PLASMA EMISSION BY WEAK TURBULENCE PROCESSES

L. F. Ziebell¹, P. H. Yoon²,³, R. Gaelzer¹, and J. Pavan⁴

¹ Instituto de Física, UFRGS, Porto Alegre, RS, Brazil; luiz.ziebell@ufrgs.br, rudi.gaelzer@ufrgs.br
² Instituto de Física & Matemática, UFPEl, Peotá, RS, Brazil; jeel.pavan@ufpel.edu.br

Received 2014 September 29; accepted 2014 October 8; published 2014 October 27

Abstract

The plasma emission is the radiation mechanism responsible for solar type II and type III radio bursts. The first theory of plasma emission was put forth in the 1950s, but the rigorous demonstration of the process based upon first principles had been lacking. The present Letter reports the first complete numerical solution of electromagnetic weak turbulence equations. It is shown that the fundamental emission is dominant and unless the beam speed is substantially higher than the electron thermal speed, the harmonic emission is not likely to be generated. The present findings may be useful for validating reduced models and for interpreting particle-in-cell simulations.

Key words: radiation mechanisms; non-thermal – solar wind – Sun: radio radiation – turbulence

Online-only material: color figures

1. INTRODUCTION

Partial conversion of electrostatic wave energy to transverse electromagnetic (EM) wave energy during the late nonlinear stage of the bump-on-tail instability is called the plasma emission process, and the resultant radiation occurs at the plasma frequency and/or its harmonic(s). It is believed that the plasma emission is responsible for the solar type II and type III radio bursts (McLean & Labrum 1985; Goldman 1983; Melrose 1986). According to the standard theoretical paradigm, based upon the first theory proposed in the 1950s (Ginzburg & Zheleznyakov 1958), an electron beam interacting with the background plasma excites Langmuir (L) waves via the bump-on-tail instability, followed by nonlinear processes of wave decay and scattering, eventually leading to the generation of EM radiation.

Such an elaborate process can, in principle, be demonstrated on the basis of EM weak turbulence theory—the fundamental equations thereof and their derivation can be found, e.g., in (Yoon 2006; Yoon et al. 2012; Ziebell et al. 2014b). Indeed, over the past several decades, the physics of plasma emission was discussed within the framework of reduced or partial EM weak turbulence theory (McLean & Labrum 1985; Goldman 1983; Melrose 1986; Robinson & Cairns 1998a, 1998b, 1998c; Li et al. 2008a, 2008b, 2009). However, the complete numerical solution of the entire set of EM weak turbulence equations has not been done hitherto. Instead, various approximations and simplifications have been made, which include the assumption of saturated wave amplitudes, predetermination of certain nonlinear processes being the most important, reduction of the equations to one-dimensional (1D) models, etc.

The above comments are not meant to diminish the value of reduced theories, however, as some of these theories, especially those of Li et al. (2008a, 2008b, 2009) and Schmidt & Cairns (2014), have advanced to the point where simple, yet reliable, plasma emission models are incorporated into macroscopic models so as to yield global-kinetic models with predictive capabilities. Nevertheless, since reduced models are based upon various assumptions, it is difficult to quantify the validity of such approaches unless there exists a benchmark full numerical solution. The present Letter reports the first ever complete numerical solutions of the EM weak turbulence equations in the presence of a beam, and the ensuing plasma emission. On the basis of the present results, detailed verification of various approximate theories can be done, but such a task is not the immediate focus of the present work.

One of the outstanding problems in the theory of plasma emission is whether the radiation is at the plasma frequency (fundamental emission) or at the first harmonic of the plasma frequency (harmonic emission). Reduced theories cannot determine the relative importance of the two emission bands.

Before we proceed, it should be mentioned that a handful of authors carried out direct EM particle-in-cell (PIC) simulations in order to characterize the nonlinear behavior of the plasma emission process (Kasaba et al. 2001; Rhee et al. 2009a, 2009b). The present full solution of EM weak turbulence equations is complementary to PIC simulation efforts.

