Spectro-photometric distances to stars: A general purpose Bayesian approach

Basilio X. Santiago\textsuperscript{1,2}, Dorothée E. Brauer\textsuperscript{3}, Friedrich Anders\textsuperscript{3,2}, Cristina Chiappini\textsuperscript{3,2}, Anna B. Queiroz\textsuperscript{1,2}, Léo Girardi\textsuperscript{1,2}, Helio J. Rocha-Pinto\textsuperscript{5,2}, Eduardo Balbinot\textsuperscript{1,2}, Luiz N. da Costa\textsuperscript{6,2}, Marcio A.G. Maia\textsuperscript{6,2}, Mathias Schultheis\textsuperscript{7}, Matthias Steinmetz\textsuperscript{3}, Andrea Miglio\textsuperscript{8}, Josefin Montalbán\textsuperscript{9}, Donald P. Schneider\textsuperscript{10,11}, Timothy C. Beers\textsuperscript{12}, Peter M. Frinchaboy\textsuperscript{13}, Young Sun Lee\textsuperscript{14}, and Gail Zasowski\textsuperscript{15}

\textsuperscript{1} Instituto de Física, Universidade Federal do Rio Grande do Sul, Caixa Postal 15051, 91501-970 Porto Alegre, Brazil
\textsuperscript{2} Laboratório Interinstitucional de e-Astronomia – LInEA, Rua Gal. José Cristino 77, 20291-400 Rio de Janeiro, Brazil
\textsuperscript{3} Leibniz-Institut für Astrophysik Potsdam (AIP), An der Sternwarte 16, 14482 Potsdam, Germany
\textsuperscript{4} Osservatorio Astronomico di Padova – INAF, Vicolo dell’Osservatorio 5, 35122 Padova, Italy
\textsuperscript{5} Universidade Federal do Rio de Janeiro, Observatório do Valongo, Ladeira do Pedro An tônio 43, 20080-090 Rio de Janeiro, Brazil
\textsuperscript{6} Observatório Nacional, Rua Gal. José Cristino 77, 20291-400 Rio de Janeiro, Brazil
\textsuperscript{7} Observatoire de la Côte d’Azur, Laboratoire Lagrange, CNRS UMR 7923, BP 4229, 06304 Nice Cedex, France
\textsuperscript{8} School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham, B15 2TT, UK
\textsuperscript{9} Institut d’Astrophysique et de Géophysique, Allée du 6 août, 17 – Bât. B5c, 4000 Liège 1 (Sart-Tilman), Belgium
\textsuperscript{10} Department of Astronomy and Astrophysics, The Pennsylvania State University, University Park, PA 16802, USA
\textsuperscript{11} Institute for Gravitation and the Cosmos, The Pennsylvania State University, University Park, PA 16802, USA
\textsuperscript{12} Department of Physics and JINA Center for the Evolution of the Elements, University of Notre Dame, Notre Dame, IN 46556, USA
\textsuperscript{13} Department of Physics & Astronomy, Texas Christian University, TCU Box 298840, Fort Worth, TX 76129, USA
\textsuperscript{14} Department of Astronomy and Space Science, Chungnam National University, 34134 Daejeon, Republic of Korea
\textsuperscript{15} Dept. of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21210, USA

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ABSTRACT

Context. Determining distances to individual field stars is a necessary step towards mapping Galactic structure and determining spatial variations in the chemo-dynamical properties of stellar populations in the Milky Way.

Aims. In order to provide stellar distance estimates for various spectroscopic surveys, we have developed a code that estimates distances to stars using measured spectroscopic and photometric quantities. We employ a Bayesian approach to build the probability distribution function over stellar evolutionary models given these data, delivering estimates of model parameters (including distances) for each star individually. Our method provides several alternative distance estimates for each star in the output, along with their associated uncertainties. This facilitates the use of our method even in the absence of some measurements.

Methods. The code was first tested on simulations, successfully recovering input distances to mock stars with ≤1% bias. We found the uncertainties scale with the uncertainties in the adopted spectro-photometric parameters. The method-intrinsic random distance uncertainties for typical spectroscopic survey measurements amount to around 10% for dwarf stars and 20% for giants, and are most sensitive to the quality of log g measurements.

Results. The code was then validated by comparing our distance estimates to parallax measurements from the HIPPARCOS mission for nearby stars (<300 pc), to asteroseismic distances of CoRoT red giant stars, and to known distances of well-studied open and globular clusters. The photometric data of these reference samples cover both optical and infrared wavelengths. The spectroscopic parameters are also based on spectra taken at various wavelengths, with varying spectral coverage and resolution: the Sloan Digital Sky Survey programs SEGUE and APOGEE, as well as various ESO instruments.

Conclusions. External comparisons confirm that our distances are subject to very small systematic biases with respect to the fundamental HIPPARCOS scale (+0.4% for dwarfs, and +1.6% for giants). The typical random distance scatter is 18% for dwarfs, and 26% for giants. For the CoRoT-APOGEE sample, which spans Galactocentric distances of 4–14 kpc, the typical random distance scatter is ≲15% both for the nearby and farther data. Our distances are systematically larger than the CoRoT distances by about 9%, which can mostly be attributed to the different choice of priors. The comparison to known distances of star clusters from SEGUE and APOGEE has led to significant systematic differences for many cluster stars, but with opposite signs and substantial scatter. Finally, we tested our distances against those previously determined for a high-quality sample of giant stars from the RAVE survey, again finding a small systematic trend of +5% and an rms scatter of 30%. Efforts are underway to provide our code to the community by running it on a public server.

Key words. stars: distances – Galaxy: structure – stars: statistics – methods: statistical – surveys

1. Introduction

A crucial step towards studying stellar populations and their variation across the Galaxy is to measure reliable stellar distances. Currently, parallax method is only viable for very local stars, although the recently launched Gaia satellite (Perryman et al. 2001) is expected to dramatically increase the number of parallax and proper motion measurements within a
significant portion of the Milky Way. Precise distances may also be obtained from asteroseismology, but only for a fraction of stars subject to systematic variability studies (Miglio 2012; Rodrigues et al. 2014). Indirect methods of distance determination, based on photometric and spectroscopic quantities and their relation to stellar absolute magnitudes, can be used for much more distant stars. Photometric distances have been applied to multiband optical and near infra-red data from different surveys to model the spatial distribution of Galactic stars and to study its substructures (e.g., Jurić et al. 2008; Correnti et al. 2010; Minniti et al. 2011). Recent Sloan Digital Sky Survey III (SDSS-III, Eisenstein et al. 2011) spectroscopic surveys, such as the Sloan Extension for Galactic Understanding and Exploration (SEGUE, Yanny et al. 2009) and the Apache Point Observatory Galactic Evolution Experiment (APOGEE, Allende Prieto et al. 2008a) have produced many more observational constraints, including stellar atmospheric parameters, kinematical and chemical data, which can be used to estimate reliable distances to tens of thousands of stars sampled from the SDSS and 2MASS (Skrutskie et al. 2006) photometric data. These surveys have already had a clear impact on our understanding of the Galaxy (Carollo et al. 2007, 2010, 2012; Lee et al. 2011a; Schlesinger et al. 2012; Cheng et al. 2012; Bovy et al. 2012b,a; Anders et al. 2014; Hayden et al. 2015). The recently completed RADial Velocity Experiment (RAVE) survey (Steinmetz et al. 2006) has also provided atmospheric parameters, radial velocities and chemical abundances for six individual elements for more than 400 000 stars (Kordopatis et al. 2013). Ongoing spectroscopic surveys, such as the Gaia-ESO Survey (GES; Gilmore 2012), Galactic Archaeology with HERMES survey (GALAH; Zucker et al. 2012), or LAMOST Experiment for Galactic Understanding and Exploration (LEGUE; Deng et al. 2013) are continuously increasing the number of stars with available spectroscopic information. In fact, these large spectro-photometric data sets will, when analysed together, enable a description of the structure and substructures of the Galaxy with unprecedented detail, excellent statistics, and full use of the 6D phase-space information. Phase-space reconstruction of stellar distributions, coupled with spectroscopic abundances for large samples, will continue to provide challenging quantitative tests to models of Galaxy formation and evolution.

