# Waves in Magnetized Dusty Plasmas With Variable Charge on Dust Particles

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***Abstract*—We present results obtained in recent years for wave propagation in a magnetized dusty plasma, including variable charge of the dust particles, and using a kinetic approach. Two forms of the dielectric tensor are obtained, which can be used depending on the application to be done. This dielectric tensor is used in some applications, in order to study the importance and influence of the variable charge on dust particles in the wave propagation characteristics. We first consider the magnetosonic wave and show that the variable charge of the dust gives the possibility of absorption. We also analyze the spatial absorption of this wave, including effects up to second order in the Larmor radius. Finally, we analyze Alfvén waves behavior in such dusty plasmas. The dispersion relation and damping rates of this mode are obtained.**

***Index Terms*—Alfvén waves, kinetic theory, magnetized dusty plasmas, variable dust charge, wave propagation.**

## I. INTRODUCTION

IN RECENT years, interest in dusty plasmas has increased significantly because it has been recognized that they are important for a number of applications in space plasmas and in the Earth's environment, as well as in laboratory plasmas and in several technologies [1]. Several analyses, treating the dust as a charged particle species of uniform mass and charge, have shown that the presence of the charged dust component leads to the appearence of new plasma modes arising from the dust particle dynamics or to modification of the existing plasma modes [2]–[4]. Several works that have taken account of the charge variation of dust particles [5]–[7] have pointed out the importance of this feature as a source of damping or amplification of waves propagating in dusty plasmas. This paper intends to report a review of our model and some applications we have done, in order to demonstrate that it is sufficiently general to apply to the distinct propagation modes, in a dusty plasma, in the frequency range $\omega > \Omega_d$. We present results of our work in recent years in the subject of wave propagation in magnetized dusty plasmas, including the possibility of variable charge for the dust particles. In particular, what is intrinsically new in the present paper is the study of the parallel propagation to $\mathbf{B}_0$, using a Maxwellian as a distribution function to describe the equilibrium, which is included in Section V. The structure of the paper is as follows. In Section II, we present the kinetic model used to study wave propagation in a dusty plasma. Section III contains general expressions for the dielectric tensor, derived from the model proposed, in two alternative forms. In Section IV, we consider the case of propagation perpendicular to the external magnetic field, in a Maxwellian plasma. The dispersion relation for magnetosonic waves is obtained in the cold plasma approach and in the case in which finite Larmor-radius effects are included. In Section V, we consider the case of propagation parallel to the external magnetic field, in a Maxwellian plasma. The dispersion relation and damping rates of Alfvén waves are obtained. Finally, in Section VI, we drawn some conclusions.

Manuscript received August 19, 2003; revised December 10, 2003. The work of M. C. de Juli was supported by the Brazilian Agency Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) (process number: 01/10921-9). The work of D. Falceta-Gonçalves and V. Jatenco-Pereira was supported by the government agency CNPq.

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Digital Object Identifier 10.1109/TPS.2004.826079

## II. THE MODEL

We consider a homogeneous plasma composed of particles of charge $q_\beta$ and mass $m_\beta$, where the subscript $\beta = e, i$ identifies electrons and ions, respectively, in a homogeneous external magnetic field $\mathbf{B}_0 = B_0\mathbf{e}_z$.

In this magnetized plasma, we consider embedded spherical dust grains with constant radius $a$ and variable charge $q$. The charging model for the dust particles must take into account the presence of an external magnetic field. This field must influence the characteristics of charging of the dust particles, because the path described by electrons and ions is modified: in this case, we have cyclotron motion of electrons and ions around the magnetic field lines. However, it has been shown by Chang and Spariosu [8] that for $a \ll \rho_G$, where $\rho_G = (\pi/2)^{1/2} r_{Le}$ and $r_{Le}$ is the electron Larmor radius, the effect of the magnetic field on the charging of the dust particles can be neglected. For the parameter values used in this work, the relation $a \ll \rho_G$ is always satisfied. We use a cross section for the charging process of the dust particles derived from the orbital motion limited (OML) theory [9], [10]. We consider that the principal process of dust charging is by capture of electrons and ions of the plasma. Since the electron thermal speed is much larger than the ion thermal speed, the dust charge will be predominantly negative.

We focus our attention on low-frequency waves in a weakly coupled dusty magnetoplasma. In our model, dust particles are assumed to be immobile, and consequently the validity of the proposed model will be restricted to waves with frequency much higher than the characteristic dust frequencies. In particular, we will consider the regime in which $\Omega_d \ll \omega \ll \Omega_\beta$, where $\Omega_d$ and $\Omega_\beta$ are the cyclotron frequency of the dust particles and of the electrons and ions, respectively. This modeling excludes the modes that can arise from the dust dynamics.

The distribution function which describes the dust particles, $f_d \equiv f_d(q, \mathbf{r}, t)$, is a solution of the equation

$$\frac{\partial f_d}{\partial t} + \frac{\partial}{\partial q}[I(\mathbf{r}, q, t) f_d] = 0 \tag{1}$$

where

$$I(\mathbf{r}, q, t) = \sum_\beta \int d^3p q_\beta \sigma_\beta(q, p) \frac{p}{m_\beta} f_\beta(\mathbf{r}, \mathbf{p}, t) \tag{2}$$

is the current on the dust particle [11], $f_\beta(\mathbf{r}, \mathbf{p}, t)$ is the distribution function of the particles labeled by $\beta$, and $\sigma_\beta$ is the charging cross section given by [12]

$$\sigma_\beta(p, q) = \pi a^2 \left(1 - \frac{2qq_\beta m_\beta}{ap^2}\right) H \left(1 - \frac{2qq_\beta m_\beta}{ap^2}\right) \tag{3}$$

where $H$ denotes the Heaviside function.

The distribution function of electrons and ions satisfies theVlasov equation with a collision term, which assures the possibility of charge variation of the dust particles

$$\frac{\partial f_\beta}{\partial t} + \frac{\mathbf{p}}{m_\beta} \cdot \nabla f_\beta + q_\beta \left[\mathbf{E} + \frac{\mathbf{p}}{m_\beta c} \times \mathbf{B}\right] \cdot \nabla_{\mathbf{p}} f_\beta = -\int \sigma_\beta \frac{p}{m_\beta}(f_d f_\beta - f_{d0} f_{\beta 0})\, dq. \tag{4}$$

The connection with the electromagnetic fields is accomplished by Maxwell equations

$$\nabla \cdot \mathbf{E} = 4\pi \left\{ \sum_\beta q_\beta \int f_\beta(\mathbf{r}, \mathbf{p}, t)\, d^3p + \int q f_d(\mathbf{r}, q, t)\, dq \right\} \tag{5}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{6}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{7}$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \sum_\beta \frac{q_\beta}{m_\beta} \int \mathbf{p} f_\beta(\mathbf{r}, \mathbf{p}, t)\, d^3p. \tag{8}$$

We note that if we exclude the charge variation of the dust particles from the proposed model, this will be equivalent to taking $\sigma_\beta(q, p) = 0$. Thus, our model will retain information about the presence of the dust particles in the plasma, only in the quasi-neutrality condition

$$\sum_\beta q_\beta n_{\beta 0} + Q_0 n_{d0} = 0 \tag{9}$$

where $Q_0 = \varepsilon_d\, e Z_d$ is equilibrium charge of the dust particle (positive, $\varepsilon_d = +1$ or negative, $\varepsilon_d = -1$) and $e = |q_e|$.