2. THEORETICAL FORMULATION

The wave intensities for plasma normal modes are defined by their electric field energy. For longitudinal modes, the intensities \( I_1 \) for \( \sigma = \pm 1 \) and \( \omega = L, S \), where \( L \) and \( S \) stand for Langmuir and ion sound, respectively, are defined by

\[
\langle \delta E_1^2 \rangle_{k,\omega} = \sum_{\omega = \pm 1} \sum_{\omega = L, S} I_1^{\sigma,\omega} \delta (\omega - \sigma \omega_0^L).
\]

The transverse mode \( T \) has both electric and magnetic fields, but it is sufficient to define the spectral wave intensity in terms of electric field,

\[
\langle \delta E_1^2 \rangle_{k,\omega} = \sum_{\omega = \pm 1} I_1^{T,\omega} \delta (\omega - \sigma \omega_0^T),
\]

as the magnetic field intensity is trivially given by \( \langle \delta B_1^2 \rangle_{k,\omega} = |e k / \omega|^2 \langle \delta E_1^2 \rangle_{k,\omega} \). The linear dispersion relations for electrostatic (Langmuir \( L \) and ion-sound \( S \)) and transverse EM (\( T \)) modes are given by

\[
\omega_0^L = \omega_{pe} (1 + 3 k^2 \lambda_D^2 / 2),
\]

\[
\omega_0^S = k c S (1 + 3 T_e / T_i)^{1/2} (1 + k^2 \lambda_D^2)^{-1/2},
\]

\[
\omega_0^T = (\omega_{pe}^2 + c^2 k^2)^{1/2},
\]

where \( \lambda_D \) is the Debye length, and \( T_e / T_i \) is the electron to ion temperature ratio. The 

where $\omega_{pe} = (4\pi n_e e^2/m_e)^{1/2}$ is the plasma frequency, $n_e$, $e$, and $m_e$ being the electron number density, unit electric charge, and electron mass, respectively, $T_P = (T_e/(4\pi n_e e^2))^{1/2}$ is the Debye length, $T_e$ and $T_i$ are electron and ion temperatures, respectively, and $c_s = (T_i/m_i)^{1/2}$ represents the ion-sound speed, $m_i$ being the ion (proton) mass. The $L$ mode wave kinetic equation is given by

$$\frac{\partial I_{k,k}^L}{\partial t} = \frac{4\pi e^2}{m_e k^2} \int d\lambda \delta(\alpha \omega_k^L - k \cdot \nu) \times \left[ n_e e^2 (F_e + \frac{\partial F_e}{\partial \nu} I_{k,k}^L) + \sum_{\sigma',\sigma=-1} \int dK V_{k,k}^{\sigma} \left( \frac{\alpha \omega_{k,k}^L I_{\sigma}^L I_{\sigma}^S}{\mu_{k-k}} - \frac{\sigma' \omega_{k,k}^L I_{\sigma}^L I_{\sigma}^S}{\mu_{k-k}} \right) \delta(\alpha \omega_k^L - \alpha' \omega_k^S - \sigma'' \omega_{\alpha k-k}^S) + \frac{n_e e^2}{\omega_{pe}^2} \left( \frac{\sigma \omega_k^L I_{\sigma}^L I_{\sigma}^S}{\mu_{k-k}} \right) + \frac{n_i e^2}{m_i} \frac{\partial F_i}{\partial \nu} \right] \right], \tag{1}$$

where

$$V_{k,k}^{\sigma} = \frac{\pi e^2 \sigma \omega_k^L \mu_{k-k}(k \cdot k)^2}{2T_e^2 k^2 k^2 |k-k|^2},$$

$$U_{k,k}^{\sigma} = \frac{\sigma \omega_k^L e^2}{n_i m_i^2 \omega_{pe}^2} \frac{(k \cdot k)^2}{k^2 k^2},$$

$$\mu_k = |k|^3 \left( \frac{m_e}{m_i} \right)^{1/2} \left( 1 + \frac{3T_i}{T_e} \right)^{1/2}. \tag{2}$$

