fig. 3. MUT in transmission line and free space with load.

The reflection coefficient at the input can be obtained from the scattering parameters and the reflection coefficient at the load from [19] [20]:

$$\Gamma_{in} = \Gamma_L \frac{S_{11} - \Delta_s \Gamma_L}{1 - S_{22}\Gamma_L} \tag{26}$$

Where $\Delta_S = S_{11}S_{22} - S_{12}S_{21}$. Considering the sample reciprocity:

$$\Gamma_{in} = \frac{S_{11} - \left(S_{11}^2 - S_{21}^2\right)\Gamma_L}{1 - S_{11}\Gamma_L} \tag{27}$$

If the load impedance is made equal to the characteristic impedance of the line loaded with the sample, the reflection coefficient at the input will be that of an infinite sample. This is due to the fact that, without reflection at the second interface, the wave will only exist in the positive direction from the input of the sample. Any load or infinite slab with the same impedance as the medium being measured will present the same reflection coefficient Γ when considered in relation to the input medium. From these considerations, it follows that if the substitution $\Gamma_{in} = \Gamma_L = \Gamma$ is done in (27), it is possible to obtain the reflection coefficient at the interface as a function of the slab scattering parameters:

$$\Gamma\left(1 - S_{11}\Gamma\right) = S_{11} - \left(S_{11}^2 - S_{21}^2\right)\Gamma \quad \therefore \quad \Gamma^2 - \frac{\left(S_{11}^2 - S_{21}^2 + 1\right)}{S_{11}}\Gamma + 1 = 0$$

$$\Gamma = \frac{S_{11}^2 - S_{21}^2 + 1 \ \pm \sqrt{\left(S_{11}^2 - S_{21}^2 + 1\right)^2 - 4S_{11}^2}}{2S_{11}} \tag{28}$$

The sign in (28) must be chosen in a way that $|\Gamma| \le 1$. Defining:

$$K \triangleq \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}} \tag{29}$$

Equation (28) can then be written as:

$$\Gamma = K \ \pm \sqrt{K^2 - 1} \tag{30}$$

Equations (29) and (30) are presented in [6] and [7] as a fundamental part of the NRW algorithm. The scattering matrix relates the electric fields in the sample, as shown in Fig. 1, in the form:

$$E_1^- = S_{11}E_1^+ + S_{12}E_3^-$$
$$E_3^+ = S_{21}E_1^+ + S_{22}E_3^-$$

The incident field in the load is E_3^+ and the reflected is E_3^-. The reflection coefficient at the load, Γ_L is given by:

$$\Gamma_L = \frac{E_3^-}{E_3^+} \tag{31}$$

Since the system is symmetrical $S_{22} = S_{11}$ and the material is isotropic and homogeneous, then $S_{12} = S_{21}$. From (31), $E_3^- = \Gamma_L E_3^+$, the above equation system can be written as:

$$E_1^- = S_{11}E_1^+ + S_{21}\Gamma_L E_3^+$$
$$E_3^+ = S_{21}E_1^+ + S_{11}\Gamma_L E_3^+$$

We can add these two equations and obtain an equivalent system with the same solution. The sum result is that:

$$E_3^+ + E_1^- = \left(S_{11} + S_{21}\right)E_1^+ + \left(S_{21} + S_{11}\right)\Gamma_L E_3^+ \tag{32}$$

Since in the proposed situation, there is not a reflected wave inside the sample and $\Gamma_{in} = \Gamma_L = \Gamma$, it follows:

$$\Gamma = \frac{E_1^-}{E_1^+} \tag{33}$$

Similarly, the propagation factor is given by:

$$T = \frac{E_3^+}{E_1^+} \tag{34}$$

From these equations we can evaluate expressions for E_3^+ e E_1^-, which are substituted in (32), with $\Gamma_L = \Gamma$:

$$TE_1^+ + \Gamma E_1^+ = \left(S_{11} + S_{21}\right)E_1^+ + \left(S_{21} + S_{11}\right)\Gamma T E_1^+$$

then:

$$T = \frac{S_{11} + S_{21} + \Gamma}{1 - \left(S_{11} + S_{21}\right)\Gamma} \tag{35}$$