## III. Dielectric Tensor

The dielectric tensor for a magnetized dusty plasma, homogeneous, fully ionized, with identical immobile dust particles and variable charge, could be written in the following way [13]–[15]:

$$\epsilon_{ij} = \epsilon_{ij}^C + \epsilon_{ij}^N. \tag{10}$$

The term $\epsilon_{ij}^C$ is formally identical, except for the $iz$ components, to the dielectric tensor of a magnetized homogeneous conventional plasma of electrons and ions, with the resonant denominator modified by the addition of a purely imaginary term which contains the collision frequency of electrons and ions with the dust particles. In the case of $iz$ components, in addition to the term obtained with the prescription above, there is a term which is proportional to the collision frequency of electrons and ions with the dust particles.

The term $\epsilon_{ij}^N$ is entirely new and only exists in the presence of dust particles with variable charge. Its form is strongly dependent on the model used to describe the charging process of the dust particles.

In the following, we present the dielectric tensor in two alternative forms. The use of one or other form depends of the application to be done.

### A. First Form of the Dielectric Tensor

To obtain what we designate as the first form of the dielectric tensor, we use the standard procedure of linearizing the set of (1), (4), (5)–(8), choose a convenient equilibrium, take the Fourier–Laplace transform of the perturbed equations, and, from the linearized Ohm's law, identify the dielectric tensor.

The expression for $\epsilon_{ij}^C$ is [13]–[15]

$$\begin{aligned}\epsilon_{ij}^C = \delta_{ij} &+ \sum_\beta \frac{X_\beta}{n_{\beta 0}} \sum_{n=-\infty}^{+\infty} \times \int d^3p p_\perp \frac{\varphi_0(f_{\beta 0})}{D_{n\beta}} \left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{iz}+\delta_{jz}} R_{ij}^{n\beta} \\ &- \delta_{iz}\delta_{jz} \sum_\beta \frac{X_\beta}{n_{\beta 0}} \int d^3p L(f_{\beta 0}) \frac{p_\parallel}{p_\perp} \\ &+ \delta_{jz} \sum_\beta \frac{X_\beta}{n_{\beta 0}} \sum_{n=-\infty}^{+\infty} \int d^3p \times \left[ i\frac{\nu_{\beta d}^0(p)}{\omega} \frac{L(f_{\beta 0})}{D_{n\beta}} \left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{iz}} \right] R_{ij}^{n\beta}\end{aligned} \tag{11}$$

where

$$D_{n\beta} \equiv 1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} - \frac{n\Omega_\beta}{\omega} + i\frac{\nu_{\beta d}^0(p)}{\omega} \tag{12}$$

$$\nu_{\beta d}^0(p) \equiv \int_{-\infty}^0 \sigma_\beta(p, q) \frac{p}{m_\beta} f_{d0}(q)\, dq = \frac{\pi a^2 n_{d0}}{m_\beta} \frac{(p^2 + C_\beta)}{p} H(p^2 + C_\beta) \tag{13}$$

$$\varphi_0(f_{\beta 0}) \equiv \frac{\partial f_{\beta 0}}{\partial p_\perp} - \frac{k_\parallel}{m_\beta \omega} L(f_{\beta 0}) \tag{14}$$

$$L(f_{\beta 0}) \equiv p_\parallel \frac{\partial f_{\beta 0}}{\partial p_\perp} - p_\perp \frac{\partial f_{\beta 0}}{\partial p_\parallel} \tag{15}$$

$$X_\beta \equiv \frac{\omega_{p\beta}^2}{\omega^2}, \quad \omega_{p\beta}^2 = \frac{4\pi n_{\beta 0} q_\beta^2}{m_\beta} \tag{16}$$

$$C_\beta \equiv -\frac{2q_\beta m_\beta Q_0}{a}, \quad \Omega_\beta = \frac{q_\beta B_0}{m_\beta c}. \tag{17}$$

The $R_{ij}^{n\beta}$ are combinations of Bessel functions and their derivatives and explicit expressions for them are given in [14]–[16]. In order to get the expressions (11) and (13), we have assumed that $f_{d0} = n_{d0}\delta(q - Q_0)$.

The expression for the term $\epsilon_{ij}^N$ is given by [13]–[15]

$$\epsilon_{ij}^N = -\frac{4\pi i n_{d0}}{\omega} U_i S_j \tag{18}$$

with

$$U_i \equiv \frac{i}{\omega + i(\nu_{\rm ch} + \nu_1)} \sum_\beta \frac{q_\beta}{m_\beta^2} \times \sum_{n=-\infty}^{+\infty} \int d^3p \frac{p_\perp \sigma'_\beta(p) f_{\beta 0} p}{D_{n\beta}} \left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{iz}} R_{iz}^{n\beta} \tag{19}$$

$$\begin{aligned} S_j \equiv &\frac{1}{\omega n_{d0}} \sum_\beta q_\beta^2 \sum_{n=-\infty}^{+\infty} \int d^3p \frac{\nu_{\beta d}^0(p)}{\omega} \\ &\times \frac{\varphi_0(f_{\beta 0})}{D_{n\beta}} \left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{jz}} R_{zj}^{n\beta} \\ &- \frac{\delta_{jz}}{\omega n_{d0}} \sum_\beta q_\beta^2 \int d^3p \frac{\nu_{\beta d}^0(p)}{\omega} \frac{L(f_{\beta 0})}{p_\perp} \\ &+ i\frac{\delta_{jz}}{\omega n_{d0}} \sum_\beta q_\beta^2 \sum_{n=-\infty}^{+\infty} \int d^3p \frac{\left[\nu_{\beta d}^0(p) \big/ \omega\right]^2}{D_{n\beta}} \\ &\times \frac{L(f_{\beta 0})}{p_\perp} R_{zj}^{n\beta} \end{aligned} \tag{20}$$

where

$$\nu_{\rm ch} = -\sum_\beta \frac{q_\beta}{m_\beta} \int d^3p\, p\sigma'_\beta(p) p f_{\beta 0} \tag{21}$$

$$\nu_1 = \sum_\beta \frac{q_\beta}{m_\beta} \sum_{n=-\infty}^{+\infty} \int d^3p \frac{\left[i\nu_{\beta d}^0(p) \big/ \omega\right]}{D_{n\beta}} \times \sigma'_\beta(p) f_{\beta 0} p R_{zz}^{n\beta}. \tag{22}$$

In (19), (21), and (22), we have used the notation $\sigma'_\beta(p) \equiv (\partial\sigma_\beta/\partial q)|_{q=-Z_d e}$.