In order to take full advantage of the large set of available spectroscopic and photometric parameters from recent surveys, probabilistic inference has been used by several authors to infer ages, absolute magnitudes, extinction, and distances, among other parameters (e.g., Pont & Eyer 2004; Jørgensen & Lindegren 2005; Baier-Jones 2011; Serenelli et al. 2013; Schönicke & Bergemann 2014). In the context of stellar distances, a series of papers with increasing levels of sophistication has been presented by the RAVE collaboration (Breddels et al. 2010; Zwitter et al. 2010; Burnett & Binney 2010). Burnett & Binney (2010) make use of a comprehensive set of measured parameters and their estimated uncertainties to infer, for each star, the probability distribution that a set of chosen stellar models generate the data. The method was further refined by Burnett et al. (2011) and Binney et al. (2014a), and later used to study Galactic chemo-dynamics in, e.g., Boeche et al. (2013a,b), Binney et al. (2014b) or Kordopatis et al. (2015).

In this paper, we follow a similar theoretical background as those authors, and implement a code that computes photometric-distances with the goal of mapping large stellar samples in three dimensions or in phase-space. We are motivated by the analyses of SDSS-III SEGUE and APOGEE data led by the Brazilian Participation Group (BPG) and the Leibniz-Institut für Astrophysik Potsdam (AIP), which are presented in Anders et al. (2014) and Brauer (2015). These papers use APOGEE giants and SEGUE G-dwarfs, respectively, to improve chemo-dynamical constraints to the Galactic components, especially the discs (see discussions in Minchev et al. 2014b; Chiappini et al. 2015). Since SEGUE and APOGEE targets were selected based on different photometric data and have different spectral coverage and resolution, our basic challenge is to ensure that accurate distances are computed in a homogeneous way for data sets of vastly different provenance. Our emphasis is therefore on confronting our distance estimates with as many reference samples as possible. A direct comparison with the RAVE distances obtained by Binney et al. (2014a) for the high-quality giant sample studied in Boeche et al. (2013a) is also provided. In Table 1 we list the main characteristics of the surveys to which we have applied our method.

The outline of the paper is as follows. In Sect. 2 we review the method, introduce our notation, and show the results of initial validation tests. An analysis of the performance of our code in terms of internal accuracy and precision is provided. We further discuss how biased stellar parameters and prior assumptions influence the estimated distances. In Sect. 3 we compare our distances to several previous distance determinations, which can be used as a reference, given their higher precision and more controlled systematics. We also compare our results for a high-quality sample of RAVE giant stars to those obtained by the RAVE collaboration. Our summary, conclusions, and future plans are provided in Sect. 4.

### Table 1. Spectroscopic stellar surveys for which distances were computed with our method.

<table>
<thead>
<tr>
<th>Survey</th>
<th>Year</th>
<th>$N_{\text{stars}}$</th>
<th>Spatial coverage</th>
<th>Typical $S/N$</th>
<th>$\lambda$ range</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEGUE (DR9 G dwarfs)</td>
<td>2004–2008</td>
<td>$\geq 120 000$ (35 000)</td>
<td>$\geq 1300$ deg$^2$</td>
<td>25 (35)</td>
<td>0.38–0.92 $\mu$m</td>
<td>2000</td>
</tr>
<tr>
<td>APOGEE (DR10 HQ giants)</td>
<td>2011–2014</td>
<td>$\geq 100 000$ (22 000)</td>
<td>$\geq 2800$ deg$^2$</td>
<td>100 (120)</td>
<td>1.5–1.7 $\mu$m</td>
<td>22 500</td>
</tr>
<tr>
<td>RAVE (DR4 HQ giants)</td>
<td>2006–2013</td>
<td>$\geq 500 000$ (9000)</td>
<td>$\geq 20 000$ deg$^2$</td>
<td>30 (70)</td>
<td>0.84–0.88 $\mu$m</td>
<td>7500</td>
</tr>
</tbody>
</table>

**Notes.** For each case we provide basic information, such as duration, number of stars for different signal-to-noise ratios ($S/N$), spatial and spectral coverage, and spectral resolving power.
In brief, assuming that the errors in the measured parameters follow a normal distribution, the probability that a measured value of some quantity, $x_i \pm \sigma_i$, is consistent with some theoretical value, $x_0$, is given by

$$P(x_i, \sigma_i | x_0) = \frac{1}{\sqrt{2\pi} \sigma_i} \exp\left[-\frac{(x_i - x_0)^2}{2\sigma_i^2}\right].$$  \hspace{1cm} (1)

We can easily extend this reasoning to a set of (independent) measured parameters, $\mathbf{x} = \{x_1, \ldots, x_n\}$, with associated Gaussian uncertainties, $\sigma_i$, whose theoretical values according to a given model are $x_0$, by writing:

$$P(\mathbf{x}, \sigma | x_0) = \prod_i N(x_i, \sigma_i | x_0) \hspace{1cm} (2)$$

where the product is taken over all the measured parameters of a single star confronted with the model values. The expression above gives the likelihood of measuring the set $\{x_i, \sigma_i\}$ given a model $x_0$. According to Bayes’s theorem, we may compute the posterior probability distribution (the probability of the model, given the data) as:

$$P(x_0 | \mathbf{x}, \sigma) = \frac{P(\mathbf{x}, \sigma | x_0) P(x_0)}{P(\mathbf{x}, \sigma)},$$  \hspace{1cm} (3)

where the numerator contains the likelihood and model prior probability, and the denominator only depends on the measured parameters and their uncertainties.$^1$

To evaluate the probability of a specific model quantity, $\theta := x_{0i}$, we consider the marginal posterior probability distribution for this quantity, which is obtained by integrating over all variables of Eq. (3), except $\theta$:

$$p(\theta) := P(\theta | \mathbf{x}, \sigma) = \int dx_0 \ldots dx_{n-1} d\sigma_0 \ldots d\sigma_n P(x_0 | \mathbf{x}, \sigma).$$  \hspace{1cm} (4)

As mentioned earlier, a typical set of measured parameters includes $\mathbf{x} = \{[M/H], T_{\text{eff}}, \log g, \text{colours}, m\}$. As for the stellar models, besides the theoretical values of the same parameters, they also involve other quantities such as mass $m_*$, age $\tau$, and absolute magnitude $M_{\text{abs}}$. In addition, we take $\theta = d$, where $d$ is the star’s model distance computed as $d [\text{pc}] = 10^d [\text{in} - M_{\text{abs}} + 5]$.  

2.1. Distance uncertainties

Our code can deliver various statistics for the desirable quantity. In this work, and following Rodrigues et al. (2014), we compute a star’s distance $d$ as the median of the marginalised posterior probability distribution, $p(d)$ (Eq. (4)). To estimate the uncertainties, we report the 68% and 95% upper and lower credible intervals of the median, $d_{68\%}, d_{68\%}, d_{95\%}$, and $d_{95\%}$. When quoting a single uncertainty value, we use the definition

$$\sigma(d) := 0.5 \cdot (d_{68\%} - d_{68\%}).$$

Whenever there is some ambiguity, we use the subscript BPG to denote our Bayesian distance estimates.$^3$

$^1$ Because we are only interested in inferring the best model parameter (in our case the distance) for a specific set of models, this term is dropped in further computations, as it merely represents a constant that can be normalised out (e.g. Ivezić et al. 2013, Chap. 5).

$^2$ BPG is short for the SDSS-III Brazilian Participation Group.

$^3$ These values can be considered typical for stellar parameters obtained from high signal-to-noise-ratio (S/N) spectra from a multi-object high-resolution spectrograph like FLAMES/GIRAFFE or APOGEE. For very high-resolution spectra from, e.g., HARPS or UVES, these values may even be lower by about a factor of 2.

Simulated Stars: Stellar parameter distribution

Fig. 1. Stellar parameter distributions of the simulated PARSEC sample in the $T_{\text{eff}}$-log $g$ diagram (without observational errors added). Upper left: solar-scaled metallicity; upper right: mass (in $M_\odot$); lower left: luminosity (in $L_\odot$); lower right: age (in years).

