Nonlinear $\sigma,\delta$ Couplings in a Relativistic Mean Field Theory for Neutron Star Matter

Sérgio S. Rocha*, César A.Z. Vasconcellos†, Moisés Razeira† and Manfred Dillig**

*Universidade do Extremo Sul Catarinense, CEP 88806-000 Criciuma, SC, Brazil
†Instituto de Física, Universidade Federal do Rio Grande do Sul, CEP 91501-970 Porto Alegre, Rio Grande do Sul, Brazil
**Institut für Theoretische Physik III, der Universität Erlangen-Nürnberg, D91058 Erlangen, Germany

Abstract. We study dense hadronic matter in a generalized relativistic mean field approach which contains nonlinear couplings of the $\sigma,\omega,\rho,\delta$ fields and compare its predictions for properties of neutron stars with the corresponding results from different models found in the literature. Our predictions indicate a substantial modification in static global properties of nuclear matter and neutron stars with the inclusion of the $\delta$ meson into the formalism.

LAGRANGIAN DENSITY MODEL

As the basic new feature of our approach, guided by a previous treatment[1], we expand and modify the scalar sector of the generalized relativistic mean field approach developed by Taurines et al.[2], by including in the lagrangian density, in addition to the scalar-isoscalar $\sigma$-meson, the scalar-isovector $\delta$-meson. Such an extension is supported mainly from the large isospin-asymmetry in neutron matter. The resulting lagrangian density describes a system of eight baryons ($B = p, n, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0$) coupled to four mesons ($\sigma, \omega, \rho$ and $\delta$) and two leptons ($e, \mu$) and contains a gradient coupling involving the baryons and the scalar-isoscalar and scalar-isovector meson fields:

$$L = \sum_{B=p,n,\Lambda,\Sigma^{-},\Sigma^{0},\Sigma^{+},\Xi^{-},\Xi^{0}} \left\{ \bar{\psi}_B \left[ \left( 1 + \frac{g_\sigma \sigma + g_\delta \vec{\delta} \cdot \vec{B}}{\alpha M_B} \right) \gamma^\mu (i\partial\mu - g_\omega \omega^\mu - \frac{1}{2} g_\rho \rho^\mu \cdot \rho^\mu) \right] \right\}_{\alpha}$$

$$- \left( 1 - \frac{g_\sigma \sigma}{M_B} \right) \bar{\psi}_B + \frac{1}{2} \left( 1 + \frac{g_\sigma \sigma}{\beta M_B} \right)^{2\gamma} \left( \frac{1}{2} m_\omega^2 \omega^\mu \omega^\mu - \frac{1}{4} \omega^\mu \omega^\nu \right)$$

$$+ \frac{1}{2} \left( 1 + \frac{g_\sigma \sigma}{\gamma M_B} \right)^{2\gamma} \left( \frac{1}{2} m_\rho^2 \rho^\mu \cdot \rho^\mu - \frac{1}{4} \rho^\mu \rho^\nu \cdot \rho^\mu \right)$$

$$+ \sum_{\lambda} \bar{\psi}_\lambda \left[ i \gamma^\mu \partial^\mu - m_\lambda \right] \psi_\lambda$$

where $\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$.

