Instantaneous and Time-Dependent Response and Strength of Jointless Bridge Beams

by

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GASTAL, FRANCISCO P.S.L. - Instantaneous and time-dependent response and strength of jointless bridge beams (Under the direction of Dr. Paul Zia and Dr. Ajaya K. Gupta).

The purpose of this study was to develop a generalized numerical solution for the analysis of deck-continuous, composite, multi-span bridge girders without joints. A finite element computer program has been developed, with the capability of performing instantaneous and time-dependent response analyses, and strength analyses, for a general type of bridge beams. Steel, reinforced or prestressed concrete girders, topped by a reinforced concrete deck-slab, under various sequences of construction and with different types of continuity may be equally analyzed by the proposed solution.

An isoparametric beam element and a connection, spring-like element, have been modified for modeling the nonlinear intrinsic characteristics of the materials, as well as accounting for the presence of mild steel reinforcements, prestressing tendons and the effects of cracking. Time-dependent properties of the comprising materials are assumed to follow the simplified models suggested by the American Concrete Institute and Prestressed Concrete Institute. The effects of a superimposed temperature gradient are included also in the analysis.

A thorough description of the problem and of the properties assumed in modeling the materials is given first, followed by the development of the finite element formulation. The proposed solution is validated by
the analyses of ten different beams, with comparisons being made with available analytical and experimental data. Two different cases of deck-continuous, jointless, multi-span beams are investigated and their performances are compared to the extreme situations of non and full continuity. Their behaviors have been found to be very satisfactory under dead and service load conditions. Under vertical loading, the response of a deck-continuous beam may be comparable to the response of a non- or fully-continuous beam, depending primarily on the imposed supporting conditions.
This work is fondly dedicated to my father, whom I shall not see again, and to my dear wife, whose love and support have made it all possible.
The author would like to express his sincere gratitude to Dr. Paul Zia and Dr. Ajaya K. Gupta, who served as co-chairmen of the author's Graduate Advisory Committee, for their most knowledgeable support, constructive criticism and continuous guidance throughout this entire study. It has been a most rewarding experience to work with Dr. Zia and Dr. Gupta in developing and reporting this research.

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1. INTRODUCTION

1.1 Background

The use of jointless construction \((1,2)\) for composite bridge beams is being recognized as a possible solution for the persistent bridge maintenance problems due to the existence of expansion joints \((3,4)\). Such joints, until not long ago taken as an indispensable design requirement for the proper behavior of bridge beams, have always been a cause for deterioration of such members \((5)\). It is virtually impossible to prevent water and debris from entering these man-made openings, damaging and eventually even completely destroying some vital parts of a bridge \((5)\), such as prestressing cable anchorage systems, beams and bearings.

Maintaining a jointed bridge is often so costly \((6,7)\) that some bridge engineers \((2)\) committed themselves into the daring task of eliminating joints of any sort, in constructing continuous composite bridges. Some short-span structures were initially built with no expansion joints at the intermediate supports, some with even no joints at all, having the beams monolithically attached to the end abutments.

In spite of the skepticism of many, the idea has been successful and those audacious designers envisioned even longer and longer structures. Nearly one thousand feet of joint-free beams have been constructed \((2)\), and no problems due to the absence of joints have been encountered.

Continuous concrete bridges have been built traditionally by using cast-in-place construction \((8)\), molding fully continuous members.
Continuous steel girders were also used with a concrete deck slab eventually cast on top of them. The use of precast concrete construction (8), proven efficient and economical in many cases, brought on the need for a new solution and many concrete bridges were made continuous for live load (9). By inserting massive concrete diaphragms over the piers, the initially simply supported girders become a continuous structure resisting the action of live load. This widely used procedure, however, is not very adequate for steel girders. Thus they have been constructed either as fully or non-continuous structures (10,11).

1.2 Objective

The idea of eliminating structural joints, presents yet another interesting possibility. Continuity could be obtained not by casting expensive fully-continuous beams, nor by casting concrete diaphragms connecting precast girders, but by simply casting a fully-continuous deck over the simply supported girders (2). Such an unconventional constructional procedure, illustrated in Figs. 3.12, 5.2 and 5.17, and referred herein as "Deck-Continuous Beams", may prove to be an economical solution not only for the construction of new bridges but also for the rehabilitation of old ones (6).

The study of the jointless bridge, by virtue of its complexity, requires the analysis of a wide range of variables. However, the study of a single group of jointless continuous composite bridge beams is undoubtedly a first step towards a better understanding of their behavior.
Simple and fully-continuous beams, composite or not, under linear and uncracked conditions, can be analyzed satisfactorily by standard methods of theory of structures. Under non-linear and cracked conditions, however, numerical solutions are generally necessary. When only partial continuity is obtained, as is the case of deck-continuous beams, conventional analytical methods are no longer applicable, even for the most simple situations.