*B. Second Form of the Dielectric Tensor*

In this section, we rewrite the components of the dielectric tensor in a new form, in order to deal more easily with the different powers of Larmor radius which are essential for the analysis of the effects of charge variation of the dust particles that are included in $\epsilon_{ij}^N$. However, we stress that this form of the dielectric tensor is as general as the form presented in the previous section.

We consider Larmor-radius effects arisen from the motion of electrons and ions and neglect this kind of the effect for the dust particles, since they are supposed to be immobile because of their large mass.

In order to obtain the new form of the dielectric tensor, the sum in $n$ from minus to plus infinity that occurs in their components, is transformed into a sum running from zero to infinity. Also, the combinations of Bessel functions and its derivatives in $R_{ij}^{n\beta}$ are expanded using the following identities [17]:

$$J_n(b_\beta) J'_n(b_\beta) = \sum_{m=0}^{\infty} (n+m) \times a(n,m) b_\beta b_\beta^{2(n+m-1)} \tag{23}$$

$$J_n^2(b_\beta) = \sum_{m=0}^{\infty} a(n,m) b_\beta^{2(n+m)} \tag{24}$$

$$J_n'^2(b_\beta) = \sum_{m=0}^{\infty} d(n,m) b_\beta^{2(n+m-1)} \tag{25}$$

where

$$a(n,m) = \frac{(-1)^m [2(n+m)]!}{[(n+m)!]^2 (2n+m)! m!} \left(\frac{1}{2}\right)^{2(n+m)} \tag{26}$$

and $d(n,m) = a(1, m-2)$ for $n = 0$ and

$$d(n,m) = \frac{1}{4}\left[a(n-1,m) - 2\frac{n+m-1}{n+m} a(n,m-1) + a(n+1,m-2)\right], \quad \text{for } n > 0 \tag{27}$$

with the convention $1/(-m)! = 0$ for $m = 0, 1, 2, \ldots$.

Using (23)–(25), the terms $\epsilon_{ij}^C$ and $\epsilon_{ij}^N$ of the dielectric tensor can be written as a series in powers of $N_\perp/Y_\beta$, where in each term we have an integral of the form [16], [18]

$$\tilde{I}_\beta(n,m,h,s;G) \equiv \frac{1}{n_{\beta 0}} \int \frac{d^3u\, u_\parallel^h u_\perp^{2(n+m-1)} u_\perp G}{g_\beta + snY_\beta} \tag{28}$$

with $\mathbf{u} = \mathbf{p}/m_\beta c$, $g_\beta \equiv 1 - N_\parallel u_\parallel + i\nu_{\beta d}^0/\omega$, $Y_\beta \equiv \Omega_\beta/\omega$, and $N_\parallel = k_\parallel c/\omega$.

As an example, we write explicitly the $xy$ component of the dielectric tensor [15], [16]

$$\epsilon_{xy}^C = \epsilon_{yx}^C = -i \sum_\beta X_\beta \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \left(\frac{N_\perp}{Y_\beta}\right)^{2(n+m-1)} \times n(n+m) a(n,m) \times \sum_{s=\pm 1} s\tilde{I}_\beta(n,m,0,s;\varphi_0(f_{\beta 0})) \tag{29}$$

$$\epsilon_{xy}^N = -\frac{4\pi i}{\omega} \frac{c^2}{\omega^2} \frac{\hat{U}_x \hat{S}_y}{1 + i\left(\frac{\nu_{\rm ch}}{\omega} + \frac{\nu_1}{\omega}\right)} \tag{30}$$

where

$$\hat{U}_x = -\sum_\beta q_\beta n_{\beta 0} \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \left(\frac{N_\perp}{Y_\beta}\right)^{2(n+m)-1} n a(n,m) \times \sum_{s=\pm 1} s\tilde{I}_\beta(n,m,0,s; u\sigma'_\beta(u) u_\perp f_{\beta 0}), \tag{31}$$

$$\hat{S}_y = \frac{i}{4\pi}\sum_\beta \frac{\omega_{p\beta}^2}{c}\left[\sum_{m=1}^{\infty}\left(\frac{N_\perp}{Y_\beta}\right)^{2m-1} m \times a(0,m)\tilde{I}_\beta\left(0,m,0,0;i\frac{\nu_{\beta d}^0}{\omega}\varphi_0(f_{\beta 0})\right) + \sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\left(\frac{N_\perp}{Y_\beta}\right)^{2(n+m)-1}(n+m) \times a(n,m)\sum_{s=\pm 1}\tilde{I}_\beta\left(n,m,0,s;i\frac{\nu_{\beta d}^0}{\omega}\varphi_0(f_{\beta 0})\right)\right] \quad (32)$$

with

$$\nu_1 = c\sum_\beta q_\beta n_{\beta 0}\left[\sum_{m=0}^{\infty}\left(\frac{N_\perp}{Y_\beta}\right)^{2m} \times a(0,m)\tilde{I}_\beta\left(0,m,0,0;i\frac{\nu_{\beta d}^0}{\omega}u\sigma'_\beta(u)u_\perp f_{\beta 0}\right) + \sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\left(\frac{N_\perp}{Y_\beta}\right)^{2(n+m)} a(n,m) \times \sum_{s=\pm 1}\tilde{I}_\beta\left(n,m,0,s;i\frac{\nu_{\beta d}^0}{\omega}u\sigma'_\beta(u)u_\perp f_{\beta 0}\right)\right]. \quad (33)$$

All other quantities, such as $\nu_{\mathrm{ch}}, \varphi_0(f_{\beta 0})$ and so on, are the same as that defined in Section III-A, but now expressed in terms of the variable $\mathbf{u} = \mathbf{p}/m_\beta c$. Full expressions for the components of the dielectric tensor are given in [15] and [16].

In this form, it is much easier to take care of the contributions in different powers of Larmor radius to the components of the dielectric tensor, especially for $\epsilon_{ij}^N$.

## IV. Propagation Perpendicular to $\mathbf{B}_0$ and Maxwellian Distribution Function

Using the dielectric tensor presented in Section III-A, we consider wave propagation in perpendicular direction to the external magnetic field. Choosing the coordinate system in such a way that $\mathbf{B}_0 = B_0\mathbf{e}_z$ and $\mathbf{k} = k_\perp\mathbf{e}_x$, assuming Maxwellian distribution functions to describe electrons and ions in equilibrium, it is easy to show that the nonnull components of the dielectric tensor $\epsilon_{ij} = \epsilon_{ij}^C + \epsilon_{ij}^N$ are given by

$$\overleftrightarrow{\epsilon} = \begin{pmatrix} \epsilon_{xx}^C + \epsilon_{xx}^N & \epsilon_{xy}^C + \epsilon_{xy}^N & 0 \\ -\epsilon_{xy}^C + \epsilon_{yx}^N & \epsilon_{yy}^C + \epsilon_{yy}^N & 0 \\ 0 & 0 & \epsilon_{zz}^C \end{pmatrix} \quad (34)$$