The first velocity integral term on the right-hand side of Equation (1) that contains the resonance factor $\delta(\alpha \omega_k^L - k \cdot \nu)$ (i.e., linear wave-particle interaction) represents the spontaneous and induced emissions of $L$ waves, which are essentially quasilinear processes; the second $K$-integral term dictated by the three-wave resonance condition $\delta(\alpha \omega_k^L - \alpha' \omega_k^S - \sigma'' \omega_{\alpha k-k}^S)$ (nonlinear three-wave interaction) represents the decay/coalescence involving $L$ mode with another $L$ mode and an $S$ mode; the double integral term $\int d\lambda \int dK \cdots$ dictated by the nonlinear wave-particle resonance condition $\delta(\alpha \omega_k^L - \alpha' \omega_k^S - (k-k) \cdot \nu)$ represents the spontaneous and induced scattering processes involving two Langmuir waves and the particles.

For the ion-sound mode, $\alpha = S$, the wave kinetic equation is given by

$$\frac{\partial I_{k,k}^S}{\partial t} = \frac{4\pi m_e e^2}{m_i k^2} \int d\lambda \delta(\alpha \omega_k^S - k \cdot \nu) \left[ n_i e^2 (F_i + \frac{\partial F_i}{\partial \nu} I_{k,k}^S) + \sum_{\sigma',\sigma=-1} \int dK V_{k,k}^{\sigma} \left( \frac{\alpha \omega_{k,k}^S I_{\sigma}^L I_{\sigma}^S}{\mu_{k-k}} - \frac{\sigma' \omega_{k,k}^L I_{\sigma}^S}{\mu_{k-k}} \right) \delta(\alpha \omega_k^S - \alpha' \omega_k^L - \sigma'' \omega_{\alpha k-k}^S) + \frac{n_i e^2}{\omega_{pe}^2} \left( \frac{\sigma \omega_k^S I_{\sigma}^L I_{\sigma}^S}{\mu_{k-k}} \right) + \frac{n_i e^2}{m_i} \frac{\partial F_i}{\partial \nu} \right] \right], \tag{3}$$

where

$$V_{k,k}^{\sigma} = \frac{\pi e^2 \sigma \omega_k^L \mu_k(k' \cdot (k' - k))^2}{4T_e^2 k^2 k^2 |k-k|^2}. \tag{4}$$

The first velocity integral term on the right-hand side of Equation (3) that contains the resonance factor $\delta(\alpha \omega_k^S - \sigma' \omega_k^L - \sigma'' \omega_{\alpha k-k}^S)$ represents the spontaneous and induced emissions of $S$ waves. The $K$-integral term dictated by the three-wave resonance condition $\delta(\alpha \omega_k^S - \alpha' \omega_k^L - \sigma'' \omega_{\alpha k-k}^S)$ corresponds to the decay/coalescence involving $S$ mode with two $L$ modes.

For the transverse mode $T$, the wave kinetic equation is given by

$$\frac{\partial I_{k,k}^T}{\partial t} = \sum_{\sigma',\sigma} \int dK V_{k,k}^{T \sigma} \left( \frac{\sigma \omega_{k,k}^T I_{\sigma}^L I_{\sigma}^T}{\mu_{k-k}} - \frac{\sigma' \omega_{k,k}^L I_{\sigma}^T}{\mu_{k-k}} \right) \delta(\alpha \omega_k^T - \alpha' \omega_k^L - \sigma'' \omega_{\alpha k-k}^L) + \int dK V_{k,k}^{T \sigma} \left( \frac{\sigma \omega_k^S I_{\sigma}^L I_{\sigma}^S}{\mu_{k-k}} - \frac{\sigma' \omega_k^L I_{\sigma}^S}{\mu_{k-k}} \right) \delta(\alpha \omega_k^T - \alpha' \omega_k^L - \sigma'' \omega_{\alpha k-k}^L) + \int dK V_{k,k}^{T \sigma} \left( \frac{\sigma \omega_k^S I_{\sigma}^L I_{\sigma}^S}{\mu_{k-k}} - \frac{\sigma' \omega_k^L I_{\sigma}^S}{\mu_{k-k}} \right) \delta(\alpha \omega_k^T - \alpha' \omega_k^L - \sigma'' \omega_{\alpha k-k}^L)$$

