Differences Between The $^3P_0$ and $^C^3P_0$ model in the Charming Strange Sector

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Differences Between The $^3P_0$ and $C^3P_0$ model in the Charming Strange Sector

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Abstract.
The goal of this work is to establish a comparison between the very well studied $^3P_0$ model and a bound-state corrected version, the $C^3P_0$ model, obtained from applying the Fock-Tani transformation to the $^3P_0$ model, in the context of the charmed-strange meson sector ($D_{sJ}$ mesons). In particular, we shall calculate the decay amplitudes and decay rates of the $D_{s1}(2460)^+ \rightarrow D_s^+ \pi^0$ and $D_{s1}(2536)^+ \rightarrow D^* (2010)^+ K^0$, showing the differences between the two models.

1. Introduction
The Fock-Tani formalism is a field theoretic method appropriated for the simultaneous treatment of composite particles and their constituents. This technique was originally used in atomic physics [1] and later in hadron physics to describe hadron-hadron scattering interactions [2, 3, 4] and meson decay [5, 6].

The $^3P_0$ model is a typical decay model which considers only OZI-allowed decay processes. The model considers a quark-antiquark pair created with the vacuum quantum numbers which
interact with a meson in the initial state. It is described as the non-relativistic limit of a pair
creation Hamiltonian [7].

The Fock-Tani transformation is applied to a $\bar{q}q$ pair creation Hamiltonian, producing a
characteristic expansion in powers of the wave function, where the $^3P_0$ model is the lowest order
term in the expansion. The corrected $^3P_0$ model ($C^3P_0$) is obtained from higher orders terms
in this expansion, where terms containing the bound state kernel appear [5].

Both the $^3P_0$ model and $C^3P_0$ model have been widely used in the study of meson
spectroscopy. Our motivation for this work is in a comparison of these models for mesons
in the charmed-strange sector. In particular, we shall calculate the decay amplitudes and decay
rates of $D_s(2460)^+ \to D^+\pi^0$ and $D_s(2536)^+ \to D^*(2010)^+K^0$.

2. Mesons in the Fock-Tani formalism: a brief outline

In the Fock-Tani formalism (FTf) we can write the meson creation operators in the following
form: $M^\dagger_{\alpha} = \Psi_{\alpha}^{\mu\nu} q_\mu \bar{q}_\nu$. A single particle state in second-quantization is
\[
|\alpha\rangle = M^\dagger_{\alpha} |0\rangle,
\]
where $\Psi_{\alpha}^{\mu\nu}$ is the bound-state wave-function for two-quarks. The quark and antiquark operators
obey the usual anticommutation relations. The composite meson operators satisfy non-canonical
commutation relations
\[
[M_\alpha, M_\beta] = 0 ; \quad [M_\alpha, M^\dagger_\beta] = \delta_{\alpha\beta} - \Delta_{\alpha\beta},
\]
where
\[
\Delta_{\alpha\beta} = \Phi_{\alpha}^{*\gamma\rho} \Phi_{\beta}^{\gamma\rho} q_\rho \bar{q}_\rho + \Phi_{\alpha}^{*\mu\gamma} \Phi_{\beta}^{\gamma\mu} q_\mu \bar{q}_\mu.
\]
The idea of the FTf is to make a representation change, where the composite particle operators
are described by "ideal particle" operators that satisfy canonical commutation relations, i.e.,
\[
[m_\alpha, m_\beta] = 0 ; \quad [m_\alpha, m^\dagger_\beta] = \delta_{\alpha\beta}.
\]
To implement this change of representation one can define a unitary transformation $U$ that maps
the composite state $|\alpha\rangle$ into an ideal state $|\alpha\rangle$. In the meson case, for example, we have
\[
U^{-1} M^\dagger_{\alpha} |0\rangle = m^\dagger_{\alpha} |0\rangle \equiv |\alpha\rangle,
\]
where $U = \exp (tF)$ and $F$ is the generator of the meson transformation given by
\[
F = m^\dagger_{\alpha} \tilde{M}_\alpha - \tilde{M}^\dagger_{\alpha} m_\alpha,
\]
with $\tilde{M}_\alpha$ defined up to third order
\[
\tilde{M}_\alpha = M_\alpha + \frac{1}{2} \Delta_{\alpha\beta} M_\beta + \frac{1}{2} M^\dagger_\beta \Delta_{\beta\gamma} M_\gamma.
\]

3. The Microscopic Model

The Hamiltonian used in this model is inspired in the $^3P_0$ model, deduced in [7]:
\[
H_I = g \int d^3x \bar{\Psi}(\bar{x}) \gamma^0 \Psi(\bar{x})
\]
where $\Psi(\vec{x})$ is the Dirac quark field, one should note that the bilinear $\Psi^\dagger \gamma^0 \Psi$ leads to the decay $(q\bar{q})_A \to (q\bar{q})_B + (q\bar{q})_C$ through the $b^d$ term. Introducing the following notation $b \to q; d \to \bar{q}$, $\mu = (p', s') e \nu = (p, s)$, after the expansion in the momentum representation, one obtains a compact notation for $H_I$:

$$H_I = V_{\mu\nu} \cdot q_\mu \bar{q}_\nu$$

where the sum (integration) is applied over repeated indeces and

$$V_{\mu\nu} \equiv -\gamma^0 \delta_{\mu \nu} \delta_{\mu \nu} \delta(n' + \nu) \chi_{\mu \nu}^* (\vec{p} \cdot \vec{p}_{\nu}) |\chi_{\nu}^* \rangle. \quad (2)$$