where

$$\epsilon_{ij}^C = \delta_{ij} + \sum_\beta \frac{X_\beta}{n_{\beta 0}}\sum_{n=-\infty}^{+\infty}\int d^3p\, p_\perp \frac{\partial f_{\beta 0}}{\partial p_\perp} \times \left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{iz}+\delta_{jz}}\frac{R_{ij}^{n\beta}}{\tilde{D}_{n\beta}} \quad (35)$$

and

$$\epsilon_{ij}^N = -\frac{4\pi i n_{d0}}{\omega}U_i S_j \quad (36)$$

with

$$U_i = \frac{i}{\omega + i(\nu_{\mathrm{ch}} + \nu_1)}\sum_\beta \frac{q_\beta}{m_\beta^2}\sum_{n=-\infty}^{\infty}\int d^3p\, p_\perp \times \frac{\sigma'_\beta(p) f_{\beta 0} p}{\tilde{D}_{n\beta}}\left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{iz}} R_{iz}^{n\beta} \quad (37)$$

$$S_j = \frac{1}{\omega n_{d0}}\sum_\beta q_\beta^2 \sum_{n=-\infty}^{+\infty}\int d^3p\, p\frac{\nu_{\beta d}^0(p)}{\omega} \times \frac{\partial f_{\beta 0}}{\partial p_\perp}\left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{jz}}\frac{R_{zj}^{n\beta}}{\tilde{D}_{n\beta}} \quad (38)$$

where

$$\tilde{D}_{n\beta} \equiv \tilde{D}_{n\beta}^C + i\frac{\nu_{\beta d}^0(p)}{\omega} \quad (39)$$

$$\tilde{D}_{n\beta}^C \equiv 1 - nY_\beta \quad (40)$$

$$Y_\beta \equiv \frac{\Omega_\beta}{\omega}. \quad (41)$$

We note that the form $\epsilon_{ij}^C$ is identical to the dielectric tensor of a plasma of electrons and ions, magnetized and homogenous, but with an additional pure imaginary term, $i\nu_{\beta d}^0(p)/\omega$, in the resonant denominator.

In the case of the wave propagation with the electric field in the parallel direction to $\mathbf{B}_0$, that is $\hat{E}_x = \hat{E}_y = 0$, the dispersion relation is

$$N_\perp^2 = \epsilon_{zz}^C. \quad (42)$$

In the case of the wave propagation with the electric field in the perpendicular direction to $\mathbf{B}_0$, that is $\hat{E}_z = 0$, the dispersion relation is

$$N_\perp^2 = \frac{\epsilon_{xx}\epsilon_{yy} - \epsilon_{xy}\epsilon_{yx}}{\epsilon_{xx}} \quad (43)$$

with $\epsilon_{ij} = \epsilon_{ij}^C + \epsilon_{ij}^N$ and $i, j = x, y$; this is the dispersion relation to be used for the magnetosonic wave.

### A. Magnetosonic Waves—Cold Plasma Approach

The magnetosonic wave propagating perpendicularly to the external magnetic field is not a damped mode in an electron-ion plasma. We will use the dielectric tensor presented in Section III-A to analyze how this wave is affected by the presence of dust with variable charge.

In order to analyze this particular wave, we make the following approximations: we retain only the contributions of the lowest harmonics, $n = 0, \pm 1$; we make small Larmor-radius expansions of the Bessel functions and their derivatives, maintaining terms up to second order and expand the denominator $D_{n\beta}^C + i\nu_{\beta d}^0(p)/\omega$, in the integrals in the variable $\mathbf{p}$, in powers of $i(\nu_{\beta d}^0(p)/\omega)/D_{n\beta}^C$, retaining terms up to second order.

Again, we write only the $xy$ component of the dielectric tensor in order to exemplify

$$\epsilon_{xy}^C = -i\frac{4\pi}{3}\sum_\beta \frac{\eta_\beta Y_\beta}{1 - Y_\beta^2}\left\{\mathcal{I}_{0\beta}^2 - iE_{3\beta}\mathcal{I}_{1\beta}^2 - E_{4\beta}\mathcal{I}_{2\beta}^2\right\} \quad (44)$$

$$\epsilon_{xy}^N = iC\left[\sum_\beta N_{1\beta}\left(\mathcal{J}_{0\beta}^3 - iE_{3\beta}I_{1\beta}^3\right)\right] \times \left[\sum_\beta N_{2\beta}Y_\beta\left(I_{1\beta}^4 - iI_{2\beta}^4\right)\right] \tag{45}$$

where

$$\mathcal{I}_{n\beta}^s \equiv I_{n\beta}^4 - \frac{s}{5}\mu_\beta^2 I_{n\beta}^6 \tag{46}$$

$$\mathcal{J}_{0\beta}^m \equiv \int_0^\infty dpp^m H(p^2 + C_\beta)f_{\beta 0} \tag{47}$$

$$I_{n\beta}^m \equiv \int_0^\infty dpp^m f_{\beta 0}\left(\frac{\nu_{\beta d}^0}{\omega}\right)^n \tag{48}$$

$$C \equiv \frac{128\pi^4 a}{9\omega^2[\omega + i(\nu_{\mathrm{ch}} + \nu_1)]} \tag{49}$$

$$\eta_\beta \equiv \frac{X_\beta}{n_{\beta 0}m_\beta T_\beta} \tag{50}$$

with

$$\nu_{\mathrm{ch}} + \nu_1 = \frac{4\pi^2 a}{3}\sum_\beta q_\beta^2\left\{6\left[\mathcal{J}_{0\beta}^1 - iI_{1\beta}^1\right] - i2\mu_\beta^2\frac{Y_\beta^2}{1 - Y_\beta^2}I_{1\beta}^3\right\} \tag{51}$$

and $\mu_\beta \equiv k_\perp/m_\beta\Omega_\beta$.

The analytical results of the integrals needed for this case and the expressions for the coefficients $E_{l\beta}(l = 1 \text{ to } 5)$ and $N_{s\beta}(s = 1, 2)$ are given explicitly in [14].

In order to study the simplest effect produced by the inclusion of dust particles with variable charge in the propagation properties of the magnetosonic wave, we make the following additional approximations in the calculation of the $\epsilon_{ij}$: we neglect contributions arising from terms that contain $(\nu_{\beta d}^0(p)/\omega)^2$; we neglect terms of the order $\mu_\beta^2$ and higher orders, which corresponds to making a small Larmor-radius approximation (in this case a cold-plasma approximation); and we work in a frequency regime such that $Y_\beta^2 \gg 1 \Rightarrow \omega \ll |\Omega_\beta|$, which is the appropriate regime for magnetosonic waves. Using these approximations, we get that

$$\epsilon_{xx}^C = \mathcal{A} + i\frac{\mathcal{B}}{\omega} \quad \text{and} \quad \epsilon_{xy}^C = i\frac{\mathcal{C}}{\omega} \tag{52}$$

where the positive quantities $\mathcal{A}, \mathcal{B}$, and $\mathcal{C}$ are given by

$$\mathcal{A} \equiv 1 + \sum_\beta \frac{c^2}{V_{A\beta}^2} \cong 1 + \frac{c^2}{V_{Ai}^2} \tag{53}$$

$$\mathcal{B} \equiv \frac{4}{3}(2\pi)^{1/2}a^2 n_{d0}\sum_\beta \frac{c^2}{V_{A\beta}^2}v_{T\beta} \times \left\{\left(2 + \frac{Z}{\tilde{\tau}}\right)\delta_{\beta i} + e^{-Z}(Z + 2)\delta_{\beta e}\right\} \tag{54}$$

$$\mathcal{C} \equiv \frac{4\pi c}{B_0}\sum_\beta n_{\beta 0}q_\beta = \frac{4\pi c}{B_0}Z_d e n_{d0} \tag{55}$$