$$+ \int dK V_{k,k}^{T \sigma} \left( \frac{\sigma \omega_k^L I_{\sigma}^T}{\mu_{k-k}} - \frac{\sigma' \omega_k^L I_{\sigma}^T}{\mu_{k-k}} \right) \delta(\alpha \omega_k^T - \alpha' \omega_k^L - \sigma'' \omega_{\alpha k-k}^L)$$

where the elements of the interaction matrix are

$$V_{k,k}^{T \sigma} = \frac{\pi e^2 \sigma \omega_k^L (k \times k)^2}{32m_i^2 \omega_{pe}^2 k^2 k^2 (|k-k|^2)^2} \left( \frac{k^2}{\sigma' \omega_k^L - \sigma'' \omega_{\alpha k-k}^L} - \frac{|k-k|^2}{\sigma' \omega_k^L - \sigma'' \omega_{\alpha k-k}^L} \right),$$

$$V_{k,k}^{T \sigma} = \frac{\pi e^2 \sigma \omega_k^L \mu_k-k(k \times k)^2}{4T_e^2 k^2 k^2 (|k-k|^2)^2},$$

$$V_{k,k}^{T \sigma} = \frac{\pi e^2 \sigma \omega_k^L (k \times k)^2}{4m_i^2 \omega_{pe}^2 (\omega_{pe}^2)^2} \left( 1 + \frac{(k \cdot k)^2}{k^2 k^2} \right),$$

$$U_{k,k}^{T \sigma} = \frac{\sigma \omega_k^L e^2 (k \times k)^2}{2n_i m_i^2 \omega_{pe}^2 k^2 k^2}. \tag{6}$$

The first $K$-integral terms in Equation (5) dictated by the three-wave resonance condition $\delta(\alpha \omega_k^S - \sigma' \omega_k^L - \sigma'' \omega_{\alpha k-k}^L)$ represents the coalescence of two $L$ modes into a $T$ mode at the second harmonic plasma frequency. This process is responsible for the harmonic emission ($T \leftrightarrow L + L$). The next $K$ integrals associated with the factor $\delta(\alpha \omega_k^L - \sigma' \omega_k^L - \sigma'' \omega_{\alpha k-k}^L)$
describe the merging of \( L \) and \( S \) modes into a \( T \) mode at the fundamental plasma frequency. This is one of the processes responsible for the fundamental emission \((T \leftrightarrow L + S)\). The third \( k' \) integrals with \( \delta(\sigma \omega_{e}^{k} - \sigma' \omega_{e}^{k'} - (k - k') \cdot v) \) depict the merging of a \( T \) mode and an \( L \) mode into the next higher harmonic \( T \) mode. This process, known as the incoherent Raman scattering, is responsible for generating higher-harmonic plasma emission \((T \leftrightarrow T + L)\). The double integral term \( \int d \nu / d k' \cdot \delta(\sigma \omega_{e}^{k} - \sigma' \omega_{e}^{k'} - (k - k') \cdot v) \) directly the thermal spread associated with the beam is taken to be the same in the actual heliospheric environment, this number should be \( \sim 10^{5} \) eV; thus, the present choice is approximately equal to the coronal value.

