**Abstract.** Having its origin in a successful mapping technique, the Fock-Tani formalism, has produced a corrected $^3P_0$ model ($C^3P_0$ model), which retains the basic aspects of the $^3P_0$ predictions with the inclusion of bound-state corrections. In high energy collisions many new mesons have been discovered in particular the enigmatic $D^+_S(2317)$ and $D^+_S(2460)$. The model is applied for $D_{SJ}$ mesons decay. The amplitude and its respective decay rates are evaluated.

**Keywords:** Decays of charmed mesons

**PACS:** 13.25.Ft

**INTRODUCTION**

Fock-Tani is a field theory formalism appropriated for the simultaneous treatment of composite particles and their constituents. The formalism was originally developed for the treatment of problems in atomic physics [1] and it was extended later on to the treatment of problems on hadron physics (hadron-hadron scattering interactions with constituent interchange [2, 3, 4], meson decay [5], glueballs [6] and exotics).

For a long time the pair creation models for strong hadronic decays have been formulated [7, 8, 9, 10]. The $^3P_0$ model is typical decay model which considers only OZI-allowed strong decays. The $^3P_0$ model considers a quark-antiquark pair creation in the presence of the initial state meson. The quark-antiquark pair is created with the vacuum quantum numbers and is defined in the non-relativistic limit of the pair creation Hamiltonian.

Applying the Fock-Tani transformation to the pair creation Hamiltonian it produces a characteristic expansion in powers of the wave function, where the $^3P_0$ model is the lowest order term of this expansion and represented by the Hamiltonian $H_{FT}$. The next step is to introduce the bound-state corrections (orthogonality corrections) to this “zero order” model. The Hamiltonian associated to this correction contains terms dependent on only one $\Delta$, called the bound state kernel. A new model is defined in order to correct the $^3P_0$, which we call the $C^3P_0$ model [5].

In high energy collisions many new mesons have been discovered in particular the enigmatic $D^+_S(2317)$ and $D^+_S(2460)$. The $C^3P_0$ model is applied for $D_{SJ}$ mesons decay, where the amplitude and its respective decay rates are evaluated.
THE MESON IN THE FOCK-TANI FORMALISM AND THE C\(^3\)P\(_0\) MODEL

In this section is presented, a brief review of the formal aspects regarding the Fock-Tani mapping procedure and how it is implemented to quark-antiquark meson states [2]. In the Fock-Tani formalism we can write the meson creation operators in the following form

\[ |\alpha\rangle = M_\alpha^\dagger |0\rangle, \]

where \(M_\alpha^\dagger\) is the meson creation operator that is defined by \(M_\alpha^\dagger = \Phi_{\alpha}^{\mu\nu} q_\mu \bar{q}_\nu\) in the Fock-Tani formalism. In this expression \(\Phi_{\alpha}^{\mu\nu}\) is the bound-state wave-functions for two-quarks and the quark and antiquark operators obey the following anticommutation relations

\[ \{q_\mu, q_\nu\} = \{\bar{q}_\mu, \bar{q}_\nu\} = \{q_\mu, \bar{q}_\nu\} = 0 \]

\[ \{q_\mu, q_\mu^\dagger\} = \{\bar{q}_\mu, \bar{q}_\mu^\dagger\} = \delta_{\mu\nu}. \tag{1} \]