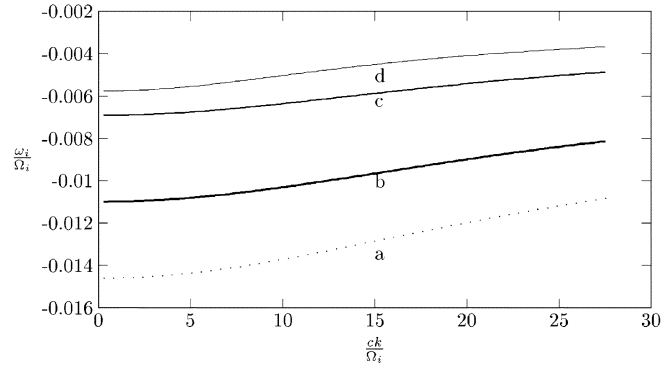


Fig. 1. $\omega_i/\Omega_i$ as a function of $ck/\Omega_i$ for $B_0 = 5 \times 10^{-3}$ T, $n_{d0} = 10^4$ cm$^{-3}$, $n_{i0} = 10^8$ cm$^{-3}$, $\alpha = 10^{-4}$ cm, and several values of the temperature $T = T_e = T_i$: (a) 100 eV, (b) 50 eV, (c) 10 eV, and (d) 5 eV.

with

$$V_{A\beta} \equiv \left(\frac{B_0^2}{4\pi n_{\beta 0}m_\beta}\right)^{1/2} \tag{56}$$

the Alfvén velocity of particles of species $\beta$ and $V_{Ai} \ll V_{Ae}, \tilde{\tau} \equiv T_i/T_e Z_i$. We will consider waves with real wave vector $\mathbf{k}$ and complex frequency $\omega$, with $|\omega_i| \ll |\omega_r|$.

From the dispersion relation (43), we then obtain that the real part of the frequency is given by

$$\omega_r^2 = \frac{-\mathcal{Q} + \left(\mathcal{Q}^2 + 4\omega_0^2\alpha_1^2\right)^{1/2}}{2} \tag{57}$$

where

$$\mathcal{Q} \equiv \alpha_1^2 - \left(\alpha_2^2 + \omega_0^2\right) \tag{58}$$

$$\omega_0^2 \equiv \frac{c^2k^2}{\mathcal{A}} = \frac{k^2V_{Ai}^2}{1 + \frac{V_{Ai}^2}{c^2}} \tag{59}$$

and $\omega_0$ is the usual frequency of the magnetosonic wave in an electron-ion plasma in the cold approach [19], $\alpha_1 \equiv \mathcal{B}/\mathcal{A}$ and $\alpha_2 \equiv \mathcal{C}/\mathcal{A}$.

The imaginary part of the frequency is given by

$$\omega_i = -\frac{\omega_r^2}{2}\alpha_1\frac{1 + F(\omega_r)}{\omega_0^2 + F(\omega_r)G(\omega_r)} \tag{60}$$

where

$$F(\omega) \equiv \frac{{\alpha_2}^2}{\omega^2 + {\alpha_1}^2} \tag{61}$$

$$G(\omega_r) \equiv \frac{\omega_r^4}{\omega_r^2 + \alpha_1^2}. \tag{62}$$

We note that for $n_{d0} = 0$ we have $\mathcal{B} = 0$ and $\mathcal{C} = 0$ and, consequently, $\omega_r^2 = \omega_0^2$ and $\omega_i = 0$. In the case $n_{d0} \neq 0$, we have two situations: the dust charge is constant so $\mathcal{B} = 0$ and $\mathcal{C} \neq 0$ which implies that $\omega_r^2 \neq \omega_0^2$ and $\omega_i = 0$, or the dust charge is variable so $\mathcal{B} \neq 0$ and $\mathcal{C} \neq 0$ which implies that $\omega_r^2 \neq \omega_0^2$ and $\omega_i < 0$. In this later case the magnetosonic wave is damped, and this damping results from the charge variation of the dust particles.

In Fig. 1, we show $\omega_i/\Omega_i$ as a function of $ck/\Omega_i$. We see that the absolute value of $\omega_i/\Omega_i$ increases with temperature of electrons and ions; this occurs because by increasing the temperature, electrons and ions get more mobility and so contribute more to currents charging the dust particles.

We note that in the above calculation in order to evaluate the integrals in the $\mathbf{p}$ variable, we make an expansion of the denominator that occurs in them, a practice very common in the literature [7], [20]. This kind of expansion is good if the involved frequencies are sufficiently high that $\omega \sim \omega_{pe} \gg \max(kv_{T_e}, \nu_{\rm ch}, \nu_{\beta d}^0)$ [7], and has as consequence that only a few terms need to be retained. If the frequency is lowered, we must retain more and more terms in the expansion, which turns out to be algebraically very expensive. In the next section, these expansions are not done, and we are interested in making a comparison between the two ways of dealing with this calculation.

### *B. Magnetosonic Waves With Finite Larmor-Radius Effects*

We use here the dispersion relation (43) and the dielectric tensor components in the form presented in Section III-B, in order to study the magnetosonic wave propagation in a magnetized dusty plasma.

The following approximations are made in the components of the dielectric tensor: in each component of the dielectric tensor, we retain only contributions up to order $N_\perp^2$, which implies retaining only the lowest order contributions in the finite Larmor radius. As an example, we write explicity the $xy$ component of the dielectric tensor [15]

$$\begin{aligned}\epsilon_{xy}^C = \epsilon_{yx}^C = &-\frac{i}{4}\sum_\beta X_\beta \sum_{s=\pm1} s\tilde{I}_\beta\left(1,0,0,s;\frac{\partial f_{\beta 0}}{\partial u_\perp}\right)\\ &-\frac{i}{4}\sum_\beta X_\beta\left(\frac{N_\perp}{Y_\beta}\right)^2\\ &\times\sum_{s=\pm1} s\left[\tilde{I}_\beta\left(2,0,0,s;\frac{\partial f_{\beta 0}}{\partial u_\perp}\right)\right.\\ &\left.-2\tilde{I}_\beta\left(1,1,0,s;\frac{\partial f_{\beta 0}}{\partial u_\perp}\right)\right]\end{aligned} \tag{63}$$

$$\epsilon_{xy}^N = -\frac{4\pi i}{\omega}\frac{c^2}{\omega^2}\frac{\hat{U}_x\hat{S}_y}{\left[1+i\left(\frac{\nu_{\rm ch}}{\omega}+\frac{\nu_1}{\omega}\right)\right]} \tag{64}$$