With the concept of ultimate strength design, attention is focused on the determination of actual safety factors along with the proper behavior of bridges not only under service loads but also under overload conditions. Desirable behavior includes the control of cracking, deflections, camber, movements, stresses and so on, which can be obtained only from a thorough analysis including the appropriate nonlinear, time-dependent material properties for all stages of loading. The objective of this study is to develop a finite element analysis capable of monitoring all of the above, so as to investigate the behavior of deck-continuous beams.

1.3 Scope

Many numerical procedures using the finite element method can be found in the literature. They are capable of analyzing simple span as well as fully-continuous structures, some proposed for steel structures, others for reinforced or prestressed concrete structures, and even some with more general applicability. The analysis of deck-continuous beams, however, requires some special treatments not found in any of the existing numerical solutions. It also needs the capability of analyzing
different types of continuities, materials and loading effects.

For such a purpose, a finite element numerical solution for the analysis of jointless beams is proposed in the present study. The beams may also be composite, continuous or not, in steel, reinforced or prestressed concrete. Girders may be pre- or post-tensioned, fully or partially prestressed, with bonded tendons. Various constructional sequences may be studied, supporting conditions may be varied, and different loading arrangements assumed. Instantaneous and time-dependent responses are obtained. Time-dependent material properties may be varied, and temperature effects can be included.

1.4 Review of Available Analytical Procedures

This section presents a brief review of the available analytical and numerical procedures which have been proposed for the analysis of reinforced and prestressed concrete beams. Some are merely methods for estimating losses of prestress, others intended for obtaining elastic deformations and also inelastic and time-dependent deflections. A few analytical and numerical techniques, based on the finite and discrete element methods, suitable for the analysis of continuous bridge beams are also mentioned. No chronological order is followed and descriptions are brief. More detailed information can be found in the original works.

1.4.1 Grouni

Grouni (12) developed a set of expressions to predict the prestress loss of noncomposite prestressed concrete beams, taking into account
creep and shrinkage of concrete and relaxation of prestressing steel. The effects of ordinary reinforcements were neglected in the procedure.

1.4.2 Huang

Huang (13) proposed a direct method for estimating prestress losses in pre-tensioned noncomposite members, avoiding a step-by-step method. Several expressions were introduced to account for the effects of creep and shrinkage of concrete and relaxation of prestressing steel.

1.4.3 Tadros

Tadros, et al (14) suggested a procedure for predicting prestress losses, taking into consideration the effects of continuous reduction of prestressing stress due to creep and shrinkage of concrete and relaxation of steel, by a recovery parameter and a relaxation reduction factor.

1.4.4 Zia

Zia, et al (15) presented equations for estimating pretress losses due to elastic shortening, creep and shrinkage of concrete and relaxation of prestressing steel for both pre- and post-tensioned members with either bonded or unbonded tendons. The equations were applicable to normal design conditions and estimated the various types of prestress loss, rather than a lump sum value.

1.4.5 Ghali and Trevino

Ghali and Trevino (16), studying the relaxation effect in prestressing steel, proposed a relaxation reduction coefficient to be
employed as a multiplier to the intrinsic relaxation for use in prestressed concrete design. A numerical example was presented showing that the prestress loss due to relaxation in prestressed concrete members is of smaller magnitude than the intrinsic relaxation presented by a free tendon.

1.4.6 TADROS, GHALI and MAYERS

Tadros, Ghali and Mayers (17) proposed a single procedure for the prediction of time-dependent deflections in pre-tensioned precast concrete members. The method accounts for the effects of creep, shrinkage of concrete and relaxation of the prestressing steel, allowing for the presence of non-prestressing steel and considering the tension stiffening effect of concrete. Deflections are obtained by numerical integration of the curvatures. The procedure has been suggested for modification of the method used by the PCI Design Handbook (18).

1.4.7 BRANSON

Based on his previous works, Branson (19) advanced a modified step function method for calculating prestress losses, cambers, and deflections of non-composite and composite prestressed beams. Formulas were given to estimate prestress losses and camber or deflections for different types of structures. The formulas were the summation of the contribution of each factor that affects prestress losses, and cambers or deflections.

The method of analysis, for both instantaneous and time-dependent analyses, is also presented in detail in the book by Branson (20). It includes the effects of aging, creep, shrinkage of concrete and
relaxation of prestressing steel. The effects of non-prestressing steel, cracking of the sections and continuity of the structure are also taken into account. The method is developed for hand calculations, but it is also suitable for computer analysis.

1.4.8 RAO and DILGER

A method of analysis called "The Varying Stiffness Method" was proposed by Rao and Dilger (21) to predict time-dependent responses of simply supported pre-tensioned members, both noncomposite and composite. The method takes into account aging, creep and shrinkage of concrete and relaxation of prestressing steel. In estimating creep strains, the method uses a modified version of the superposition principle. It is also assumed that creep is proportional to stress. Stress-strain relationships of concrete and prestressing steel are assumed linear in the range of stress level of interest.