In Eq. (2) $\gamma$ is the free parameter pair production strength with $\gamma = g/2m_q$, where $m_q$ is the quark mass of the pair creation. Applying the Fock-Tani transformation to $H_I$ one obtains the effective Hamiltonian

$$H_{FT}^{C_P0} = U^{-1} H_I U = H_0 + \delta H_1$$

The decay amplitude $h_{fi}$ for $m_\gamma \to m_\alpha + m_\beta$, is given by

$$\langle f | H_{FT}^{C_P0} | i \rangle = \delta(P_\gamma - P_\alpha - P_\beta) h_{fi} \quad (3)$$

where $|i\rangle = m_{i\alpha}^\dagger|0\rangle$, $|f\rangle = m_{f\beta}^\dagger|0\rangle$ and

$$h_{fi} = -\frac{1}{2} \sum_{LS} C_{LS} (\Omega) \Phi_L^{\alpha \beta}$$

where $\Phi_L^{\alpha \beta}$ is the spin wave function for the particle from the initial state to the final state. In this kernel a sum is performed over mesons with the quantum numbers of the initial state.

4. Applications and Results

Now we shall consider some specific processes for a comparative study between the $^3P_0$ and $C^3P_0$ models. In particular, the decay processes studied are: $D_{s1}(2460)^+ \to D^+\pi^0$ and $D_{s1}(2536)^+ \to D^*(2010)^+ K^0$. The full expressions for the decay amplitudes $h_{fi}$ has the following form

$$h_{fi} = \left[ \frac{\gamma}{\pi^2 (\rho + 1)^2} \right] \sum_{LS} C_{LS} Y_{LM} (\Omega) \, ,$$

where $\chi$ is spin; $f$ is flavor and $C$ are color coefficients. The spatial part is given by the SHO wave-functions [8].
where the coefficients $C_{LS}$ are polynomials which have a dependence on the momentum $P$ and in $\beta$ gaussian width of the mesons involved in the processes. The decay amplitude $h_{fi}$ can be combined with relativistic phase space to give the decay rate [5, 7]:

$$\Gamma_{A \rightarrow BC} = 2\pi P \frac{E_BE_C}{M_A} \left( \frac{\gamma}{\pi^{1/4}(\rho + 1)^2} \right)^2 \sum_{LS} (C_{LS})^2.$$

The experimental values are extracted from “Particle Data Group 2010” (PDG) [9] and the theoretical values obtained with $3P_0$ and $C^3P_0$ model for these processes are shown in tables 1 and 2. In Tab. 1 we can see that the $3P_0$ model is zero for the decay processes with final state $D_{SJ} \pi$. In Tab. 2 the two models obtain the equal results. For the theoretical results presented in tables the $\gamma$ and $\beta$ (in GeV) are $\gamma = 0.420$, $\beta_{K} = 0.410$, $\beta_{K^{0}} = 0.399$, $\beta_{D^{*(2010)+}} = 0.280$, $\beta_{D^{++}} = 0.200$ and for the intermediate state $\beta_8 = 0.100$ and $\beta_7 = 0.630$ is the state $1^1S_0$ and the intermediate state $\beta_8 = 0.300$ and $\beta_9 = 0.200$ is the state $1^3S_1$.

### Table 1. Experimental values of the total decay rates and branching ratios for the meson $D_{s1}(2460)^+$.  

<table>
<thead>
<tr>
<th>Process</th>
<th>Exp. (PDG)</th>
<th>$3P_0$</th>
<th>$C^3P_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{D^{++},\pi^0}/\Gamma_{tot}$</td>
<td>0.48 ± 0.11</td>
<td>0</td>
<td>0.014</td>
</tr>
</tbody>
</table>

### Table 2. Experimental values of the total decay rates and branching ratios for the meson $D_{s1}(2536)^+$.  

<table>
<thead>
<tr>
<th>Process</th>
<th>Exp. (PDG)</th>
<th>$3P_0$</th>
<th>$C^3P_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{D^{<em>(2010)+,K^0}/\Gamma_{D^{</em>(2010)+,K^0}}$</td>
<td>0.72 ± 0.05 ± 0.01</td>
<td>0.995</td>
<td>0.995</td>
</tr>
</tbody>
</table>

5. Conclusions  
Briefly we presented a comparison of the $3P_0$ model and the Corrected $3P_0$ model applied for two meson decay processes of the charmed-strange sector. In this sector the decay processes are of two forms: $D_{SJ}^+ \rightarrow D_{SJ}\pi$ and $D_{SJ}^+ \rightarrow D^+K$. The first can not be obtained in the $3P_0$ model, but the $C^3P_0$ model can be applied for both decay processes. The next step will be to consider the other $D$ decay channels in the Corrected $3P_0$ model.

6. Acknowledgements  
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7. References