where

$$\begin{aligned}\hat{U}_x = &-\frac{1}{4}\sum_\beta q_\beta n_\beta \frac{N_\perp}{Y_\beta}\\ &\times\sum_{s=\pm1} s\tilde{I}_\beta(1,0,0,s;u\sigma'_\beta u_\perp f_{\beta 0})\end{aligned} \tag{65}$$

$$\begin{aligned}\hat{S}_y = &-\frac{i}{8\pi}\sum_\beta \frac{\omega_{p\beta}^2}{c}\frac{N_\perp}{Y_\beta}\\ &\times\left[\tilde{I}_\beta\left(0,1,0,0;\frac{i\nu_{\beta d}^0}{\omega}\frac{\partial f_{\beta 0}}{\partial u_\perp}\right)\right.\\ &\left.-\frac{1}{2}\sum_{s=\pm1}\tilde{I}_\beta\left(1,0,0,s;\frac{i\nu_{\beta d}^0}{\omega}\frac{\partial f_{\beta 0}}{\partial u_\perp}\right)\right]\end{aligned} \tag{66}$$

with

$$\begin{aligned}\nu_1 = c\sum_\beta q_\beta n_\beta &\left\{\tilde{I}_\beta\left(0,0,0,0;\frac{i\nu_{\beta d}^0}{\omega}u\sigma'_\beta u_\perp f_{\beta 0}\right)\right.\\ &-\frac{1}{2}\left(\frac{N_\perp}{Y_\beta}\right)^2\left[\tilde{I}_\beta\left(0,1,0,0;\frac{i\nu_{\beta d}^0}{\omega}u\sigma'_\beta u_\perp f_{\beta 0}\right)\right.\\ &\left.\left.-\frac{1}{2}\sum_{s=\pm1}\tilde{I}_\beta\left(1,0,0,s;\frac{i\nu_{\beta d}^0}{\omega}u\sigma'_\beta u_\perp f_{\beta 0}\right)\right]\right\}.\end{aligned} \tag{67}$$

The dielectric tensor components of interest for the dispersion relation (43) will be rewritten in such a way as to make clear the dependence on $N_\perp^2$. Again, to save space, we write only the $xy$ component in complete form [14]–[16]

$$\epsilon_{xy} = \epsilon_{xy}^C + \epsilon_{xy}^N \tag{68}$$

with

$$\epsilon_{xy}^C = \left(\chi_{xy}^{0R} + N_\perp^2\chi_{xy}^{1R}\right) + i\left(\chi_{xy}^{0I} + N_\perp^2\chi_{xy}^{1I}\right) \tag{69}$$

$$\epsilon_{xy}^N = -N_\perp^2 C\left(\chi_{xy}^{NR} + i\chi_{xy}^{NI}\right) \tag{70}$$

where

$$C \equiv \frac{1}{36\omega^4}\frac{n_{d0}a^3}{(d^R)^2+(d^I)^2} \tag{71}$$

and

$$d^R \equiv 1 - \frac{\nu_1^I}{\omega}, \quad d^I \equiv \frac{\nu_{\rm ch}}{\omega} + \frac{\nu_1^R}{\omega} \tag{72}$$

$$\nu_{\rm ch} = \frac{a}{(2\pi)^{1/2}c}\left[\omega_{pi}^2\mu_i^{1/2} + \omega_{pe}^2\mu_e^{1/2}e^{-\frac{Z_d e^2}{Ta}}\right] \tag{73}$$

$$\nu_1 \cong c\sum_\beta q_\beta n_\beta \tilde{I}_\beta\left(0,0,0,0;i\frac{\nu_{\beta d}^0}{\omega}u\sigma'_\beta u_\perp f_{\beta 0}\right). \tag{74}$$

The quantities $\chi_{ij}^{0R}, \chi_{ij}^{1R}, \chi_{ij}^{0I}, \chi_{ij}^{1I}, \chi_{ij}^{NR}$, and $\chi_{ij}^{NI}$ are combinations of the integrals [16], [18]

$$i_l \equiv \int_0^{\sqrt{-\tilde{C}_e}} u^l e^{-\frac{\tilde{\mu}_e}{2}u^2}\,du \tag{75}$$

$$I(p,l) \equiv \int_0^\infty \frac{u^l e^{-\frac{\tilde{\mu}_i}{2}u^2}\,du}{(1+pY_i)^2u^2 + A^2(u^2+\tilde{C}_i)^2} \tag{76}$$

$$E(p,l) \equiv \int_{\sqrt{-\tilde{C}_e}}^\infty \frac{u^l e^{-\frac{\tilde{\mu}_e}{2}u^2}\,du}{(1+pY_e)^2u^2 + A^2(u^2+\tilde{C}_e)^2} \tag{77}$$

where $A \equiv \pi a^2 n_{d0}c/\omega, \tilde{\mu}_\beta \equiv m_\beta c^2/T_\beta, T_\beta$ being the temperature of particles of species $\beta$, and $l, p$ are integer numbers. The complete expressions for these quantities are given in [16]. We note that in the case of Maxwellian distribution and $N_\parallel = 0$, the part $\epsilon_{ij}^N$ of the dielectric tensor will be of the order $N_\perp^2$ or higher.

In Fig. 2, we show the imaginary part of the refractive index $N_i$ as a function of $\omega/\Omega_i$. The forbidden region of values of $N_i$ is generated by the cutoff of $N_r$ obtained using the approximated expressions of Section IV-A. We notice that the result in (a) curve comes from a formulation in which the expansion of the denominator of the integrals in $\mathbf{p}$-variable has not been used, and in which, therefore, the effects of charge variation are dealt with in a more complete way. This figure shows also that

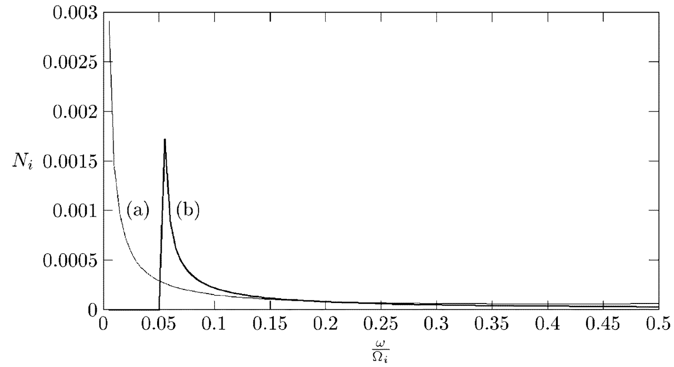


Fig. 2. The imaginary part of the refractive index $N_i$ as a function of $\omega/\Omega_i$, using the cold-plasma model, for $\alpha = 10^{-4}$ cm, $n_{i0} = 10^8$ cm$^{-3}$, $n_{d0} = 10^4$ cm$^{-3}$, $B_0 = 0.2$ T, and $T = T_e = T_i = 1$ eV. Curve (a) was calculated using the cold-plasma limit of the procedure presented in Section IV-B, while curve (b) corresponds to the calculation using the procedure presented in Section IV-A.