In Figure 1, we showcase a typical numerical result. For this case, we considered the initial beam speed \( v_{b}/v_{th} = 6 \), and have numerically integrated the complete equations up to the normalized time step \( \alpha_{pe} t = 2 \times 10^{5} \). The description can be found in the figure caption. The electron distribution function shown in Figure 1(a) features a broad range of velocity-space plateau in the velocity space initially occupied by a beam, which is a result of quasilinear diffusion. The Langmuir turbulence spectrum in Figure 1(b) shows an enhanced forward-propagating component (the primary \( L \)), which is the result of initial bump-on-tail instability, and the backscattered component, which is the result of combined three-wave decay and nonlinear wave-particle interaction. Note that the morphology of the 2D \( L \) mode spectrum compares favorably with the simulated spectrum—see, e.g., Figure 2 of Rhee et al. \( 2009b \).

Nonlinear decay processes generate the ion-sound mode by decay instability, as indicated in Figure 1(c) by the “enhanced” and “weakly enhanced” range of \( 2D k \) space. Panel (d) of Figure 1 is the transverse EM mode spectrum. We plot the electric field intensity associated with the radiation, \( I_{T}(k) \). The small \( k \) (or long wavelength) regime corresponds to the fundamental (F) emission near \( \omega_{pe} \), while the outer ring corresponds to the harmonic (H) emission with frequency in the vicinity of \( 2\omega_{pe} \). In short, the present Letter reports the first ever theoretical demonstration of F/H pair emission starting from the beam-plasma instability, by allowing all the relevant nonlinear processes to operate. This is in contrast to other related works, e.g., \( \text{Li et al. 2008a, 2008b, 2009} \), where simplified approaches are taken at the outset. Our rigorous numerical solutions are comparable to the full PIC simulations \( \text{Kasaba et al. 2001; Rhee et al. 2009a, 2009b} \), and indeed we find that the comparison with simulation results is favorable. For instance, the present Figure 1(d) can be compared against the simulated radiation spectral pattern, e.g., Figures 3 and 4 of Rhee et al. \( 2009b \). Upon visual inspection of the present Figure 1(d) and the simulated radiation pattern, it is quite obvious that the agreement is excellent. This shows that the present weak turbulence simulation is a reliable research tool for quantitative investigations of plasma emission phenomena, yet it is computationally much more efficient when compared with the full PIC code scheme.

We have repeated the self-consistent calculations for other parameters, but in Figure 2, we choose to show the angle-averaged electric field spectrum (in normalized form) versus the frequency \( \omega / \omega_{pe} \), for four different initial beam speeds, while other input parameters are held constant as in Figure 1. For \( v_{b}/v_{th} = 5 \), Figure 2 shows that the F emission completely dominates over H emission, but as the normalized beam speed increases to \( v_{b}/v_{th} = 6 \) we observe that the H component becomes more apparent. Proceeding to higher beam velocities, one can discern that the H component becomes progressively more intense, but it never becomes higher than the F component.

Figure 1. Panel (a) depicts the electron distribution function \( F_e(\mathbf{v}) \) vs. \( v_x/v_{th} \) and \( v_z/v_{th} \); panel (b) shows the Langmuir wave spectral intensity \( I_L(k) \) vs. \( k_x v_{th}/\omega_{pe} \) and \( k_z v_{th}/\omega_{pe} \); panel (c) plots the wave spectral intensity corresponding to the ion-sound mode, \( I_S(k) \); and panel (d) is the transverse mode electric field spectrum \( I_T(k) \). The above results are at normalized time \( \omega_{pe} t = 2 \times 10^3 \), with the initial condition \( V_b/v_{th} = 6 \). Other input parameters are described in the text.

(A color version of this figure is available in the online journal.)