Equation (35) shows the propagation factor as a function of the scattering parameters and the reflection coefficient at the interface presented in [5] and [6]. The NRW algorithm to determine the permittivity from the MUT scattering parameters is fulfilled when (2) is considered and then:

$$\varepsilon_r = \left(\frac{1 - \Gamma}{1 + \Gamma}\right)^2 \tag{36}$$

And from (4):

$$\varepsilon_r = \left(j\frac{c}{\omega d}\ln(T)\right)^2 \tag{37}$$

The equations (36) and (37), isolated or combined, can be used for permittivity determination [21]. The use of (36), as described in [12] will result in a permittivity explicit expression, independent of the sample size. However this leads to indeterminations when the sample length is a multiple of half wavelength in low loss materials. The authors conducted an uncertainty analysis as a function of the permittivity of the measured material and of the sample size. Equation (37) does not show these problems, but it depends on the sample length, which leads to phase ambiguity problems since T is complex and its logarithm may have infinite solutions [22].

B. Reflection only methods

If in fig. 3, since $d_L = 0$ and the load is a short-circuit, we have the method known as SCTL (short-circuit transmission line). This method is also applied to the free-space [4] where the short-circuit is made through a metal back (metal-back method). Other load types are possible. The model in fig. 3 can be used with any load. The sample scattering parameters, as functions of the propagation factor *T* and of the reflection coefficient at the interface Γ, are given in (19) and (25). In a distinct approach from the deduction of the NRW algorithm, which is intended to write Γ as a function of scattering parameters only, we now want an expression for the input reflection coefficient Γ_{in}, given by (27), as a function of the factor *T* and of the coefficient at the interface Γ. When substituting (19) and (25) in (27) (obtained from (26)), then:

$$\Gamma_{in} = \frac{\Gamma\left(1 - T^2\right) - \Gamma_L\left(\Gamma^2 - T^2\right)}{1 - \Gamma^2 T^2 - \Gamma_L\Gamma\left(1 - T^2\right)} \tag{38}$$

C. Double reflection methods – same size samples and different loads.

It is possible to obtain an explicit equation for the permittivity from (38) through the double reflection method [11][15] (also known as double impedance method [14]) or when considering the

same sample with two different loads. These measurements result in the input reflection coefficients Γ_1 e Γ_2 from the respective loads Γ_{L1} e Γ_{L2}. For each one of the loads the propagation factor T can be isolated in (38) with Γ given by (2):

$$T^2 = \frac{\left[\Gamma_L\left(\sqrt{\varepsilon_r}-1\right)+\sqrt{\varepsilon_r}+1\right]\left[\Gamma_{in}\left(\sqrt{\varepsilon_r}+1\right)+\sqrt{\varepsilon_r}-1\right]}{\left[\Gamma_L\left(\sqrt{\varepsilon_r}+1\right)+\sqrt{\varepsilon_r}-1\right]\left[\Gamma_{in}\left(\sqrt{\varepsilon_r}-1\right)+\sqrt{\varepsilon_r}+1\right]} \qquad (39)$$

Thus, if $\Gamma_L=\Gamma_{L1}=$-1 (short circuit) in the first measurement and $\Gamma_L=\Gamma_{L2}=1$ (open circuit) in other measurement, are applied to equation (39) and compared, the permittivity as a function of two reflection coefficients Γ_1 and Γ_2 is:

$$\varepsilon_r = \frac{\left(\Gamma_1-1\right)\left(\Gamma_2-1\right)}{\left(\Gamma_1+1\right)\left(\Gamma_2+1\right)} \qquad (40)$$

The normalized input admittance of a transmission line is given by [19]:

$$y = \frac{Y}{Y_0} = -\frac{\Gamma-1}{\Gamma+1} \qquad (41)$$

Therefore, the permittivity of a sample, when obtained from two measurements, one terminated in a short-circuit and the other terminated in an open-circuit, is given by:

$$\varepsilon_r = y_a y_c \qquad (42)$$

The permittivity as a product of a short-terminated line admittance (y_c) by an open-circuit terminated line admittance (y_a) has already been shown in [14].