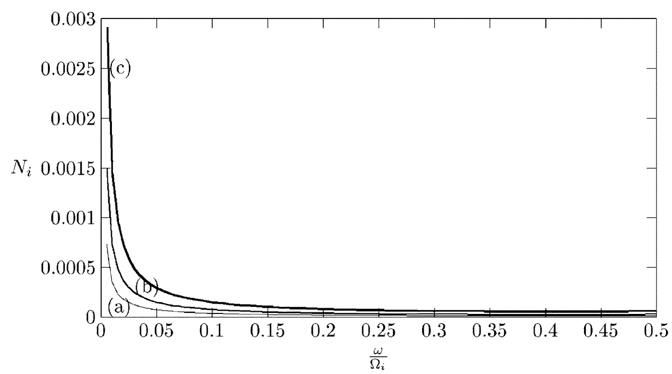


Fig. 3. The imaginary part of the refractive index $N_i$ as a function of $\omega/\Omega_i$, including Larmor-radius effects, for $n_{d0} = 10^4$ cm$^{-3}$, $n_{i0} = 10^8$ cm$^{-3}$, $\alpha = 10^{-4}$ cm, and $B_0 = 0.2$ T. The curves labeled by (a), (b), and (c) correspond to $T = T_e = T_i = 0.06, 0.25$, and 1 eV, respectively.

the effects of charge variation of the dust particles become more important for lower values of $\omega/\Omega_i$.

In Fig. 3, we see that for fixed values of $\omega/\Omega_i, N_i$ increases with increasing temperature. For $\omega/\Omega_i > 0.2$, the values of $N_i$ are approximately equal and very small. Again, we observe that the influence of the charge variation of the dust particles is enhanced at low frequencies. We remark that when $\omega$ increases relative to the charging frequency $\nu_{\rm ch}$, the importance of the charge variation of the dust particles decreases, and accordingly $N_i$ must decrease, because it is directly connected with charge variation.

From the numerical analysis [16], we observe that the dominant contributions of the Larmor radius arise from the part $\epsilon_{ij}^C$ of the dielectric tensor. The contributions of order $N_\perp^2$ arising from $\epsilon_{ij}^N$ are, for the situations tested, negligible.

## V. Propagation Parallel to $\mathbf{B}_0$ and Maxwellian Distribution Function

Again, if $f_{\beta 0}$ is a Maxwellian distribution, we get, from (15), $L(f_{\beta 0}) = 0$ and the effects of charge variation in the term $\epsilon_{ij}^C$, given by (11), only occurs in the resonant denominator. The resonant denominator is modified by the addition of a purely imaginary term which contains the collision frequency of electrons and ions with dust particles.

In the case of the propagation exactly parallel to the external magnetic field, the term $\epsilon_{ij}^N$, given by (18), only occurs for $i = j = z$ (whether $f_{\beta 0}$ is or is not a Maxwellian distribution).

It can be shown that the nonzero contributions to $\epsilon_{ij}$, in the case of propagation parallel to the external magnetic field and Maxwellian distributions for the electrons and ions, are

$$\overset{\leftrightarrow}{\epsilon} = \begin{pmatrix} \epsilon_{xx}^C & \epsilon_{xy}^C & 0 \\ -\epsilon_{xy}^C & \epsilon_{xx}^C & 0 \\ 0 & 0 & \epsilon_{zz}^C + \epsilon_{zz}^N \end{pmatrix} \tag{78}$$

where

$$\epsilon_{xx}^C = 1 + \frac{1}{4}\sum_\beta X_\beta [\hat{I}_\beta^+ + \hat{I}_\beta^-] \tag{79}$$

$$\epsilon_{xy}^C = -\frac{i}{4}\sum_\beta X_\beta [\hat{I}_\beta^+ - \hat{I}_\beta^-] \tag{80}$$

$$\epsilon_{zz}^C = 1 + \sum_\beta X_\beta \hat{I}_\beta^0 \tag{81}$$

with

$$\hat{I}_\beta^\pm \equiv \frac{1}{n_{\beta 0}} \int d^3p \frac{p_\perp \partial f_{\beta 0}/\partial p_\perp}{1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} \pm \frac{\Omega_\beta}{\omega} + i\frac{\nu_{\beta d}^0(p)}{\omega}} \tag{82}$$

$$\hat{I}_\beta^0 \equiv \frac{1}{n_{\beta 0}} \int d^3p \frac{p_\perp \partial f_{\beta 0}/\partial p_\perp (p_\parallel/p_\perp)^2}{1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} + i\frac{\nu_{\beta d}^0(p)}{\omega}} \tag{83}$$

and

$$\epsilon_{zz}^N = -\frac{4\pi i n_{d0}}{\omega} U_z S_z \tag{84}$$

where

$$U_z \equiv \frac{i}{\omega + i(\nu_{\rm ch} + \nu_1)} \sum_\beta \frac{q_\beta}{m_\beta^2} \times \int d^3p \frac{p_\perp \sigma_\beta'(p) f_{\beta 0} p}{1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} + i\frac{\nu_{\beta d}^0(p)}{\omega}} \left(\frac{p_\parallel}{p_\perp}\right) \tag{85}$$

$$S_z \equiv \frac{1}{\omega n_{d0}} \sum_\beta q_\beta^2 \int d^3p \frac{\nu_{\beta d}^0(p)}{\omega} \times \frac{\partial f_{\beta 0}/\partial p_\perp}{1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} + i\frac{\nu_{\beta d}^0(p)}{\omega}} \left(\frac{p_\parallel}{p_\perp}\right) \tag{86}$$

with

$$\nu_{\rm ch} = -\sum_\beta \frac{q_\beta}{m_\beta} \int d^3p \sigma_\beta'(p) p f_{\beta 0} \tag{87}$$

$$\nu_1 = \sum_\beta \frac{q_\beta}{m_\beta} \int d^3p \frac{\left[i\nu_{\beta d}^0(p) \big/ \omega\right] \sigma_\beta'(p) f_{\beta 0} p}{1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} + i\frac{\nu_{\beta d}^0(p)}{\omega}}. \tag{88}$$

In the case of wave propagation with electric field parallel to $\mathbf{B}_0$, that is $\hat{E}_x = \hat{E}_y = 0$, we obtain

$$\epsilon_{zz}^C + \epsilon_{zz}^N = 0 \tag{89}$$

which is the dispersion relation for longitudinal electrostatic modes.

In the case of wave propagation with the electric field in the perpendicular direction to $\mathbf{B}_0$, that is $\hat{E}_z = 0$, we obtain the dispersion relation

$$\left[N_\parallel^2 - \epsilon_{xx}^C\right]^2 + \left(\epsilon_{xy}^C\right)^2 = 0. \tag{90}$$

### A. Alfvén Waves

In this section, we study the effects of the dust particles on the propagation and damping of the Alfvén waves.