1.4.9 NAAMAN

Naaman (22) proposed a simplified method for the prediction of time-dependent deflections of prestressed concrete beams, called "The Pressure Line Method". The method considers the beams subjected to a compressive force following the profile of the pressure line instead of that of the tendons. A lump sum estimate of the total time-dependent prestress loss, and its percentage value with time, are assumed a priori, and a single sustained loading comprising the combined effects of sustained external moments and prestressing moments is considered.
1.4.10 PCI

The PCI Committee on Prestress Losses (23) also recommended a step-by-step method of estimating prestress losses in both pre-tensioned and post-tensioned concrete members. Total prestress losses could also be computed using a simplified method where several equations were given for different types of materials and methods of tensioning.

For composite sections, the effects of differential shrinkage and differential creep between deck slab and precast beams are difficult to obtain. The change of stiffneses of indeterminate structures due to time-dependent properties of materials may change the bending moments caused by external loads in any section. These effects, however, are not considered in most of the above mentioned prestress loss methods.

Adapting the step-by-step method, the time dependent response problem of prestressed members is reduced to a series of instantaneous, age-corrected, response problems. The instantaneous load responses of a general prestress beam can be best estimated numerically through the use of a digital computer.

1.4.11 SINNO and FURR

Sinno and Furr (24) developed a computer program to predict the time-dependent responses of noncomposite and composite simply supported pre-tensioned beams. The step-by-step procedure together with the rate of creep method were used in estimating prestress losses and camber of the members. The separations method for predicting responses of composite sections, as proposed by Branson (25), was used in estimating the effects of the differential shrinkage and creep. The method assumed
linear stress-strain relationships for both concrete and steel, it did not take into account the effects of concrete aging nor the effects of external loading.

1.4.12 MOSSOSIAN and GAMBLE

Mossosian and Gamble (26) developed a computer program based on a step-by-step procedure which iteratively estimates the time-dependent behaviors of both noncomposite and composite prestressed concrete members. The superposition method and the rate of creep method, taking into account the effect of concrete aging, were used in estimating the creep strains. In developing the computer program, linear stress-strain relationships for both concrete and steel were assumed. The aging effects, creep and shrinkage of concrete were considered, but relaxation of prestressing steel was neglected. The program was later revised and updated by Fadl and Gamble (27), to predict the time-dependent behaviors of noncomposite and composite pre-tensioned girder bridges.

Various versions of the direct stiffness method of structural analysis, in which structural elements are modeled as finite elements or discrete elements, are used widely to solve nonlinear structural problems by an iterative approach. Many finite element computer programs have been written for the analysis of reinforced concrete beams and frames (28,29,30,31,32), although many of them are not suitable for the analysis of precast composite members nor for the analysis of time-dependent behaviors. An extensive list of references may be found in the report by the American Society of Civil Engineers (33).
1.4.13 ATKINS, PIERCE, CHANG, LO and WANG

The discrete element method for the analysis of elastic and inelastic prestressed concrete members was used by Atkins (34), of elastic or inelastic prestressed concrete members with unbonded tendons by Pierce (35), and of composite prestressed concrete members by Chang (36), Lo (37) and Wang (38). These analyses, however, were based on linear stress-strain relationships for both concrete and prestressing steel, and were proposed for obtaining instantaneous responses only.

1.4.14 FREYERMUTH

Based on the work by Mattock and Kaar (39), Freyermuth (9) developed a simplified procedure to account for the effects of differential creep and differential shrinkage in the analysis and design of precast, composite, prestressed concrete girders, made continuous for live load only. In this type of construction, positive moments may develop over the piers due to creep in the prestressed girders, as well as due to loads in the remote spans. This may be partially counteracted by negative moments resulting from differential shrinkage between the cast-in-place deck slab and the precast girders.

Freyermuth's procedure assumes that the effects of creep under prestress and dead load can be evaluated by an elastic analysis assuming that the girders and slab are cast and prestressed as a monolithic continuous beam. The conjugate beam theory is used to calculate the various fixed end restraint moments, due to shrinkage, prestressing and dead loads, and the final restraining moments are then obtained by moment distribution. The moments so obtained are, then, multiplied by
creep factors to allow for the effects of creep. Those creep factors are evaluated in terms of intrinsic creep potential of the concrete materials, volume to surface ratio of the member, and ages of casting and prestressing. It is worth noting that this procedure, if correct, is only applicable to members under service loads with linear and uncracked conditions. Furthermore, as any other elastic analysis, it is not suitable for the case of deck-continuous beams being considered herein.

