PROBING THE SPIN-ORBIT COUPLING OF NUCLEAR PROTONS★

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The strong absorption in quasi-free scattering opens a possibility of measuring the spin-orbit coupling of nuclear protons. Cross sections for polarized incoming protons are expected to exhibit pronounced asymmetries, with size and sign depending on the selected momentum and the spin-orbit coupling in the nuclear state involved.

In this letter we show that the spin-orbit coupling experienced by nuclear protons should be directly measurable through quasi-free scattering [1, 2], if the usual description of this process by the distorted wave impulse approximation [3] is valid.

We first explain the idea involved qualitatively. Consider the example of a coplanar quasi-free (p, 2p) process in ^{16}O depected in fig. 1. For the kinematics shown in this figure, protons stemming from quasi-free collisions at the right side of the nucleus have on the average to traverse less nuclear matter than the ones originating from the left side. Because the mean free path of a medium energy proton is of the order of the ^{16}O radius, the right side of the nucleus contributes considerably more to the quasi-free cross section than the left side does. This is in particular true if the mean free path in nuclear matter corresponding to the energy E_2 is smaller than those corresponding to the energies E_0 and E_1, a situation which can actually be achieved. To avoid unessential complications we take the extreme single particle model and consider the knock-out of protons from the $j = 3/2$ and $j = 1/2$ states. If the momentum of the knocked out proton is selected, through momentum conservation, to have not too small a value k_3 in about the direction indicated in fig. 1, then the dominance of the right side of the nucleus will for single-particle states classically emphasize a definite orientation of the orbital angular momentum orthogonal to the scattering plane (clockwise in the situation of the figure). This again favours, through the spin-orbit coupling, a definite spin orientation of the nuclear proton (in fig. 1, clockwise for $j = 3/2$ and counterclockwise for $j = 1/2$). Thus the combined result of absorption and spin-orbit coupling is that the nuclear proton in the quasi-free scattering process has an effective polarization orthogonal to the scattering plane. This polarization can be

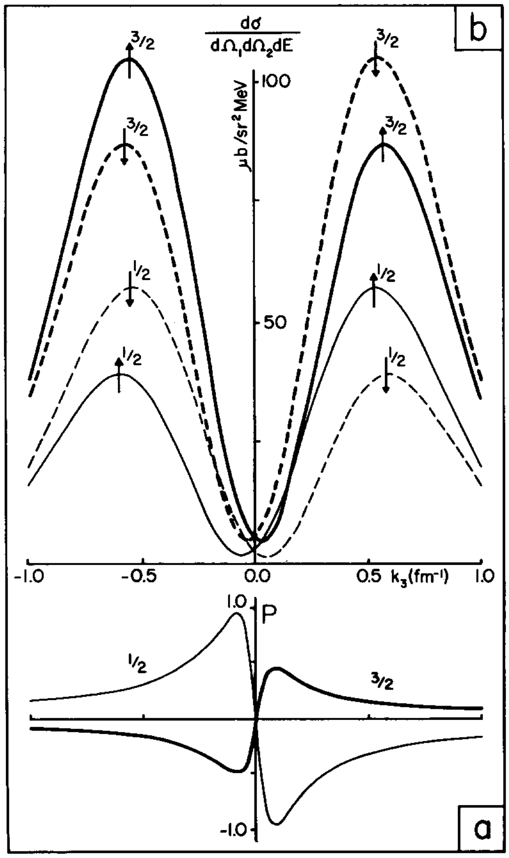

Fig. 1. The approximate geometry relevant to the discussion.

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measured using polarized incoming protons and exploiting the considerable difference between the spin up-spin up and spin down-spin up proton-proton cross sections caused by the large difference in triplet and singlet scattering.

The proposed experiment consists therefore of measurements of the coplanar quasi-free cross section in a suitable asymmetric geometry for the two polarization directions (orthogonal to the scattering plane) of the incoming proton, or, equivalently by rotational invariance, in two suitable mirror geometries keeping the polarization of the incoming proton fixed†.

Quantitatively, the quasi-free cross section for polarized protons is given by [5]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_1\mathrm{d}\Omega_1\mathrm{d}E_2\mathrm{d}\Omega_2} = \frac{4}{(\hbar c)^2}\frac{k_1k_2\bar{E}_0^2}{k_0E_3}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(1,2;0,3) \qquad (1)$$

$$\times\sum_m\sum_n |g_m'^{(n)}(k_3)|^2\,\delta(E_1+E_2+E_{A-1}-E_0-E_A),$$

where $\mathrm{d}\sigma(1,2;0,3)/\mathrm{d}\Omega$ is the free cross section (in the centre of mass system) of protons having initial polarizations P_0 and $P_3(k_3)$, with

$$P_3(k_3) = \frac{\sum_m\{|g_m'^{(+)}(k_3)|^2 - |g_m'^{(-)}(k_3)|^2\}}{\sum_m\sum_n |g_m'^{(n)}(k_3)|^2}. \qquad (2)$$

The distorted single particle momentum-spin wave functions $g'^{(\pm)}(k_3)$ in eqs. (1) and (2) replace, for our case, the more general overlap integrals of refs. [2] and [5], where also the rather obvious notation is defined.