The different steps of our formulation may be synthesized in the following. We first introduce a re-scaling of the baryons and the $\omega$ and $\rho$ meson fields:
\[
\phi \rightarrow \left[ 1 + \frac{(g_{\sigma\sigma} + g_{\delta} \tau_{3B} \cdot \delta)}{\alpha M_B} \right]^{-\alpha/2} \phi
\]
with \( \phi = \psi_B, \omega^\mu, \rho^\mu \). Then we combine the resulting expressions with the lagrangian density (1) and define new coupling constants: 
\[
g_{\alpha\sigma B} = m_{\alpha B}^* \sigma; g_{\beta\omega B} = m_{\beta B}^* \omega; g_{\gamma\rho B} = m_{\gamma B}^* \rho; g_{\alpha\delta B} = m_{\alpha B}^* \delta
\]
with 
\[
m_{\kappa B}^* = (1 + (g_{\sigma\sigma} + g_{\delta} \tau_{3B} \cdot \delta_3) / \kappa M)^{-\kappa}; \text{ and with } (\kappa = \alpha, \beta, \gamma) \text{ where } \tau_{3B} \text{ is the third component of the baryons isospin vectors } \vec{\tau}_B \text{ and } \sigma \text{ and } \delta_3 \text{ are the mean field components of the } \sigma \text{ and } \delta \text{ mesons. Then, we determine the equation of state (EoS) for hadron matter and the values of the coupling constants of the theory which allows our model to reproduce static global properties of nuclear matter. As the final step we integrate the Tolman-Oppenheimer-Volkoff equations combined with the EoS of our approach to obtain standard plots of global static properties of neutron stars. The effective baryon mass is defined as}
\[
M_N^* = M_N \left(1 - g_{\sigma\sigma} \sigma \right) \eta \left[ 1 + \frac{(g_{\sigma\sigma} + g_{\delta} \tau_{3N} \cdot \delta)}{\alpha M_N} \right]^{-\alpha}.
\]

We note that in the limit \( \alpha = 1, \eta = 0 \) and \( g_{\delta} = 0 \), up to first order in \( \sigma \), the re-scaled lagrangian gives for the effective nucleon mass the well known Yukawa minimal coupling \( M_N^* \approx M_N - g_{\sigma\sigma} \sigma \). Similarly, taking \( \alpha = 0 \) and \( \eta = 1 \) the same result holds.

**COMMENTS AND RESULTS**

The inclusion of the \( \delta \) meson deserves some comments. Its mass is very high when compared to the nucleon mass, corresponding to a length scale of the strong interaction of the order of less than 0.4 Fermi. In most models found in the literature, the relevant physical phenomena are restricted to length scale \( > 0.5 \text{Fermi} \) with the dynamics at shorter length scales — after integrating out the contributions corresponding to heavier meson degrees of freedom — implicitly taken into account in the various coupling

![FIGURE 1](image-url). On the left, predictions for the maximum mass of neutron stars. On the right, mass-radius relation for neutron stars.
parameters of the theory. Assuming the coupling constants are natural (see discussion in Ref.[3]), it is possible in a consistent way to truncate in any desired order any expansion of the lagrangian density just by counting powers of the expansion parameters. A crucial aspect in order to accomplish convergence is to assume the expansion parameters are small in the desired physical domain. The inclusion of the $\delta$ meson in this formalism may represent — as it may test the dynamics at shorter length scales — a more accurate indication of the degree of naturalness of current effective models. Moreover, similarly to the $\sigma$ meson, the $\delta$ meson couples to the baryon masses introducing splittings whose signals depend on the value of the third component of the baryon isospin. These splittings correspond to mixed attractive and repulsive strong interaction components into the formalism and may cause a modification in the isospin-asymmetry in neutron matter. Even in case we do not consider the presence of the $\delta$ meson, our approach is different from the one developed by Zimanyi and Moszkowski (ZM model): a) our model contains different coupling terms in comparison to the ZM model; b) the coupling of the $\sigma$ meson to the $\omega$ has a different parametrization; c) there is an additional parameterized coupling of the $\sigma$ meson to the $\rho$ meson; d) the mass term is also different from the corresponding one in the ZM model; e) finally, our model approach contains the fundamental baryon octet, leptons and the equations of chemical equilibrium and charge neutrality.

**CONCLUSIONS**

Typical results in our formulation, considering chemical equilibrium and charge neutrality in neutron stars, are shown in the figures. In particular, our predictions for neutron star properties are in good agreement with recent results of the Vela pulsar (see Ref.[4]). Our main conclusion, after comparing the predictions of our model for global static properties of neutron stars with the corresponding results from different models found in the literature, indicate a substantial modification in the mentioned static global properties of nuclear matter and neutron stars with the inclusion of the $\delta$ meson into the formalism. Finally, we wish to mention that an accurate investigation of the degree of naturalness of our approach is presently one of our central topics of investigation and we leave for a future publication the discussion of this important topic.

**REFERENCES**

3. M. Razeira et al. (contribution to this volume).