2.2. A test suite of simulated stars

We use a set of simulated stars drawn from a grid of PARSEC 1.2S stellar models (Bressan et al. 2012; Tang et al. 2014; Chen et al. 2015), with a narrow metallicity step of $\Delta[M/H] = 0.1$, spanning the range $-2.2 \leq [M/H] \leq 0.6$, and age steps of $\log \tau [\text{yr}] = 0.05$ in the range $7.5 \leq \log \tau [\text{yr}] \leq 10.15$, to verify the method and our implementation. Specifically, we randomly select 5000 models that fall into the parameter regime targeted by large-scale spectroscopic surveys, by restricting model space to $3000 < T_{\text{eff}} < 7000$ K and $1 < \log g < 5$ (see Fig. 1). These simulated stars are assigned random distances between 0.1 and 6 kpc, drawn from a uniform distribution. The simulated photometry is in the 2MASS $JHK_s$ system (Cutri et al. 2003), with no extinction applied.

By adding typical (conservative) Gaussian random errors to the model observables, we then create two sets of simulated spectrophotometric observations from this file: a set of spectroscopically measured quantities resembling high-resolution observations ($\delta T_{\text{eff}} = 100$ K, $\delta \log g = 0.1$ dex, $\delta [Z/H] = 0.1$ dex)$^3$, and a set resembling low-resolution observations ($\delta T_{\text{eff}} = 200$ K, $\delta \log g = 0.2$ dex, $\delta [Z/H] = 0.15$ dex), hereafter referred to as high- and low-resolution-like. We assume that the errors are uncorrelated and the observational uncertainties are correctly estimated. Photometric errors are also assumed to be Gaussian and of the order of 0.01 mag.

Some of the effects of possible deviations from these assumptions are discussed in the next sections (stellar parameter biases, incomplete parameter sets, extinction, Galactic density priors). Other effects (e.g. [Fe/H]-enhanced isochrones, correlated and non-Gaussian stellar parameter errors) are beyond the scope of this paper, and deferred to a future investigation.
2.3. Internal accuracy and precision

Figure 2 summarises our results from the test with the simulated PARSEC sample with high-resolution-like observational errors, when using the following set of measured quantities as an input: \( x = ([\text{M}/\text{H}], T_{\text{eff}}, \log g, m, \text{colours}) \). In each panel except for the last, we show the relative deviations of our estimated values from the true distances, \( (d_{\text{BPG}} - d_{\text{true}})/d_{\text{true}} \), as a function of the model parameters.

The first row of this figure demonstrates that our distance errors are typically below 20\% and do not show any strong systematics with the measured quantities \( T_{\text{eff}}, [\text{Z}/\text{H}] \) and \( \log g \) themselves. However, the upper right panel (distribution of distance errors in the \( T_{\text{eff}}-\log g \) diagram) in particular reveals that our distances for main-sequence and subgiant stars are much better recovered than for giants (mean deviation \(-0.5\%\), rms scatter 10.6\% vs. 1.8\% and 23.3\%, respectively). This is a known problem because of the larger overlap of models with very different luminosities in the giant region of the Hertzsprung-Russell diagram (see Fig. 1).

The second row of Fig. 2 examines the dependency of our distance errors with the model parameters which are not directly measured through spectroscopic observations: mass, age, and distance. While there is no correlation between the distance errors and the true distances themselves, we do see some systematic trends with mass and age. In particular, our code tends to slightly overpredict the distances to very old, very low-mass giant stars, while the code underpredicts the distances for very young (\( \tau < 100 \) Myr) giants with super-solar masses. However, when taking the Galactic star formation history and initial mass function (IMF) into account, these groups typically represent a negligible minority of the stars targeted by large spectroscopic stellar surveys; their fraction is clearly overrepresented in our test sample. We can therefore be confident that our code delivers unbiased distance estimates for the vast majority of stars.

Finally, in the lower right panel of Fig. 2, we show the distance uncertainties for the simulated stars, again as a function of \( T_{\text{eff}} \) and \( \log g \). Evidently, the internal precision of our method is much better for main-sequence stars than for giants (excluding the red-clump/horizontal-branch stars at \( \log g \sim 2.5 \), for which we also find accurate and precise distances). A comparison between the upper and lower right panels of this figure also shows that our code determines reliable and unbiased statistical uncertainties.

2.4. Sensitivity to input observables

Initial tests showed that, among the input measured quantities, \( \log g \) is most critical to estimate unbiased distances, since it is the quantity that best discriminates between low-luminosity dwarfs and more luminous giants, which otherwise share similar values of temperature, metallicities, and colours. Dropping \( \log g \) from the fitting procedure leads to overestimated distances for dwarf stars, and underestimated distances to giants, respectively. Similar tests also showed that cutting models with \( |x_0 - x| > 3\sigma_x \) decreases the processing time without appreciably changing the final distance estimates.
Based on these initial assessments, we implemented a code that estimates distances using the method described above and based on four distinct sets $x$:

1. $x = [(M/H), T_{\text{eff}}, \log g, m, \text{colours}]$
2. $x = [(M/H), T_{\text{eff}}, \log g, m]$
3. $x = [(M/H), \log g, m, \text{colours}]$
4. $x = [(M/H), m, \text{colours}]$, but with a cut in models whose $\log g$ differ by more than 0.5 dex from the measured value.

The code can accommodate other sets if so desired. The advantage of implementing different sets of parameters for distance determination is to provide flexibility: distances can be estimated even in the absence of one or two parameters from the set with the largest number of parameters (set 1). Furthermore, the alternative distance estimates may be compared to provide a means to evaluate the sensitivity of the method to the particular combination of spectroscopic and photometric parameters adopted.

Often, the data set in a given sample only includes [Fe/H] values, which are used in place of $[M/H]$. When [$\alpha$/Fe] values are available, we adopt the empirical relation $[M/H] = [\text{Fe/H}] + [\alpha$/Fe] for the comparison with PARSEC isochrones (e.g., Anders et al. 2014). Another important issue is extinction: the photometric data, $m$, and colours, must be corrected for extinction and reddening before comparing with the models. The exact procedure depends on the bandpasses used, on the available extinction estimates and on the assumed extinction coefficients.

The top panels of Fig. 3 show the results of applying the method to our simulated observations (see Sect. 2.2). The distribution of the relative distance errors, $(d_{\text{BPG}} - d_{\text{true}})/d_{\text{true}}$, are shown for the four parameter sets listed above, for both the high-resolution-like (left panel) and the low-resolution-like simulations, and for giants and dwarfs separately.

The four distance estimates proposed lead to reasonable results as attested by these error distributions. Surprisingly, there is only a very small systematic degradation in the precision of our distances as we move down from sets 1 to 4. As noted above, set 1 in the high-resolution case yields mean distance errors of $-0.5\%$ with an rms scatter of 10.6% for main-sequence stars ($\log g > 4.0$), while for giants the numbers are expectedly a bit worse (+1.8% bias and 23.3% precision).

For the other three sets, these numbers are $-0.2\% \pm 11.3\%$, $-0.7\% \pm 13.1\%$, $-2.9\% \pm 21.5\%$ for dwarfs, and $+1.7\% \pm 24.3\%$, $+1.8\% \pm 23.4\%$, $+2.8\% \pm 25.1\%$ for giants, respectively. This trend is expected, at least in the context of simulations, since the usage of more measured parameters tends to better constrain the stellar models that best describe each star.

As is shown in Fig. 3 (top right panel), these numbers are slightly worse for the low-resolution case. The relevant numbers for this and all other comparison samples are listed in Table 3. In the following, we use only the full input parameter set (set 1: $x = [(M/H), T_{\text{eff}}, \log g, m, \text{colours}]$).

The bottom panels of Fig. 3 show the distributions of our distance uncertainties, $\sigma(d_{\text{BPG}})/d_{\text{BPG}}$, for each of the four input sets, and for dwarfs and giants separately. Again, we also show the numbers for the low-resolution case. As expected, the uncertainty distributions for dwarfs peak at $<10\%$, while the distributions for giants are broader and peak at larger values.

### 2.5. Effect of systematic stellar parameter errors

We studied the effect of constant shifts in each of the spectroscopic input parameters on our distance estimates to gain some insight into the robustness of our method to systematics in the stellar parameters used. Figure 4 shows the results of this experiment: in each panel, all except for one input parameter are kept constant for the whole simulated high-resolution sample, while the remaining parameter is shifted by a certain amount. The figure shows the effect on the relative distance error (observed – true) as a function of this shift. As before, we treat dwarfs and giants separately because these two groups display distinct systematic trends.