D. New double reflection explicit equations

When measuring the reflection coefficient of a short terminated sample and, then, of an impedance matched terminated sample ($\Gamma_{L1}=$-1 e $\Gamma_{L2}=0$), it is possible to obtain another explicit equation from (39):

$$\varepsilon_r = \frac{\Gamma_2\Gamma_1 - 3\Gamma_2 + \Gamma_1 + 1}{\Gamma_2\Gamma_1 + \Gamma_2 + \Gamma_1 + 1} \qquad (43)$$

Applying the same procedure, but with an open-circuited load in place of the short-circuited one and, then, of an impedance matched terminated sample ($\Gamma_{L1}=1$ e $\Gamma_{L2}=0$), the permittivity is now given by:

$$\varepsilon_r = \frac{\Gamma_2\Gamma_1 - \Gamma_2 + 1 - \Gamma_1}{\Gamma_2\Gamma_1 + 3\Gamma_2 + 1 - \Gamma_1} \qquad (44)$$

This equation has been derived earlier [15], but its derivation uses a different procedure.

The equations (43) and (44) are particular cases of a general equation. Given any two loads Γ_{L1} and Γ_{L2} (with two measurements Γ_1 and Γ_2 being done with these two loads), the general explicit equation for permittivity is:

$$\varepsilon_r = \frac{\Gamma_{L1}\Gamma_{L2}\Gamma_1 - \Gamma_{L1}\Gamma_{L2}\Gamma_2 - \Gamma_{L1}\Gamma_2\Gamma_1 + \Gamma_{L2}\Gamma_2\Gamma_1 + 2\Gamma_{L1}\Gamma_2 - 2\Gamma_{L2}\Gamma_1 - \Gamma_{L1} + \Gamma_{L2} - \Gamma_2 + \Gamma_1}{\Gamma_{L1}\Gamma_{L2}\Gamma_1 - \Gamma_{L1}\Gamma_{L2}\Gamma_2 - \Gamma_{L1}\Gamma_2\Gamma_1 + \Gamma_{L2}\Gamma_2\Gamma_1 - 2\Gamma_{L1}\Gamma_2 + 2\Gamma_{L2}\Gamma_1 - \Gamma_{L1} + \Gamma_{L2} - \Gamma_2 + \Gamma_1} \quad (45)$$

E. Double reflection methods – same loads and different size samples

Using two samples with different lengths, arranged on a short-circuited line, as shown in Figure 4, it is possible to obtain the permittivity and the permeability. This procedure appears in [11] and [23].
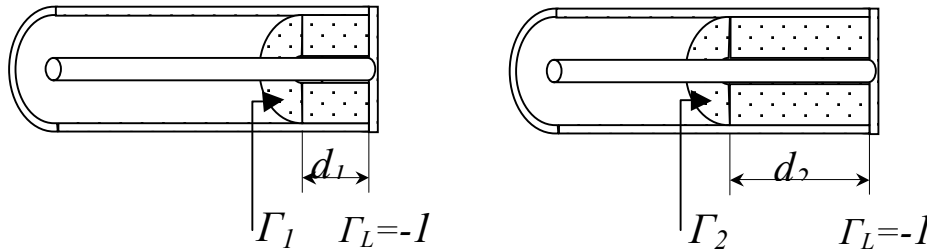

Fig. 4. Short-circuited lines with different sizes samples.

An explicit equation for the permittivity can be obtained considering $\Gamma L = -1$ in (39):

$$T^2 = -\frac{\left[\Gamma_1\left(\sqrt{\varepsilon_r}+1\right)+\sqrt{\varepsilon_r}-1\right]}{\left[\Gamma_1\left(\sqrt{\varepsilon_r}-1\right)+\sqrt{\varepsilon_r}+1\right]} \quad (46)$$

By measuring with sample widths $d2 = \alpha\, d1$ the squared propagation factor is given by:

$$T^2 = \left(-\frac{\left[\Gamma_2\left(\sqrt{\varepsilon_r}+1\right)+\sqrt{\varepsilon_r}-1\right]}{\left[\Gamma_2\left(\sqrt{\varepsilon_r}-1\right)+\sqrt{\varepsilon_r}+1\right]}\right)^{1/\alpha} \quad (47)$$

where **α** is a scaling fator.