We have made a crude distorted wave calculation for the p-states of ^{16}O, in the geometry of fig. 1, taking single particle harmonic oscillator wave functions and central, purely imaginary, square well optical potentials, all corresponding to a radius of 3.4 fm for ^{16}O. The other parameters used are: T_0 = 215 MeV, T_1 = 153 MeV respectively 146 MeV for $j = 1/2$ and $j = 3/2$, T_2 = 50 MeV, $\theta_1 = 30°$, $\theta_2 \approx 60°$, λ_0 = 4.0 fm, λ_1 = 3.6 fm and λ_2 = 2.8 fm. In calculating the proton mean free paths λ_i from proton-nucleon cross sections [6] the correction due to the Pauli principle for the appropriate Fermi momentum has been applied [7]. For the free proton-proton cross sections with parallel and antiparallel spins, which enter in $\mathrm{d}\sigma(1,2;0,3)/\mathrm{d}\Omega$, we took for simplicity the ones for the helicity states, i.e., we have symmetrized the asymmetric spin up-spin up cross section of protons polarized orthogonal to the scattering plane. (However, it will be interesting to observe, e.g. from the cross sections for s hole-states, to what degree this asymmetry is influenced by the nuclear environment). The cross section parametrization and phase shifts of ref. [8] have been used; relativistic effects on polarizations have been neglected. The analysing power of the free proton-proton scattering may be defined as

$$\left\{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\uparrow\downarrow) - \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\uparrow\uparrow)\right\}\Big/\left\{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\uparrow\downarrow) + \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\uparrow\uparrow)\right\},$$

which, with our choice of parameters has a value of about 2/3. For the two free cross sections we have taken average values, neglecting the relatively small variations caused by changes in the geometry.

The effective polarizations $P_3(k_3)$ for the p-states of ^{16}O ($j = 1/2$ and $j = 3/2$) are shown in fig. 2a, and figure 2b gives the corresponding angular correlation cross sections. In the calculations the momentum k_3 was varied by changing the direction of k_2 in the scattering plane keeping k_0, k_1 and $|k_2|$ fixed; consequently k_3 is approximately orthogonal to k_2. The incoming protons are assumed to be fully polarized orthogonal

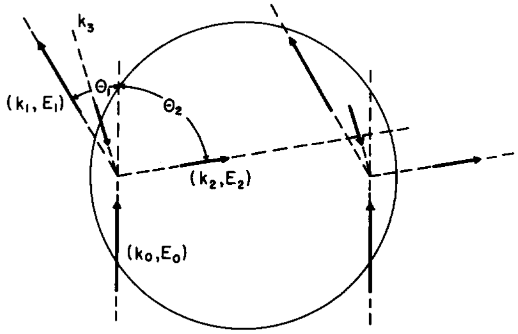

Fig. 2. Polarizations (a) and angular correlation cross sections (b) for coplanar quasi-free scattering of totally polarized 215 MeV protons in ^{16}O leading to p hole-states. The geometry is the one of fig. 1; k_0, k_1 and $|k_2|$ are fixed, θ_2 is varied and $k_3 = \pm |k_3|$ with the sign defined so that $\mathrm{d}k_3/\mathrm{d}\theta_2 < 0$. Thick (thin) lines refer to $j = 3/2$ ($j = 1/2$) and full (dashed) lines to spin up (spin down) of the incoming proton.

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† Experiments in which polarizations of the final or initial nuclei are measured have been proposed in ref. [4].

to the scattering plane. The calculated overall absorption is about 80%, which is a reasonable value [9, 10].

Because of the approximations made, the results of the present calculation should be considered as order of magnitude estimates; also the exact reflection and mirror symmetries in fig. 2 of the curves for one j-value are the result of our approximations. As harmonic oscillator wave functions have too small a tail at the nuclear surface, the calculated asymmetries are probably underestimates. For a quantitative comparison with future experiments more elaborate distorted wave computations are necessary.

Such a comparison would have a two-fold interest. First it might give meaningful and detailed answers to the questions: "To what degree is the optical model realistic if it is used to calculate the initial and final state interactions after a violent interaction in the nucleus has taken place? How good is the approximation of using optical potentials as obtained from elastic scattering in such cases? And how good is the impulse approximation for the prOcess under discussion?" The answers to these questions are at present uncertain and are of interest for reaction theory in general and essential for the quantitative interpretation of many medium and high energy processes occurring inside the nucleus, in particular of the incoherent ones. The comparison of any such a reaction with the corresponding free one will necessarily involve an estimate of the influence of the nucleus, which as a rule will be strong (e.g., an absorption of 80% was calculated above). It is natural to study this influence for such quasi-free (often inelastic) reactions first in detail for one of the simplest quasi-free processes, namely the elastic one which is discussed in this letter.

Second, if the usual distorted wave impulse approximation is reasonably good, the measured asymmetries will show characteristic variations in size and sign, depending on the state and momentum of the nuclear proton involved (see fig. 2). They would therefore be useful for the identification and investigation of the hole states resulting from quasi-free processes, which are at present the only means of studying holes in strongly bound nuclear shells.

From cross section estimates we believe that the discussed measurements, although they will certainly not be easy, can well be performed at most existing accelerators in the required energy range, for example by polarizing the incoming protons through elastic scattering on carbon. (Incidentally, this effect is in lowest order also the combined result of spin-orbit coupling and absorption [11]).

Similar discussions can be made for other types of quasi-free experiments. For example, in (e, e'p) scattering [12, 13] with unpolarized electrons leading to a hole state with definite non-zero orbital angular momentum, the knocked out proton will in general be polarized because of spin-orbit coupling and absorption, and this polarization is strongly dependent on the state and the momentum of the nuclear proton involved. The polarizations of outgoing protons with energies of 50, 100 and 150 MeV were calculated in the distorted wave Born approximation for the $p_{1/2}$ state in ^{16}O using single particle harmonic oscillator wave functions. In each case the resulting value for the polarization varies in the relevant momentum range between 1/4 and 1, but for intensity reasons such experiments would seem to be still inpractical.

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