The most striking observation from Fig. 4 is that even a substantial systematic shift in the temperature ($|T_{\text{eff}}| \leq 200$ K) or metallicity scale ($|\Delta[Z/H]| \leq 0.2$ dex) does not terribly affect neither accuracy nor precision of our distances. This is in stark contrast to systematic shifts in the surface gravity parameter (which are in fact quite commonly found even in high-resolution spectroscopic surveys; e.g. Holtzman et al. 2015).

#### 2.6. The model priors

In order to determine distances to stars in the Milky Way, we use our current knowledge about the Galaxy and its components to build the model prior probability distribution, $P(x_0)$. This includes basic information concerning the distributions of stars as a function of position, mass, age, and metallicity.
In particular, we assume that all stars follow a Chabrier initial mass function (Chabrier 2003) and incorporate the probability that a randomly selected star falls within a given mass range, \( P(m_\text{true}) \). This involves an integral over the mass function which is provided by the model isochrones.

We take the effect of observing a given star in a small but finite solid angle into account, by applying a geometric prior \( s^2 \text{d}s \) called the volume element, where \( s \) is the model distance. We further assume different age and spatial distributions for the basic Galactic components, namely the thin disc, thick disc, and spheroid. We here adopt the same Galactic structural parameters as Binney et al. (2014a), and refer to this work for a detailed justification.

The age distributions, \( p(\tau) \), are taken to be uniform with an upper limit of 10 Gyr for the thin disc, and lower limits of 8 and 10 Gyr for the thick disc and halo, respectively.

The spatial priors for the thin and thick discs are given, respectively, by

\[
p_\text{thin}(R,Z) \propto s^2 \text{d} s \cdot \exp(-|R/h_{\text{R, thin}}|) \cdot \exp(|Z/h_{\text{Z, thin}}|),
\]

\[
p_\text{thick}(R,Z) \propto s^2 \text{d} s \cdot \exp(-|R/h_{\text{R, thick}}|) \cdot \exp(-|Z/h_{\text{Z, thick}}|),
\]

where \((R,Z)\) are cylindrical coordinates with origin at the Galactic centre, whose values are given computed under the model distance \( s \) and a direction \((l,b)\), and \( h_{\text{R}} \) and \( h_{\text{Z}} \) are the respective scale lengths and heights. In computing the cylindrical coordinates, we use \( R_0 = 8.33 \text{kpc} \) as the distance from the Sun to the Galactic centre.

For the halo, we assume a power-law density profile with spherical symmetry:

\[
p_\text{halo}(r,l,b) \propto s^2 \text{d} s \cdot r^{-\gamma},
\]

where \((r,l,b)\) is a radial spherical coordinate. Again, as in Binney et al. (2014a), we adopt \( \gamma = 3.39 \).

The three density profiles are normalised at the solar location, which, for simplicity, we take to be \((R,Z) = (R_0,0)\) and \( r = R_0 \), in cylindrical and spherical coordinates, respectively. The normalisation values are again from Binney et al. (2014a).

We assume that the three basic Galactic components follow Gaussian metallic distribution functions (MDFs), \( p_\text{}\left(|Z/\text{H}\right]\) ), exactly as described by Binney et al. (2014a). Our final model prior is then given as

\[
P(x_0) = P(m_\text{true}) \sum_i p_i(\tau) p_i(R) p_i(|Z/\text{H}|).
\]
Fig. 5. Relative distance residuals (estimated – reference) as a function of the main spectroscopic parameters $T_{\text{eff}}$, $[Z/H]$ and $\log g$ as well as the reference distances $d_{\text{ref}}$. Top rows: comparison to the astrometric distance scale provided by the HIPPARCOS satellite, using the Bensby et al. (2014) FGK dwarf sample (1st row) and the APOGEE-HIPPARCOS giant sample (2nd row). 3rd row: APOGEE clusters. 4th row: CoRoT-APOGEE sample. 5th row: SEGUE clusters. The central dashed line is the identity line, whereas the upper and lower lines indicate deviations at the 20% level.

The top row of Fig. 5 shows the results of our distance code for the HIPPARCOS–Bensby sample. There is little or no systematics in the residuals with the parameters themselves. The mean and rms residuals over all stars are 0.4% and 18.2%. In fact, the observed rms residual is comparable to the expected errors in the parallaxes and in our distances based on the high-resolution simulation (presented in Sect. 2.2) combined together. Because the HIPPARCOS–Bensby sample combines high-resolution spectroscopic data for nearby dwarf stars with accurate parallaxes, it is arguably the best reference sample to test our code against.

We have validated that, as in our simulation tests described in Sect. 2, the four sets of parameters yield acceptable distances. In our subsequent validation analyses, we concentrate on the full parameter set {$T_{\text{eff}}$, $[Z/H]$, $\log g$, colours}, since it makes full use of the set of spectroscopic and photometric parameters which is common to the surveys to which we are applying our method.

3.2. APOGEE

The SDSS-III project APOGEE has acquired high-resolution, near infrared (NIR) spectra ($R \approx 22,500$) of >100,000 stars selected from the 2MASS Point Source Catalogue (Cutri et al. 2003), most of which are red giant stars located at low Galactic latitudes. APOGEE is providing a unique spectroscopic sample of disc-dominated stars with unprecedented volume coverage, for which precision kinematical and abundance measurements are available. For more details about APOGEE data, we refer to Ahn et al. (2014), Alam et al. (2015), and Holtzman et al. (2015).
Validation of APOGEE stellar parameters partly relies on calibrating star clusters with many probable member stars with well-determined parameters (Mészáros et al. 2013; Holtzman et al. 2015). In this section, we use two APOGEE subsamples to test our distances. In Sect. 3.2.2, we use a set of 678 CoRoT stars in the exoplanet fields LRa01 and LRc01 (e.g., Miglio et al. 2013), which have been co-observed within an APOGEE ancillary campaign, and whose asteroseismic analysis has provided high-precision surface gravities (Anders et al. 2015). In Sect. 3.2.3, we make use of the APOGEE calibration cluster data set.

In the case of APOGEE, we set $m = K_{\alpha}$, $M_{\text{abs}} = M_{K_{\alpha}}$, and colours = $\{(J - H), (H - K_{\alpha})\}$, all of which are corrected for extinction before computing the distances. For the cluster stars, extinction values are based on the Rayleigh-Jeans Colour Excess method (RICE, Majewski et al. 2011; Nidever et al. 2012; Zasowski et al. 2013). This method is based on stellar colour excesses measured using fluxes in near- and mid-infrared bands, where almost all stellar spectra are in the Rayleigh-Jeans regime and therefore have very similar intrinsic colours. For the stars in common with CoRoT, we use the precise extinctions calculated by Anders et al. (2015), using the method described in Rodrigues et al. (2014).

We use the calibrated stellar parameters determined by the APOGEE Stellar Parameters and Chemical Abundances Pipeline (ASPCAP; Holtzman et al. 2015; García-Pérez et al. 2015). Some minor improvements on the DR12 calibrations of metallicity and surface gravity have been applied; they are described in Appendix A. Since $\alpha/Fe$ is again available, we assume that $[M/Fe] = [Fe/H] + [\alpha/Fe]$ for APOGEE stars.

For the comparison with the CoRoT sample (Sect. 3.2.2), APOGEE surface gravities were replaced by those from asteroseismic scaling relations (e.g., Miglio et al. 2013). Table 2 summarises the data provenance.

### 3.2.1. The HiPPARCOS–APOGEE sample

Through a fiber feed from the New Mexico State University 1m telescope at Apache Point Observatory to the APOGEE spectrograph, it is also possible to use APOGEE to observe smaller numbers of bright stars, one at a time (Holtzman et al. 2015, Sect. 2.3). Apart from calibration, this set-up is also being used within an ancillary campaign to observe several hundred red giant stars with measured HiPPARCOS parallaxes.