Knowing the relationship between the widths and measuring the reflection coefficients, it is possible, for certain α values, to obtain explicit expressions for the permittivity considering the equation:

$$\left(-\frac{\left[\Gamma_1\left(\sqrt{\varepsilon_r}+1\right)+\sqrt{\varepsilon_r}-1\right]}{\left[\Gamma_1\left(\sqrt{\varepsilon_r}-1\right)+\sqrt{\varepsilon_r}+1\right]}\right)^{\alpha} = -\frac{\left[\Gamma_2\left(\sqrt{\varepsilon_r}+1\right)+\sqrt{\varepsilon_r}-1\right]}{\left[\Gamma_2\left(\sqrt{\varepsilon_r}-1\right)+\sqrt{\varepsilon_r}+1\right]} \quad (48)$$

In [11], the widths are set as d2 = 2 d1 (or α = 2). Equation (48) can then be solved explicitly, obtaining the permittivity:

$$\varepsilon_r = \frac{\left(\Gamma_1-1\right)\left(\Gamma_1\Gamma_2 - 3\Gamma_1 + 3\Gamma_2 - 1\right)}{\left(\Gamma_1+1\right)^2\left(\Gamma_2+1\right)} \quad (49)$$

Employing the same procedure, starting from equation 39 but forcing the samples to end with a matched load or an absorbing material in free space, another explicit equation for the permittivity can be obtained:

$$\varepsilon_r = \frac{(\Gamma_1 - 1)(\Gamma_1 \Gamma_2 - 2\Gamma_1 + \Gamma_2)}{(\Gamma_1 + 1)(\Gamma_1 \Gamma_2 + 2\Gamma_1 - \Gamma_2)} \tag{50}$$

V. ACCURACY

To estimate the uncertainty of the new equations, the Monte Carlo method is applied. The error sources considered are the finite accuracies of the measured reflection coefficient (within 3% of the nominal value for amplitude and phase) and of the load impedance (taken to be within 1% of nominal value). The combined effect of these error sources is computed for a population of 5000 samples in a rectangular distribution. A low-loss material with ε= 4 – 0,2j and 25 mm width was used.

The standard deviation in permittivity generated by these error sources when applied to equations (36) (obtained from (28), NRW method), (40) and (43) are shown in *Fig.* 5 and *Fig.* 6, for the real and imaginary parts of the permittivity, respectively. When the same errors sources are applied to (49) and (50), the results are shown in *Fig.* 7 and *Fig.* 8, for the real and the imaginary parts of the permittivity, respectively.
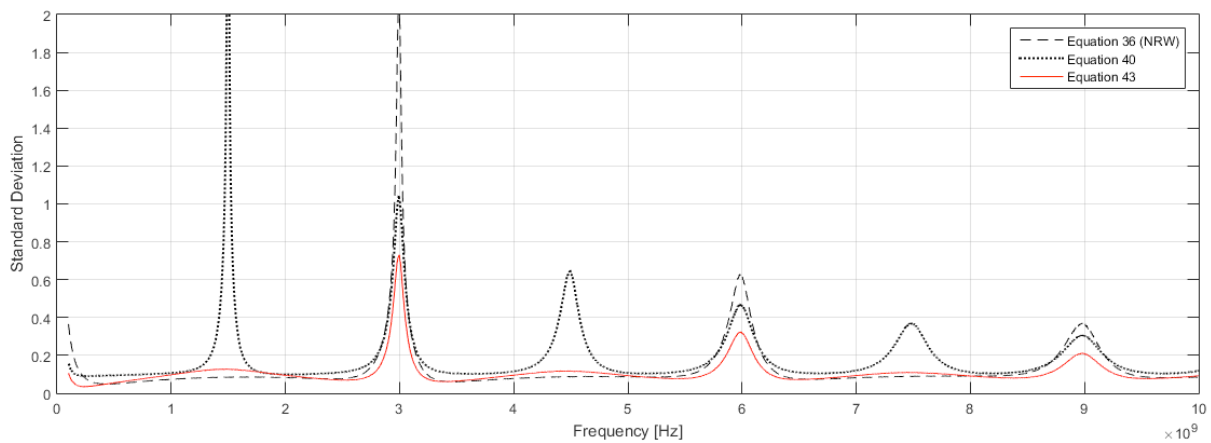