Note that for the case of \( V_b/v_{th} = 8 \), there is a small, barely visible, enhancement at the third harmonic, \( \omega \simeq 3 \omega_{pe} \). This is the result of the incoherent Raman process, \( T \leftrightarrow T + L \), that generates higher harmonics. However, as one can see, the third-harmonic peak is extremely low so that, in general, higher harmonic plasma emission is not expected to be a common feature, which is consistent with observations. Also note that the transverse EM radiation at the plasma frequency and its harmonic takes place over a background level, as indicated in Figure 2. The theory of the background radiation for isotropic plasma in the absence of the beam was recently put forth by the present authors (Ziebell et al. 2014a), and it is supported by results obtained from PIC simulations (Ziebell et al. 2014b). It is this background level of radiation that partially hides the harmonic emission peak in the case of \( V_b/v_{th} = 5 \). The theory of background radiation (Ziebell et al. 2014b) is based upon the nonlinear wave-particle interaction, which some reduced theories have ignored at the outset (Li et al. 2008a, 2008b, 2009; Schmidt & Cairns 2014) can be tested, and it may also be employed to interpret the PIC simulations (Kasaba et al. 2001; Rhee et al. 2009a, 2009b). However, such a task is beyond the scope of the present work.

4. FINAL REMARKS

To summarize, despite many decades of theoretical research on the plasma emission, the actual numerical solution of the equations of EM weak turbulence theory that forms the basis of the plasma emission had not been available in the literature. In the absence of exact numerical solution, it is difficult to quantitatively verify the validity of various approximate models. The present Letter reports the first ever complete numerical solution of electromagnetic weak turbulence equations with which, the validity of various approximate theories (Li et al. 2008a, 2008b, 2009; Schmidt & Cairns 2014) can be tested, and it may also be employed to interpret the PIC simulations (Kasaba et al. 2001; Rhee et al. 2009a, 2009b). However, such a task is beyond the scope of the present work. One of the most interesting findings according to our complete numerical solutions is that the plasma emission must be dominated by the fundamental emission, and that the presence of harmonic emission may imply high average beam velocities. In fact, weak harmonic emission may be obscured by the background EM emission, which implies that if our numerical example is the...
norm, then most of the plasma emission has to take place at the fundamental plasma frequency. We should caution that this conclusion is based upon the weak turbulence paradigm. Alternative models of plasma emission, e.g., one that is based upon the concept of strong turbulence (Robinson, 1997), are not the focus of the present Letter.

This work has been partially supported by the Brazilian agencies CNPq and FAPERGS. P.H.Y. acknowledges NSF grant AGS1242331 and the BK21 plus grant from the National Research Foundation (NRF) of the Republic of Korea.

REFERENCES

Ginzburg, C. L., & Zheleznyakov, V. V. 1958, SVA, 2, 653
Goldman, M. V. 1983, SoPh, 89, 403
Kasabu, Y., Matsumoto, H., & Omura, Y. 2001, JGR, 106, 18693

Li, B., Cairns, I. H., & Robinson, P. A. 2008a, JGR, 113, A06104
Li, B., Cairns, I. H., & Robinson, P. A. 2008b, JGR, 113, A06105
Li, B., Cairns, I. H., & Robinson, P. A. 2009, JGR, 114, A02104
Rhee, T., Woo, M., & Ryu, C.-M. 2009b, JKPS, 54, 313
Robinson, P. A. 1997, RvMP, 69, 507
Yoon, P. H. 2006, PhPl, 13, 022302
Yoon, P. H., Ziebell, L. F., Gaelzer, R., & Pavan, J. 2012, PhPl, 19, 102303
Ziebell, L. F., Yoon, P. H., Gaelzer, R., & Pavan, J. 2014a, PhPl, 21, 012306

Figure 2. Panel (a) shows the normalized electric field intensity \(\langle \delta E^2(k) \rangle\) vs. normalized frequency \(\omega/\omega_{pe}\) for initial beam speed \(V_b/v_{th} = 5\); panel (b) corresponds to \(V_b/v_{th} = 6\); panel (c) corresponds to \(V_b/v_{th} = 7\); and panel (d) is the case of high beam speed, \(V_b/v_{th} = 8\). The above results are for normalized time \(\omega_{pe}t = 2 \times 10^3\).

(A color version of this figure is available in the online journal.)