### 3.2.2. APOGEE – comparison with asteroseismology: the CoRoT-APOGEE sample

Another important test for our distance method is the comparison with asteroseismically derived distances. It has been shown by recent studies (e.g., Miglio 2012; Silva Aguirre et al. 2012, 2013) that stellar distances (or equivalently, radii) determined from CoRoT and Kepler lightcurves, either via asteroseismic scaling relations or by comparing asteroseismic parameters to predicted values from a grid of models, agree within a few percent with HiPPARCOS parallaxes and eclipsing binary data. For example, Silva Aguirre et al. (2012) show that distances can be derived with 5% precision for solar-like stars by coupling the infrared flux method to determine $T_{\text{eff}}$ and bolometric fluxes to the grid-modelling of the measured values of the frequency of maximum oscillation power, $f_{\text{max}}$, and of the frequency difference between dominant oscillation modes, $\Delta v$. We therefore compared our spectro-photometric distances based on APOGEE stellar parameters with the distances obtained from CoRoT data for 678 stars in the fields LRa01 and LRc01, which have been successfully observed by both instruments. The reference distance scale is that of Anders et al. (2015) who used the well-tested stellar parameter estimation code PARAM (da Silva et al. 2006; Rodrigues et al. 2014), which is based on a similar theoretical background as this paper, but differs in a number of details. PARAM utilises the parameter set {$T_{\text{eff}}, [Z/H], \Delta v, f_{\text{max}}$} to estimate the posterior probability over the stellar models first (i.e. it delivers mass, radius, age, and absolute magnitude pdfs). In a second separate step, it compares the derived absolute magnitude pdfs with a number of broad-band photometric measurements to derive individual stellar distances and extinctions. The code has

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**Table 2. Summary of the reference data: parameter ranges, uncertainties, and provenance.**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Dist. range [kpc]</th>
<th>$\sigma(T_{\text{eff}})$ [K]</th>
<th>$T_{\text{eff}}$ range [K]</th>
<th>$\sigma\log g$</th>
<th>$\log g$ range</th>
<th>$\sigma[\text{Fe/H}]$</th>
<th>[Fe/H] range</th>
<th>photom.</th>
<th>Magnitude range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated</td>
<td>0.1–6</td>
<td>100</td>
<td>3000–7000</td>
<td>0.1</td>
<td>1.0–5.0</td>
<td>0.1</td>
<td>–2.2–0.5</td>
<td>JHKs</td>
<td>distance-limited</td>
</tr>
<tr>
<td>HIPPARCOS-Bensby</td>
<td>$&lt;0.3^4$</td>
<td>30–100</td>
<td>4800–7000</td>
<td>0.1</td>
<td>3.0–4.8</td>
<td>0.1</td>
<td>–2.5–0.5</td>
<td>BV+V+J</td>
<td>V $&lt; 9^7$</td>
</tr>
<tr>
<td>HIPPARCOS-APOGEE</td>
<td>$&lt;0.3^3$</td>
<td>90</td>
<td>3800–5500</td>
<td>0.2</td>
<td>1.0–4.0</td>
<td>0.1</td>
<td>–1.0–0.5</td>
<td>JHKs, 5</td>
<td>(J – Ks) $&gt; 0.5$; H $&lt; 7$</td>
</tr>
<tr>
<td>CoRoT-APOGEE</td>
<td>1–6</td>
<td>100</td>
<td>4000–5300</td>
<td>0.05</td>
<td>1.6–3.0</td>
<td>0.1</td>
<td>–0.8–0.5</td>
<td>JHKs, 5</td>
<td>(J – Ks) $&gt; 0.5$; H $&gt; 13.8^4$</td>
</tr>
<tr>
<td>APOGEE Clusters</td>
<td>1–12,67</td>
<td>100</td>
<td>3700–7000</td>
<td>0.2</td>
<td>1.0–3.4</td>
<td>0.1</td>
<td>–1.0–0.5</td>
<td>JHKs</td>
<td>(J – Ks) $&gt; 0.5$; H $&gt; 13.8^4$</td>
</tr>
<tr>
<td>SEGUE Clusters</td>
<td>1–12,67</td>
<td>130</td>
<td>4700–7000</td>
<td>0.21</td>
<td>1.6–5.0</td>
<td>0.11</td>
<td>–3.0–0.0</td>
<td>ugriz $^{11}$</td>
<td>0.48 $&lt; (g - r)_0 &lt; 0.55$; r $&lt; 20.2^{12}$</td>
</tr>
</tbody>
</table>

**Notes.** $\log g$ values in the CoRoT-APOGEE sample are based on asteroseismic measurements. All other non-photometric parameters were derived spectroscopically; the relevant references are given in the footnotes indicated in the third column.

**References.** (1) van Leeuwen (2007); (2) Bensby et al. (2014); (3) Høg et al. (2000); (4) Anders et al. (2015) and references therein; (5) Cutri et al. (2003); (6) WEBDA Database; (7) Harris (1996); (8) Ahn et al. (2014); (9) Zasowski et al. (2013); (10) Ahn et al. (2012); (11) Fukugita et al. (1996); (12) Lee et al. (2006a).
been extensively tested in Rodrigues et al. (2014) and Anders et al. (2015); it delivers very precise distances for asteroismic targets ($\sigma(d) \leq 2\%$). The distance scale of Anders et al. (2015) does not provide a completely independent benchmark because we use the same data and similar methods to derive stellar distances, which does not provide a completely independent benchmark because we use the same data and similar methods to derive stellar distances. However, the code serves as an important cross-check for the implementation of our code.

The comparison is shown in the fourth row of Fig. 5. We again show the relative distances residuals between our distances (BPG) and those from CoRoT-APOGEE. The general picture is reassuring, despite the limited number statistics. Our code converged for 644 stars, the mean and rms relative distance residuals are $+8.7\%$ and $14.9\%$, respectively. No strong trends are seen with metallicity, effective temperature, or surface gravity.

The systematic differences between the two methods may arise from the different handling of priors, while the rms scatter is more likely to be attributed to our use of only three photometric passbands instead of the multi-wavelength photometry used by PARAM, and to minor differences in building the distance pdf. For simplicity, and because of the high quality of acquired data, Anders et al. (2015) opted to use an IMF prior plus flat priors on Galactic structure to derive distances. However, their sample extends over a large range of Galactocentric distances, so that small systematic shifts like that measured here are not excluded.

In summary, we find a very good agreement between the two methods, modulo a small systematic trend which we attribute to our choice of more comprehensive priors, and a scatter of $\sim15\%$ which could be reduced by the inclusion of more photometric observations in our data set.