The composite meson operators satisfy non-canonical commutation relations

\[ [M_\alpha, M_\beta] = 0; \ [M_\alpha, M_\beta^\dagger] = \delta_{\alpha\beta} - \Delta_{\alpha\beta}, \tag{2} \]

where \(\Delta_{\alpha\beta} = \Phi_{\alpha}^{*\mu\gamma} \Phi_{\beta}^{\gamma\rho} q_\rho \bar{q}_\mu + \Phi_{\alpha}^{\mu\gamma} \Phi_{\beta}^{*\rho\gamma} \bar{q}_\rho q_\mu\). The idea of the Fock-Tani formalism is to make a representation change, where the composite particles operators are described by operators that satisfy canonical commutation relations, i. e., which obey canonical relations

\[ [m_\alpha, m_\beta] = 0; \ [m_\alpha, m_\beta^\dagger] = \delta_{\alpha\beta}, \tag{3} \]

where \(m_\alpha^\dagger\) is the operator of the “ideal particle” creation. This way one can transform the single-meson state \(|\alpha\rangle\) into an elementary-meson state \(|\alpha\rangle\) by \(|\alpha\rangle \rightarrow |\alpha\rangle = U^{-1} |\alpha\rangle\), where the \(U\) operator must be unitary so that

\[ \langle \alpha | \alpha \rangle = (\alpha | \alpha \rangle; \ \langle \alpha | O | \alpha \rangle = (\alpha | O_{FT} | \alpha \rangle. \tag{4} \]

This transformation can be applied on the creation meson operators and the microscopic Hamiltonian. In this model the pair creation microscopic Hamiltonian, reduced to a compact form, is given by \(H_I = V_{\mu\nu} q_\mu^\dagger \bar{q}_\nu\) where

\[ V_{\mu\nu} = g \sum_{ss'} \int d^3p d^3p' \delta (\vec{p} + \vec{p}') u_{s'}^\dagger (\vec{p}') \gamma^0 u_s (\vec{p}), \tag{5} \]

and \(g = \frac{g}{2m_q}, m_q\) is the mass of both produced quarks and the sum (integration) is implied over repeated indexes. Applying the Fock-Tani transformation to \(H_I\) one obtains the effective Hamiltonian

\[ H_{FT}^{C^3P_0} = U^{-1} H_I U = H_0 + \delta H_1 \tag{6} \]

Now considering the transition \(m_\gamma \rightarrow m_\alpha + m_\beta\), our interest here is to calculate the \(h_{fi}\) (decay amplitude) which is given by

\[ \langle f | H_{FT}^{C^3P_0} | i \rangle = \delta (P_\gamma - P_\alpha - P_\beta) h_{fi} \tag{7} \]
where \(|i\rangle = m_i^\uparrow |0\rangle \) and \(|f\rangle = m_f^\uparrow m_f^\uparrow |0\rangle \). Developed the calculation, finally we find that the matrix element (7) is

\[
\langle f | H_{ET}^{C3P0} | i \rangle = -V_{i\mu} \left\{ \Phi^\mu_{\beta\tau} \Phi^\nu_{\alpha\eta} \Phi^\eta_{\beta\gamma} + \Phi^\nu_{\beta\tau} \Phi^\nu_{\alpha\rho} \Phi^\eta_{\gamma\tau} + \Phi^\nu_{\beta\rho} \Phi^\nu_{\alpha\eta} \Phi^\eta_{\gamma\tau} \right\}
- \frac{1}{4} V_{i\mu} \left\{ \Phi^\nu_{\alpha\tau} \Phi^\nu_{\beta\eta} \Delta(\rho \eta; \lambda \nu) \Phi^\lambda_{\gamma\tau} + \Phi^\nu_{\beta\tau} \Phi^\nu_{\alpha\eta} \Delta(\rho \eta; \lambda \nu) \Phi^\lambda_{\gamma\tau} \right\}
- \frac{1}{4} V_{i\mu} \left\{ \Phi^\nu_{\beta\tau} \Phi^\nu_{\alpha\eta} \Delta(\rho \eta; \mu \xi) \Phi^\sigma_{\gamma\tau} + \Phi^\nu_{\beta\rho} \Phi^\nu_{\alpha\eta} \Delta(\rho \eta; \mu \xi) \Phi^\sigma_{\gamma\tau} \right\}
+ \frac{1}{2} V_{i\mu} \left\{ \Phi^\nu_{\beta\tau} \Phi^\nu_{\alpha\eta} \Delta(\rho \eta; \mu \nu) \Phi^\sigma_{\gamma\tau} + \Phi^\nu_{\beta\rho} \Phi^\nu_{\alpha\eta} \Delta(\rho \eta; \mu \nu) \Phi^\sigma_{\gamma\tau} \right\}.
\]