Fig. 5. Same size – different loads. Standard deviation of the real part of the permittivity.
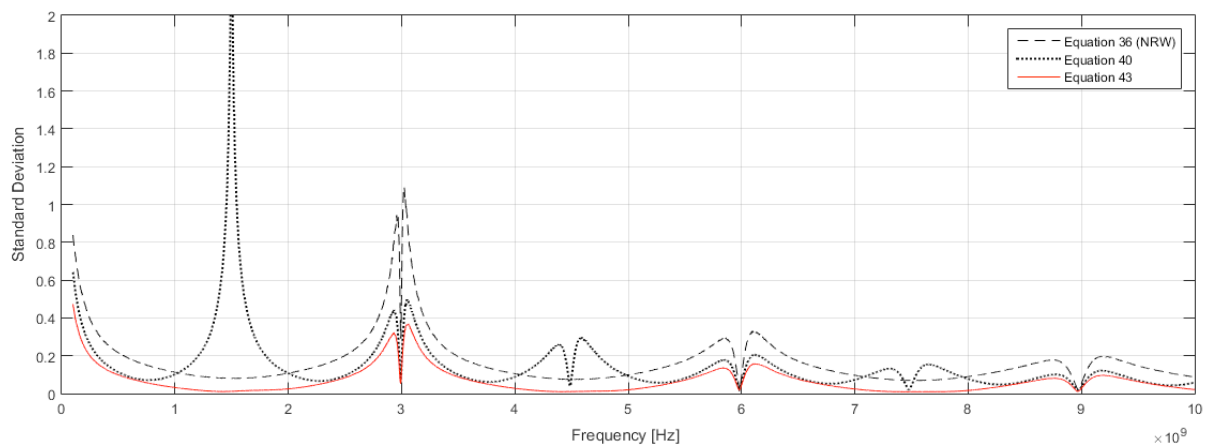

Fig. 6. Same size – different loads. Standard deviation of the imaginary part of the permittivity.

In the *Fig.* 5 and *Fig.* 6 it can be observed that the new explicit equation, (43), has smaller uncertainty in the frequencies which are multiple of half-wavelengths (3, 6 and 9 GHz) in comparison

to the traditional method of (40). Minimal uncertainty in frequencies 1.5, 4.5 and 7.5 GHz is found for (43), whereas (40) has an instability. It also can be noted that (43) has, in the entire band, a lower uncertainty for the imaginary part (when compared to the NRW method). The uncertainty for the real part of permittivity in quarter-wavelength frequencies (1.5, 4.5 and 7.5 GHz) is slightly higher for (43) than for the NRW method, but for half-wavelength frequencies the precision of (43) is higher than the NRW.
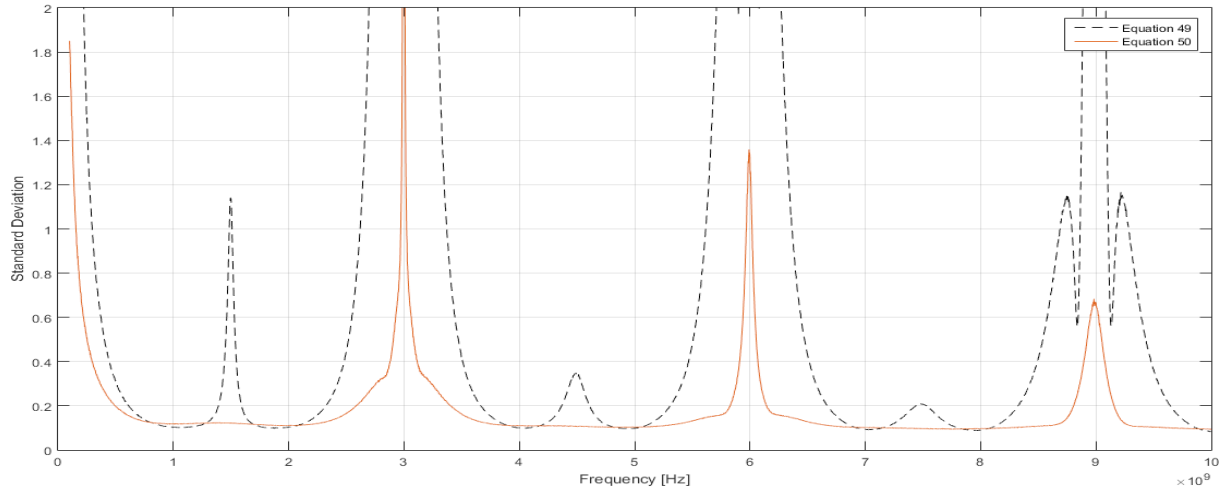