### 3.2.3. APOGEE – cluster comparison

In the third row of Fig. 5, we compare our spectro-photometric distances with those obtained from isochrone fitting of star cluster colour-magnitude diagrams (CMDs). We again restrict the comparison to the spectro-photometric distances based on the parameter set 1 in Sect. 2. We use a subsample of the $\sim400$ open and globular cluster stars that are used for calibration of ASPCAP, as described in Mészáros et al. (2013), Holtzman et al. (2015). We again refer to Table 2 for a summary of the data used. Most of the stars with reliable parameters belong to the globular clusters M 5, M 13, M 15, M 92, M 107, and to the open cluster M 67. The cluster distances and ages are adopted from the WEBDA cluster database in the case of open clusters and Harris (1996) for the globular clusters. Table 4 summarises the cluster properties.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$(m-M)_V$</th>
<th>$\log(t)$</th>
<th>[Fe/H]</th>
<th>$E(B-V)$</th>
<th>$N_{\text{pro}}$</th>
<th>$N_{\text{agnoe}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 188</td>
<td>11.55</td>
<td>9.63</td>
<td>-0.02</td>
<td>0.082</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>NGC 2158</td>
<td>13.52</td>
<td>9.02</td>
<td>-0.23</td>
<td>0.36</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>NGC 2420</td>
<td>12.45</td>
<td>9.05</td>
<td>-0.26</td>
<td>0.029</td>
<td>16</td>
<td>125</td>
</tr>
<tr>
<td>NGC 5466</td>
<td>16.02</td>
<td>10.13</td>
<td>-1.98</td>
<td>0.0</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>NGC 6791</td>
<td>13.06</td>
<td>9.64</td>
<td>+0.15</td>
<td>0.117</td>
<td>22</td>
<td>37</td>
</tr>
<tr>
<td>NGC 6819</td>
<td>11.86</td>
<td>9.17</td>
<td>+0.07</td>
<td>0.238</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>NGC 7789</td>
<td>11.84</td>
<td>9.23</td>
<td>-0.08</td>
<td>0.217</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>M 2</td>
<td>15.50</td>
<td>10.11</td>
<td>-1.65</td>
<td>0.06</td>
<td>3</td>
<td>64</td>
</tr>
<tr>
<td>M 3</td>
<td>15.07</td>
<td>9.69</td>
<td>-1.5</td>
<td>0.12</td>
<td>18</td>
<td>35</td>
</tr>
<tr>
<td>M 5</td>
<td>14.46</td>
<td>10.02</td>
<td>-1.29</td>
<td>0.03</td>
<td>84</td>
<td></td>
</tr>
<tr>
<td>M 13</td>
<td>14.33</td>
<td>10.06</td>
<td>-1.53</td>
<td>0.02</td>
<td>46</td>
<td>149</td>
</tr>
<tr>
<td>M 15</td>
<td>15.39</td>
<td>10.07</td>
<td>-2.37</td>
<td>0.10</td>
<td>31</td>
<td>78</td>
</tr>
<tr>
<td>M 35</td>
<td>10.37</td>
<td>7.97</td>
<td>-0.16</td>
<td>0.262</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>M 53</td>
<td>16.32</td>
<td>10.10</td>
<td>-2.1</td>
<td>0.02</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>M 67</td>
<td>9.79</td>
<td>9.41</td>
<td>0.00</td>
<td>0.059</td>
<td>48</td>
<td>50</td>
</tr>
<tr>
<td>M 71</td>
<td>13.01</td>
<td>10.00</td>
<td>-0.78</td>
<td>0.25</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>M 92</td>
<td>14.65</td>
<td>10.14</td>
<td>-2.31</td>
<td>0.02</td>
<td>34</td>
<td>39</td>
</tr>
<tr>
<td>M 107</td>
<td>15.05</td>
<td>10.14</td>
<td>-1.02</td>
<td>0.33</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>Be 29</td>
<td>15.86</td>
<td>9.025</td>
<td>-0.44</td>
<td>0.157</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

**Notes.** Almost all of the listed clusters were used for APOGEE calibration, while SEGUE targeted a larger number of cluster members in some selected calibration clusters.

The right panel in this comparison shows a hint that our distances are being underestimated for more distant clusters. The global mean residual is $-16.5\%$, with an rms scatter of $30\%$. All clusters, except for NGC 2420 ($d_{\text{ref}} \approx 3$ kpc), have most of their stars with residuals smaller than $30\%$. Instead of the field star priors discussed in Sect. 2.6, we have used a simple age prior for each cluster. This adopted cluster prior simply assumes a log-normal distribution of cluster stellar ages, whose mean is the literature age estimate for the cluster and the dispersion is taken as $d\log\tau = 0.1$. We find that for most APOGEE clusters this prior is irrelevant, but the age prior improves the agreement with the reference distances for NGC 2420 and NGC 2158, while it negatively affects M 71, which is the oldest and most metal poor cluster on the list.

The comparison suggests that our spectro-photometric method, despite the scatter, yields distances that are in general agreement with the cluster scale, but subject to significant systematics, especially as a function metallicity (see Fig. 5, third column).
row, second panel). The fact that we do not see these systematic trends in our comparison with the HIPPARCOS scale suggests that the cluster distances might be subject to improvements in the underlying stellar physics (e.g. the inclusion of rotation; Brandt & Huang 2015b), and less of a gold standard than commonly assumed. Another important caveat is the possible contamination of the cluster sample with non-members.

Regarding the uncertainty of isochrone-based cluster distances, it has been shown by Pinsonneault et al. (2000) that open cluster distances, which are usually determined via measuring the main-sequence shift relative to the Hyades cluster, may be subject to zero-point shifts. Such shifts may result from changes in the adopted distance to the Hyades and interstellar reddening, as well as to the metal and helium abundance, and the age of the Hyades (e.g. Brandt & Huang 2015a). However, assuming a conservative error of 0.1 mag for the cluster distance moduli, the uncertainties in the spectroscopic distances are still by far the most important.

### 3.3. SEGUE clusters

The Sloan Extension for Galactic Understanding and Exploration (SEGUE) is a large optical spectroscopic survey at low resolution ($R \approx 2000$, Smeee et al. 2013). The goal of SEGUE is to deepen our knowledge about the Galactic structural components and their stellar content, sampling them mostly at high latitudes (Yanny et al. 2009; Eisenstein et al. 2011). In this sense, SEGUE is largely complementary to APOGEE in terms of coverage of the Galactic components. For more information on the spectra and instruments we refer to Gunn et al. (2006) and Smeee et al. (2013). SEGUE data have been processed through the SEGUE Stellar Parameter Pipeline (SSPP), which is described, along with its improvements, in a series of papers (Lee et al. 2008a,b, 2011; Allende Prieto et al. 2008b; Smolinski et al. 2011). Particularly important to SSPP is the validation of the derived stellar parameters. Field stars with high-resolution spectra and known members of well-studied star clusters have been used for that purpose (Allende Prieto et al. 2008b; Lee et al. 2008b; Smolinski et al. 2011).

As in the case of APOGEE (see Sect. 3.2.3), we use the sample of cluster stars, whose distances are well known from isochrone fitting, to further test our distance estimates. In the case of SEGUE data, we set $m_g = g$, $M_{\text{abs}} = M_g$, and colours $=(u-g), (g-r), (r-i), (i-z)$ (see Gunn et al. 1998 for a description of the SDSS camera and Fukugita et al. 1996 for information on the photometric system). The spectroscopic and photometric data are from the SDSS ninth data release (DR9; Ahn et al. 2012) database. Since $\alpha/\text{Fe}$ is available for all SEGUE stars in our reference clusters, we assume that $\text{[M/H]} = \text{[Fe/H]} + [\alpha/\text{Fe}]$ in order to compare the data to the model $\text{[M/H]}$ values. As previously mentioned, all measured quantities have associated uncertainties. The measured photometric quantities also must be corrected for extinction and reddening to allow for a direct comparison to the models. Again, the data for the clusters used here are listed in Table 4.

The original sample contained eleven clusters, among open and globular, totalling a bit over 1000 stars. For 593 stars, a complete set of spectroscopic and photometric parameters and associated uncertainties allowed us the use of parameter set 1 to estimate distances. As the sample is spectroscopic, in all other cases the missing parameter(s) was (were) photometric. Distances were successfully computed, i.e. at least one model was found within $3\sigma$ of all the measured quantities, for 425 out of the 593 stars, pertaining to the eight clusters listed in the table. We include only those clusters with at least five confirmed members and successfully derived distances using our approach. Their distances, ages, and metallicities were again taken from the WEBDA database in the case of open clusters, and from Harris (1996) in the case of the globular clusters.

In the last row of Fig. 5, we show the distance residuals of individual stars against $\text{[Fe/H]}, T_{\text{eff}}, \log g$, and reference cluster difference, similar to what we did with the other samples in this section. The scatter is larger than in previous samples, attesting to the lower resolution of the data. Still, most of the stars have residuals close to or within the 20% lines and do not exhibit any strong systematics with the spectroscopic parameters. The most deviant points correspond to a fraction of the low-metallicity ($\text{[Fe/H]} < -1.0$) giant and subgiant ($\log g \leq 4.0$) stars, located beyond $\approx 5$ kpc. The mean and rms residuals over all stars are $0.14 \pm 0.43$ after applying a 3$\sigma$ clipping. Our individual distances are biased towards large values for a significant fraction of the stars in the more distant globular clusters. This trend is opposite to what was observed in the case of APOGEE clusters (see Sect. 3.2.3). If we consider only the 3 clusters with $d_{\text{ref}} < 5$ kpc, the mean and rms scatter are reduced to $0.03 \pm 0.27$.

We further investigated the cause of this systematic overestimate of the distances to part of the SEGUE stars in the more distant clusters. In analysing the distribution of different SEGUE spectroscopic parameters, it became clear that the log $g$ parameter for our sample, as published in DR9, appears to be underestimated for most stars in those clusters. Recalling that log $g$ is the most important quantity to efficiently separate dwarf and giant stellar models, we decided to correct for this shift heuristically.