The dispersion relation for Alfvén waves is given by (90). We can write the components of the dielectric tensor that occurs in this dispersion relation in the following way [21]:

$$\epsilon_{xx}^C = \hat{\epsilon}_{xx} - i\epsilon_{xx}^q \tag{91}$$

$$\epsilon_{xy}^C = \epsilon_{xy}^q + i\hat{\epsilon}_{xy} \tag{92}$$

where

$$\hat{\epsilon}_{xx} \equiv 1 + \frac{1}{4}\sum_\beta X_\beta \left[I_\beta^{(+)L} + I_\beta^{(-)L}\right] \tag{93}$$

$$\hat{\epsilon}_{xy} \equiv -\frac{1}{4}\sum_\beta X_\beta \left[I_\beta^{(+)L} - I_\beta^{(-)L}\right] \tag{94}$$

$$\epsilon_{xx}^q \equiv \frac{1}{4}\sum_\beta X_\beta \left[I_\beta^{(+)q} + I_\beta^{(-)q}\right] \tag{95}$$

$$\epsilon_{xy}^q \equiv -\frac{1}{4}\sum_\beta X_\beta \left[I_\beta^{(+)q} - I_\beta^{(-)q}\right] \tag{96}$$

with

$$I_\beta^{(\pm)L} \equiv \int \frac{d^3p}{n_{\beta 0}} \frac{p_\perp \partial f_{\beta 0}/\partial p_\perp \left(1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} \pm \frac{\Omega_\beta}{\omega}\right)}{\left(1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} \pm \frac{\Omega_\beta}{\omega}\right)^2 + \left(\frac{\nu_{\beta d}^0(p)}{\omega}\right)^2} \tag{97}$$

$$I_\beta^{(\pm)q} \equiv \int \frac{d^3p}{n_{\beta 0}} \frac{p_\perp \partial f_{\beta 0}/\partial p_\perp \left(\nu_{\beta d}^0 \big/ \omega\right)}{\left(1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} \pm \frac{\Omega_\beta}{\omega}\right)^2 + \left(\frac{\nu_{\beta d}^0(p)}{\omega}\right)^2}. \tag{98}$$

Equations (93) and (94) have some similarity to the corresponding equations in an electron-ion plasma. However, in the present case there are modifications introduced via the quasi-neutrality condition $(n_{i0} \neq n_{e0})$, that occurs in terms with $X_\beta \equiv \omega_{p\beta}^2/\omega^2$ and also effects of charge variation of the dust particles present in terms with the frequencies ratio $\nu_{\beta 0}/\omega$.

Integrals $I_\beta^{(\pm)q}$ in (95) and (96) are different of zero only in the case of a dusty plasma with dust particles with variable charge. Thus, $\epsilon_{ij}^q$, with $i = x$ and $j = x, y$, do not have a similar counterpart in an electron-ion plasma.

Using (91) and (92) in (90), it is possible to split the dispersion relation in its real and imaginary contributions [21]

$$\begin{aligned}\Lambda_r(N_\parallel, \omega) \equiv {} & [N_\parallel^2 - \hat{\epsilon}_{xx} + \hat{\epsilon}_{xy}] \\ & \times [N_\parallel^2 - \hat{\epsilon}_{xx} - \hat{\epsilon}_{xy}] \\ & + \left[\epsilon_{xy}^q - \epsilon_{xx}^q\right]\left[\epsilon_{xy}^q + \epsilon_{xx}^q\right]\end{aligned} \tag{99}$$

$$\Lambda_i(N_\parallel, \omega) \equiv 2\left[\epsilon_{xx}^q N_\parallel^2 + \epsilon_{xy}^q \hat{\epsilon}_{xy} - \epsilon_{xx}^q \hat{\epsilon}_{xx}\right]. \tag{100}$$

The last term in the expression to $\Lambda_r(N_\parallel, \omega)$, (99), only contributes for a dusty plasma with dust particles with variable charge. Others terms in (99) have their limiting counterpart in an electron-ion plasma, but for a dusty plasma these terms are modified by dust particles. The imaginary part of the dispersion relation, given by (100), is nonvanishing only for dusty plasmas with dust particles with variable charge.

We will consider that the wave vector $\mathbf{k}$ is real and that frequency $\omega$ is complex, with $\omega = \omega_r + i\omega_i$ and $|\omega_i| \ll |\omega_r|$.

The real and imaginary parts of the wave frequency are obtained, respectively, from the equations [19]

$$\Lambda_r(N_\parallel, \omega_r) \simeq 0, \tag{101}$$

$$\omega_i \simeq -\frac{\Lambda_i(N_\parallel, \omega_r)}{\left.\frac{\partial \Lambda_r(N_\parallel, \omega)}{\partial \omega}\right|_{\omega=\omega_r}}. \tag{102}$$

We note from (99) that the effects of charge variation of dust particles are present in terms with $\epsilon_{ij}^q$ where $i = x$ and $j = x, y$. From (100) and (102), we can see that the imaginary part of the dispersion relation provide a new damping mechanism connected to charge variation of dust particles.

It is possible to rewrite the integrals $I_\beta^{(\pm)L}$ and $I_\beta^{(\pm)q}$, (97) and (98), respectively, in terms of the basic integrals

$$\begin{aligned}\mathcal{I}_{m,n}^{(\pm,\pm),\beta} \equiv {} & \int_0^\infty p^m e^{-p^2/2m_\beta T_\beta} \left(\frac{\nu_{\beta d}^0}{\omega}\right)^n \\ & \times \ln\left[\left(1 \pm \frac{k_\parallel p}{m_\beta \omega} \pm \frac{\Omega_\beta}{\omega}\right)^2 + \left(\frac{\nu_{\beta d}^0}{\omega}\right)^2\right] dp\end{aligned} \tag{103}$$

$$\begin{aligned}\mathcal{G}_{m,n}^{(\pm,\pm),\beta} \equiv {} & \int_0^\infty p^m e^{-p^2/2m_\beta T_\beta} \left(\frac{\nu_{\beta d}^0}{\omega}\right)^n \\ & \times \arctan\left[\frac{1 \pm \frac{k_\parallel p}{m_\beta \omega} \pm \frac{\Omega_\beta}{\omega}}{\nu_{\beta d}^0/\omega}\right] dp\end{aligned} \tag{104}$$

and use these to obtain more suitable expressions to numerical analysis, of the real and imaginary part of wave frequency. Details about the calculations are given in [21].

## VI. Conclusion

In this paper, we have discussed dusty plasma waves in magnetized plasmas whose constituents are electrons, ions, and charged dust grains with variable charge. Starting from the Vlasov–Maxwell equations and the charging equations, we have obtained a compact dielectric response function for electromagnetic disturbances in the magnetized dusty plasma. It is found that the presence of the dust in the plasma and the possibility of variable charge for the dust is capable of producing damping or amplification of waves which propagate in this plasma. A few applications are presented. We intend to explore applications more deeply in astrophysical, space, and laboratory plasmas, trying to understand the importance of the charge variation of the dust particles as a mechanism of damping of the different modes of propagation.

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