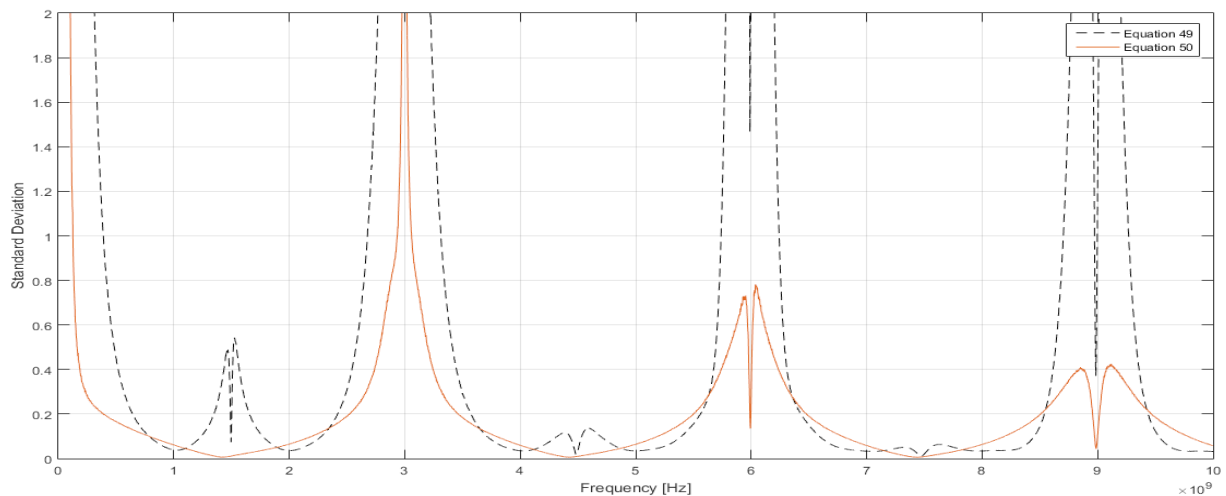Fig. 7. Same load – different sizes. Standard deviation of the real part of the permittivity.

Fig. 8. Same load – different sizes. Standard deviation of the imaginary part of the permittivity.

In *Fig.* 7 and *Fig.* 8, it can be observed, that the different-size-samples method, which uses matched loads, shows a lower error along most part of the band.

For both equations the uncertainties get smaller (and closer to one another) as the frequency increases. This can be due to the larger number of wavelengths inside the material sample width (a virtual thickening), resulting in larger attenuation and less signal being reflected at the termination.

VI. CONCLUSION

A new general model for the non-resonant permittivity measurement method was presented. From

this model the equations for classical NRW algorithm and SCTL method can be derived. Explicit equations to determine the permittivity were obtained from the new model. In addition to this, two new equations for the double reflection method were evaluated. One of them uses a short circuit load and a matched load and the other uses an open-circuit load and a matched load. A new equation is also obtained for the method with different sizes terminated in the same load, in this case, a matched one. The uncertainty of the new equations is calculated using the Monte Carlo method and, in both cases, it is lower than the classical methods. These methods were used for TEM waves in the free-space and in transmission lines. However, they could be easily extended for waves and samples in rectangular waveguides.

REFERENCES

[1] S. Kharkovsky and R, Microwave and Millimeter Wave Nondestructive Testing and Evaluation," IEEE Instrumentation and Measurement Magazine, vol. 10 (2), pp. 26-38, April 2007.

[2] K. Kupfer, Electromagnetic Aquametry – Electromagnetic Wave Interation with Water and Moist Substances, Berlin: Springer-Verlag, 2005, 529p.

[3] A. Von Hippel, editor. Dieletrics Materials and Applications. Cambridge,MA: Technology Press of MIT., 1954, 438p.

[4] L. F. Chen, et al., Microwave Electronics Measurement and Materials Characterization. Chichester: John Wiley & Sons, 2004. 537 p.

[5] D. M. Pozar, Microwave Engineering – Third Edition. New York, NY: John Wiley & Sons. 2005. 700 p.

[6] A. M. Nicolson, and G. F. Ross. "Measurement of the Intrinsic Properties of Materials by Time-Domain Techniques". IEEE Trans. Instrum. Meas., vol. IM-19, No. 4, pp. 377-382, Nov. 1970.