The contours (black) in Fig. 7 show the distribution of a large sample of SEGUE G-dwarf stars (sample compiled from DR9 and described and analysed in Brauer 2015) in the plane. The lower panel shows the distance residuals, and the upper panel shows a subsection for the clusters used here. The lower panel shows the distance residuals, and the upper panel shows a subsection for the clusters used here.
For comparison, we also present the density distributions for a mock sample of G-dwarf stars obtained from TRILEGAL simulations (Girardi et al. 2005, 2012) covering the same SEGUE plates as present in the SEGUE G-dwarf data sample. These simulations were carried out using ADDSTAR, a web-based tool that uses parallel processing to efficiently run TRILEGAL with many independent pointings (Balbinot et al. 2012). The TRILEGAL G-dwarf mock sample shown here has been compiled by applying the same overall cuts to the simulation that were used for the SEGUE G-dwarf data sample including the colour–magnitude limits for SEGUE G-dwarfs. For a detailed description of the TRILEGAL mock sample see Brauer (2015).

A clear shift is seen in both panels in the sense that the data have systematically lower values of log \( g \) for a fixed \( T_{\text{eff}} \) or metallicity by \( \Delta \log g \) = 0.25 dex. The comparison with the population synthesis model reveals that the SSP log \( g \) values published in DR9 are underestimated and that shifting the log \( g \) values for the SEGUE stars is clearly needed, at least for the purpose of deriving reliable distances. Further details about the TRILEGAL mock G-dwarf sample, the discrepancies seen in stellar parameters between data and simulation, and the underestimated surface gravity values published with DR9 are given in Brauer (2015).

We thus decided to test the effect of applying a shift of 0.25 dex to the log \( g \) values when computing the distances. The results are shown in Fig. 6, in which we compare the median spectro-photometric distances of each cluster with the reference cluster distances for different log \( g \) calibrations and/or priors used. The open diamonds correspond to median cluster distances using the SEGUE data as they come from DR9. The black pentagons show the effect of shifting log \( g \) as explained above. The global mean offset relative to the reference distances has been reduced to –2.4%, although the rms relative residual remains high (35%). For the three more nearby clusters, the distances become systematically underestimated by –18% although the scatter around the mean residual has decreased considerably to 12%. The result of adopting a simple age prior for the clusters, as done for APOGEE data, is shown with the red circles of Fig. 6. This choice has a sizable effect on the distances in that they are increased for the more distant clusters, yielding a mean and rms relative residual of +0.13 ± 0.39.

We conclude that, despite the large scatter in distances of individual cluster stars, there is a general agreement between our median distances for SEGUE cluster stars and those from CMD fitting. The distances of fraction of the stars at lower metallicities and log \( g \), located in the more distant globular clusters, are overestimated via our method. This seems to be because the SEGUE log \( g \) scale is systematically too low, as attested by simulated samples of SEGUE stars.

Finally, we emphasise that the SEGUE data are very distinct from APOGEE in terms of spectral range and resolution. These data also probe much larger distances than the HIPPARCOS-based data used in Sect. 3.1.

### 3.4. RAVE

RAVE collected medium-resolution \( (R \sim 7500) \) CaII triplet spectra of \( \gtrsim 400 000 \) stars with \( 9 < I < 13 \). For \( > 200 000 \) of these stars, Burnett et al. (2011) have been able to derive spectro-photometric distances with spectroscopic parameters from the RAVE pipeline (Kordopatis et al. 2011) as well as near-infrared photometry from 2MASS (Skrutskie et al. 2006; Cutri et al. 2003). Their method has provided the background for this, which is largely based on the Bayesian approach proposed by Burnett et al. (2011). More recently, Binney et al. (2014a) presented an improved version of the method. In this section, we use their distances for a high S/N sample containing \( \gtrsim 9000 \) giant stars (Boeche et al. 2013a) to cross-check the implementation of our method (details in Appendix C). The spectroscopic parameters are taken from the RAVE data release 4 (DR4; Kordopatis et al. 2013), the photometry is from 2MASS.

In order to derive distances for RAVE stars, we set \( m = M_{\text{w}} \), \( M_{\text{abs}} = M_{Ks} \), and colours \( \Delta(J - H), (H - K) \). We adopt the \( A_V \) extinction values that have been computed for individual RAVE objects by Binney et al. (2014a) to deredden the NIR magnitudes of the stars. The conversion between the extinction in the optical and near-IR wavelength range is performed following Rieke & Lebofsky (1985). We adopted the \( [\text{Fe}/\text{H}] \) and \( [\alpha/\text{Fe}] \) estimates from the chemical RAVE pipeline (Boeche et al. 2011; Kordopatis et al. 2013) to make our metallicity scale more comparable with the stellar models, as in the previous sections.

Despite the use of a very similar theoretical framework, there are some differences between the way distances are estimated by Binney et al. (2014a) and in this paper. The Padova isochrone grid Binney et al. (2014a) used (Bertelli et al. 2008) is not the same isochrone grid that we used. And the Binney et al. (2014a) grid was restricted to \( [\text{Fe}/\text{H}] \gtrsim -1 \). Also, they use a slightly different IMF (Aumer & Binney 2009), and apparently they did not use the volume element elements \( s^3 ds \) in the density priors presented in Sect. 2.6. They also apply a kinematic correction for their distances (Schönrich et al. 2012). From the data side, we have...
used a slightly different metallicity scale, namely that of the RAVE chemical pipeline instead of the RAVE stellar parameter pipeline, and we handled the $\alpha$-enhancement differently.

Figure 8 shows the comparison of our distance estimates with the distances from Binney et al. (2014a) for the giant sample analysed in Boeche et al. (2013a). To make this comparison compatible with our approach described in Sect. 2, we use the $(s)$ distance estimate from the calibrated DR4 catalogue from Binney et al. (2014a). This distance estimate is the expectation value computed from their model pdf over distances instead of their corresponding parallaxes. Since neither of the distances being compared are supposed to be at the same level of precision as those in our reference samples, we have changed the way we compare them. Instead of computing the relative residual with respect to any one of the estimates, we normalise the residual by the combined uncertainty from both, which are added in quadrature. Our distance scale seems to be slightly compressed relative to RAVE DR4: we find systematically larger (smaller) distances as those in our reference samples, we have changed the way we estimating compared are supposed to be at the same level of precision.

The e-RAVE sample ($d_{\text{RAVE}} \leq 1 \text{kpc}$) and e-DR4: we find systematically larger (smaller) distances as those in our reference samples, we have changed the way we estimating compared are supposed to be at the same level of precision.

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4. Summary and conclusions

We implemented a code to estimate distances to field stars based on their measured spectroscopic and photometric parameters. The theoretical background is very similar to that of Burnett & Binney (2010) and subsequent papers from the same authors. Briefly, given a set of measured quantities, these are compared to model values and the posterior probability distribution function is derived for each star using a Bayesian approach. The distance is then determined as the median value of the posterior pdf marginalised over the other parameters.

We used simulated stars from PARSEC models as a first validation test of our distances. We showed that distances based on four different sets of parameters are well recovered, without significant systematic biases, and statistical uncertainties that scale with the uncertainties of the input parameters. Distances to dwarf stars are more accurately recovered, with $<1\%$ mean residual and $10\%$ rms errors for a high-resolution simulated sample; typical errors for giant stars are about twice those values. The method most sensitive to log $g$, and starts breaking down if this parameter is removed from the data set, also yielding strong systematic distance errors as a function of bias in this parameter.

Our simulations also show that the estimated distance uncertainties are in general agreement with the errors, with the uncertainty distribution peaking at $\approx 10\% \,(\approx 20\%)$ for dwarf (giant) stars in a mock high-resolution sample, reaching significantly larger values when spectroscopic parameters are less precise.
We demonstrated that distances of nearby stars with parallaxes from HIPPARCOS and precise atmospheric parameters are successfully recovered, with only very small systematic trends, and random errors consistent with the expected combination of errors in the parallaxes and in our distances. The relative random errors are of the order of 20% or less, with <1% systematic errors, for nearby FGK dwarf stars. For giants with HIPPARCOS parallaxes, the random and systematic residuals in our distance estimates are also as expected based on our mock samples.