The bound state kernel \(\Delta\), in this expression, represents an intermediate state of the decay process. As so, it assumes possible quantum numbers consistent with the overall symmetries. The meson wave function is defined as

\[
\Phi_{\alpha}^{\mu \nu} = \chi_{S_{\alpha}} f_{fa} C_{C_{l}C_{j}} \Phi_{nl}^{\bar{F}_{\alpha} - \bar{p}_{1} - \bar{p}_{2}}. \tag{8}
\]

where \(\chi, f\) and \(C\) are spin, flavor and color coefficients respectively. The spatial part of the wave function have the following form

\[
\Phi^{\bar{F}_{\alpha} - \bar{p}_{1} - \bar{p}_{2}}_{nl} = \delta(\vec{p}_{\alpha} - \vec{p}_{1} - \vec{p}_{2}) \Phi_{nl}(\vec{p}_{1}, \vec{p}_{2}) \tag{9}
\]

where \(\Phi_{nl}(\vec{p}_{1}, \vec{p}_{2})\) is given by simple harmonic wave functions.

**APPLICATIONS OF THE CORRECTED \(^3P_0\) MODEL**

In this work we apply, as an example, the \(C^3P_0\) model to the following decay process of the \(D_{s1}(2460)^+ \rightarrow D_S^+ \pi^0\) of the \(D_{SJ}\) mesons. The full expression for \(h_{fi}\) is

\[
h_{fi}^{C3P0} = \mathcal{C}_{21} Y_{20}(\Omega),
\]

where the \(\mathcal{C}_{21}\) polynomial is given by

\[
\mathcal{C}_{21} = \left( \frac{2^{3/2}}{3 \sqrt{3}} \right) \left\{ \frac{\sqrt{\beta} (\beta^2 + \beta_D^2)}{(3 \beta^2 + 4 \beta_D^2)^{7/2}} \exp \left[ -\frac{(3 \beta^4 + 5 \beta^2 \beta_D^2 + \beta_D^4) P^2}{4 \beta^2 \beta_D^2 (3 \beta^2 + 4 \beta_D^2)} \right] \right\}.
\]

and where \(\beta\) and \(\beta_D\) are respectively the Gaussian width of the mesons decay process and the intermediate state "\(D^+\)-meson" type contribution that arises from bound-state kernel. This \(h_{fi}\) decay amplitude is combined with relativistic phase space to give the decay rate

\[
\Gamma_{D_{s1}(2460)^+} = 2\pi P \frac{E_{D_S} E_{\pi}}{M_{D_{s1}(2460)}} \mathcal{C}_{21}^2
\]

The experimental value for this process is \(\Gamma < 1.68\) MeV. In the Corrected \(^3P_0\) model we obtain the decay rate of \(\approx 1.627\) MeV for this decay process, with \(\beta = 0.4\) GeV, \(\beta_D = 0.407\) GeV and \(\gamma = 1.4\).
SUMMARY AND CONCLUSIONS

We have calculated the $D_{S1} (2460)^+ \rightarrow D_S^{*+} \pi^0$ decay in the context of the $C^3P_0$ model. This channel is consistent with the experimental data, but the coupling parameter $\gamma$ was very above of the usual value of 0.4 [7]-[10]. A possible improvement, in order to obtain a lower $\gamma$ value, is the inclusion of different $\beta$ values for each particle in the decay. The next step will be to consider the other $D_{SJ}$ decay channels in the $C^3P_0$ model.

ACKNOWLEDGMENTS

This research was supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and Universidade Federal do Rio Grande do Sul (UFRGS).

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