[7] W. B. Weir. "Automatic Measurement of Complex Dielectric Constant and Permeability at Microwave Frequencies", Proceedings of the IEEE, vol. 62, No. 1, pp. 33-36, Jan. 1974.

[8] S. Roberts and A. von Hippel. "A New Method for Measuring Dielectric Constant and Loss in the Range of Centimeter Waves". J. Appl. Phys., vol. 17, pp. 610-616, April 1946.

[9] M. G. Corfield, J. Horzelski and A. H. Price."Rapid method for determining v.h.f. dielectric parameters for liquids and solutions using standing wave procedures" British Journal of Applied Physics, vol. 12, pp. 680-682. Dec. 1961.

[10] S. O. Nelson, L. E. Stetson, and C. E. Schlaphoff. "A General Computer Program for Precise Calculation of Dielectric Properties From Short-Circuited-Waveguide Measurements". IEEE Trans. Instrum. Meas., vol. IM-23, No. 4, pp. 455-460, Dec. 1974.

[11] U. C. Hasar, J. J. Barroso, C. Sabah, and Y. Kaya."Resolving Phase Ambiguity in the Inverse Problem of Reflection-only Measurement Methods". Progress In Electromagnetics Research , vol. 129, pp. 405-420, June 2012.

[12] S. S. Stuchly and M. Matuszewski. "A Combined Total Reflection-Transmission Method in Application to Dielectric Spectroscopy". IEEE Trans. Instrum. Meas., vol. IM-27, No.3, pp. 285-288, Sept. 1978.

[13] D. K. Ghodgaonkar, V. V. Varadan, and V. K. Varadan. "Free-Space Measurement of Complex Permittivity and Complex Permeability of Magnetic Materials at Microwave Frequencies". IEEE Trans. Instrum. Meas., vol. 39, No. 2, pp. 387-394, April 1990.

[14] M. A. Stuchly, and S. S. Stuchly. "Coaxial Line Reflection Methods for Measuring Dielectric Properties of Biological Substances at Radio and Microwave Frequencies – a Review", IEEE Trans. Instrum. Meas., vol. IM-29, No. 3, pp.176-183, Sept. 1980.

[15] S. L. S. Severo, Aquametria por microondas: desenvolvimento de transdutor em microfita, Programa de pós-graduação em engenharia Elétrica, Dissertação de Mestrado, Universidade Federal do Rio Grande do Sul. Porto Alegre 2003.

[16] J. Baker-Jarvis, E. J. Vanzura, and W. A. Kissick."Improved Techique for Determining Complex Permittivity with the Transmission/Reflection Method", IEEE Trans. on Microwave Theory and Techniques, vol. 38, no. 8, pp. 1096-1103, Aug. 1990.

[17] U. C. Hasar, J. J. Barroso, M. Bute, Y. Kaya, M. E. Kocadagistan, and M. Ertugrul. "Attractive method for thickness-independent permittivity measurements of solid dielectric materials". Sensors and Actuators A: Physical, vol. 206, pp. 107-120, Feb. 2014.

[18] U. C. Hasar and M. T. Yurtcan, "A microwave method based on amplitude-only reflection measurements for permittivity determination of low-loss materials," Measurement , vol. 43, no. 9, pp. 1255-1265, Nov. 2010.

[19] R. J. Weber, Introduction to microwave circuits – Radio frequency and design applications, New York, NY: IEEE Press, 2001,432p.

[20] D. M. Pozar, Microwave Engineering – Third Edition, New York, NY: John Wiley & Sons. 2005. 700 p.

[21] A. Boughriet, C. Legrand and A. Chaponton, "Noniterative Stable Transmission/Reflection Method for Low-Loss Material Complex Permittivity Determination", IEEE Transactions on Microwave Theory and Techniques, vol. 45, no. 1, pp. 52-57, January 1997

[22] J. J. Barroso, and H. C. Hasar, "Resolving Phase Ambiguity in the Inverse Problem of Transmission /Reflection Measurement Methods", Journal Infrared Milli. Terahz Waves. Vol. 32. pp. 857-866., 201

[23] J Baker-Jarvis,M. D. Janezic, J. H. Grosvernor and R. G. Geyer. "Transmission / Reflection and Short-Circuit Line Methods for Measuring Permittivity and Permeability". NIST Technical Note 1355-R - NIST National Institute of Standards and Technology. Boulder, CO, 236 p. December 1993.