For more distant stars, we have validated the code using giant stars with asteroseismic observations, as well as certain members of well-studied star clusters. In the latter case, we compared our distances to the cluster distances taken from the literature.

Our distances for giant stars in common between APOGEE and CoRoT typically agree within 15%. A systematic overestimate in our distances in comparison to those inferred from asteroseismology at the level of ≈9% may result from the use of a more set of comprehensive Galactic priors and a less comprehensive set of photometric measurements than our reference.

We used star clusters included in the SDSS-III SEGUE and APOGEE samples for a final set of validation tests, this time covering a larger range in distances out to ≈12 kpc. The distance scatter among individual cluster stars is larger than the distance residuals for the more nearby stars with parallax measurements. For APOGEE clusters, there is a small trend with distance itself, in the sense of underestimating the distances to the more distant cluster stars (d ≳ 6 kpc).

In the case of SEGUE, scatter tends to be higher than APOGEE, which is consistent with the lower resolution of the spectra. In particular, for a fraction of the low-metallicity giant and subgiant stars that predominate in the more distant globular clusters used, our distances tend to be overestimated, which is in contrast to the trend seen in APOGEE. We have identified the probable origin for this discrepancy as a systematic bias in the SEGUE log g values. Applying a shift of 0.25 dex to the SEGUE log g reduces the mean distance residual over individual stars from 14% to −2% when all clusters are considered. The scatter is also significantly reduced for the more nearby cluster SEGUE stars. Despite the large scatter observed for a subset of the SEGUE clusters, the inferred average distances to the SEGUE clusters are in good agreement with those quoted in the literature.

We also compared our distances to those for a sample of high-quality RAVE stars, which are determined with similar approaches to the method presented here (Binney et al. 2014a). Despite the large scatter and a slight systematic trend, our distances for the same sample of red giant stars studied in Boeche et al. (2013a) agree, within the expected uncertainties, with those derived by Binney et al. (2014a).

The validation results are all summarised in Table 3, where we show the data set used, the number of stars and range of reference distances in each, and the mean and rms relative residuals. The table allows us to evaluate the distances globally, specially when confronted to those from astrometry, asteroseismology, and well-calibrated star clusters. The systematic residuals are usually less than 9% while the scatter scales with the quality of the input spectro-photometric parameters, where the most distant SEGUE clusters are our worst case.

The Brazilian Participation Group on SDSS-III is using the code we presented and validated here to derive distances for tens of thousands of stars belonging to the SEGUE and APOGEE surveys. These surveys are complementary in many ways, including the directions and Galactic components they probe more efficiently. When analysed together, provided that reliable distances
Appendix A: APOGEE distances and calibration of stellar parameters

For the computation of distances to APOGEE stars, we make use of the ASPCAP stellar parameters bundled in the twelfth data release of the Sloan Digital Sky Survey (SDSS DR12; Alam et al. 2015). The APOGEE data products contained in this release are described in detail in Holtzman et al. (2015). While we generally use the stellar parameter calibrations provided by these authors, we have adopted slightly different calibrations for metallicity and surface gravity, which are explained below.

Our APOGEE distances will be released within an SDSS DR12 value-added APOGEE distance catalogue.

Surface gravity for red-clump stars: Holtzman et al. (2015) use the very accurate and precise surface gravities from the APOGEE-Kepler sample (Pinsonneault et al. 2014) to calibrate out systematic shifts in the surface gravities delivered by ASPCAP. They demonstrate (their Fig. 4) that these systematics are different for red-giant branch (RGB) and core-He burning red-clump stars (RC). The offset between both groups is about 0.2 dex. However, the authors only provide a surface gravity calibration for RGB (their formula 3), and for RC stars defer to the APOGEE RC catalogue described by Bovy et al. (2014), who provide very precise distances to these stars. Because we wanted to test our distance code on all APOGEE giant stars, we have separately fit the calibration relation for the stars contained in the RC catalogue, i.e.

\[
\log g_{\text{RC}} = \log g_{\text{calib}} - \left(7.21 \times 10^{-5} \cdot (T_{\text{eff}}[K] - 4400) + 0.129\right)
\]

Metallcities in the super-solar regime: ASPCAP metallicities have been calibrated using open and globular cluster metallicities from the literature. Figure 6 of Holtzman et al. (2015) shows the result of the external metallicity calibration: in the metal-poor regime, the raw ASPCAP metallicities are up to 0.3 dex too high compared to the literature, while the values are similar at solar metallicities. The authors have therefore opted to fit a second-order polynomial to the data (their formula 6). Since there are very few calibration clusters above solar metallicity, however, the quadratic calibration relation may be overfitting the data at the metal-rich end, and, therefore, overestimate metallicities in this regime. We have thus opted to use the calibrated values only when \([Z/H]< 0.0\), and to otherwise use the uncalibrated values.

Appendix B: SEGUE distances

For the computation of SEGUE distances, we use the results of the SEGUE Stellar Parameter Pipeline (SSPP; Lee et al. 2008a,b, 2011a; Smolinski et al. 2011) for a clean sample of G dwarfs (details will be provided in Brauer 2015). From SDSS DR8 to DR9, it underwent a couple of modifications providing improved stellar parameters for all SEGUE objects within the scope of DR9. In particular, there are some obvious systematic shifts in the individual parameters when comparing DR8 and DR9 SSPP values for the same objects.

The most significant deviation is observed for the surface gravity showing a clear overall decrease of the parameter by 0.2 dex to 0.3 dex to lower values. This finding is in agreement with the fact that the DR9 surface gravity is in general lower by about -0.2 dex (Rockosi et al., in prep.). Possible explanations for the decrease are the following: (1) the surface gravity estimates, from the reanalysis of the high-resolution spectra that are used to calibrate the DR9 SSPP results are in general lower by 0.13 dex than those from the older high-resolution analysis; (2) for some of the \[log g\] estimation methods utilised by the SSPP, new synthetic spectral grids were used; and (3) the line index methods \[MgH\] and \[CaI2\] were not considered any longer – which may have had a substantial effect since those estimators produced higher \[log g\] values compared to others. The overall lower gravity is certainly caused by a combination of the above facts.

Besides the shift in surface gravity, the temperatures in DR9 trend to be systematically larger than in DR8 by \(-50\) K. This temperature shift occurs because of the IRFM temperature scale being utilised to calibrate the SSPP DR9 temperature estimators.

While the metallicity scale did not change significantly, the \([\alpha/Fe]\) abundances shifted to higher values by about 0.1 dex.

Log g calibration: as shown in Sect. 4, accurate absolute surface gravity values are crucial for computing spectro-photometric distances. Any systematic under- or overestimation of the surface gravity leads to significant under- or overestimation of our distances. Hence, as justified in Sect. 3.3, we decided to calibrate the SEGUE DR9 surface gravity values by applying a shift of 0.25 dex and thus increasing the \[log g\] of each object in the G-dwarf sample by the same amount. Apart from this modification, we use the SEGUE DR9 values delivered by the SSPP.

Appendix C: RAVE distances

We used the RAVE giant sample assembled by Boeche et al. (2013a). This sample comprises stars with the highest quality spectra and abundances. The following quality criteria have been applied: (1) only spectra with a signal-to-noise ratio of at least 60 were selected on which the RAVE pipeline converged to a single point of the parameter space (Flag \(\text{Alg}_{\text{Conv}} = 0\)); (2) the chemical pipeline converged (\(\chi^2 < 1000\)); (3) the number of defective pixels along the spectrum is small (\(\text{frac} > 0.99\)); and (4) every object that is not classified as a normal star, according to the morphological classification described in Matijevič et al. (2012), was excluded. Like Binney et al. (2014a), we use the stellar parameters from DR4 (Kordopatis et al. 2013). The applied cuts to select RAVE giants are 0.5 dex < \[log g\] < 3.5 dex and 4000 K < \(T_{\text{eff}}\) < 5500 K. This ensures that only cool giants are selected and problems with the grid limits of the automated pipeline are avoided by setting the lower limit in \[log g\]. The resulting RAVE giant sample comprises 9098 stars.

We will also provide distances for the full RAVE DR4 sample in the near future.