1.4.15 QIU, CHUNFU and JINHUAN

Qiu, Chunfu and Jinhuang (40) have very recently reported a new version of a computer program for the analysis and design of long span prestressed concrete bridges, the SABRIJ-B83. The program, which also uses the "CAD" techniques, is suitable for both static linear and nonlinear analyses of any structural system of bridges which could be considered as plane frames, e.g. continuous-beams, -arches, -trusses, T-frames, cable-stayed and enclosed frames. It also accounts for the effects of various loading conditions, temperature, shrinkage, creep, prestress losses and support settlements. Although showing remarkable versatility, the program SABRIJ-B83, as by its description, may not be suitable for the problem in question, i.e. deck-continuous beams.

1.5 Outline of Remaining Chapters

In the next chapter the mechanical properties of the materials used in this study are described. Methods used in estimating instantaneous uniaxial load responses of concrete, reinforcing steel and prestressing
steel, and time-dependent responses, i.e. aging, creep under constant and variable stresses, and shrinkage of concrete and relaxation of steel, are presented.

Chapter III reviews the fundamental concepts which are used as the basis for the development of the proposed solution. The formulation of two finite elements used are thoroughly described, as well as the methods adopted for the analysis of loading, prestressing, support displacements, temperature and time-dependent effects.

Various example problems illustrating the capability and the validity of the proposed numerical solution are presented in Chapter IV. The instantaneous and time-dependent responses, and strength, of several different beams are analyzed and compared to experimental data reported in the literature.

Chapter V illustrates the applicability of the proposed solution to the study of deck-continuous beams. Two distinct example problems are investigated and particular conclusions are drawn. A summary of the study, final conclusions and some suggestions for further research are found in Chapter VI. Appendices A, B, C and D show input and output details, a complete listing of the developed program and a sample input and output, respectively.
A composite bridge beam is generally composed of concrete, mild steel, for the girders or for reinforcement, and prestressing steel. In order to accurately predict the behavior of the beam under any loading condition, constitutive relations, i.e. stress-strain-time relationships for the materials that comprise the beam, must be available. In the following sections the expressions representing the uniaxial constitutive laws of concrete, prestressing steel, and reinforcing steel will be briefly reviewed. The presentation will be helpful in determining the material properties for estimating the responses of the structures when only limited experimental data are available.

2.1 Concrete
2.1.1 Constitutive Relationship for Loading and Unloading
2.1.1.1 Compressive Behavior

The instantaneous compressive stress-strain relationship of concrete depends on so many factors that it is not feasible to establish the relationship for a particular concrete from its constituents, curing conditions, and ambient conditions. The common method for determining the relationship is to perform a series of compression tests on cylinders taken from the same mix and stored in the same ambient conditions as the structure. But a complete stress-strain curve of concrete in compression is not easily obtained (41,42,43). Concrete cylinders under uniaxial compression fail suddenly when loaded beyond their ultimate strength. A specially designed equipment is needed to obtain the descending branch of the stress-strain curve, i.e. the portion
of the stress-strain curve exceeding the strain corresponding to the ultimate strength.

Many empirical expressions representing the complete concrete stress-strain curves have been proposed by several investigators (44,45,46,47), some will be listed herein, but more extensive references can be found in the reports by Popovics (48) and Sargin (49). Note that the expressions will be changed such that they are consistent with the sign convention used in this thesis, i.e., compressive stress and strain are negative and tensile stress and strain are positive.

Hognestad (44,50), after extensive tests of concrete columns subjected to combined bending and axial load, proposed a stress-strain relationship composed of a parabolic ascending branch and a straight descending branch, shown in Fig. 2.1.

For \( 0 \geq \varepsilon \geq \varepsilon_0 \)

\[
\begin{align*}
    f_c &= f''_c \left[ 2 \frac{\varepsilon}{\varepsilon_0} - \left( \frac{\varepsilon}{\varepsilon_0} \right)^2 \right] \\
    E_{ci} &= 1\ 800\ 000 + 460\ |f''_c| \\
    E_c &= 2\ |f''_c| \left( 1 - \frac{\varepsilon}{\varepsilon_0} \right) \left( \frac{1}{\varepsilon_0} \right)
\end{align*}
\]

For \( \varepsilon_0 > \varepsilon > \varepsilon_u \)

\[
\begin{align*}
    f_c &= f''_c \left( \varepsilon_u - 0.85\varepsilon_0 - 0.15\varepsilon \right) / \left( \varepsilon_u - \varepsilon_0 \right) \\
    E_c &= 0.15\ f''_c / \left( \varepsilon_u - \varepsilon_0 \right)
\end{align*}
\]

Where

- \( \varepsilon \) = Concrete compressive strain
- \( f'_c \) = Concrete compressive stress at strain \( \varepsilon \)
- \( f''_c \) = Maximum compressive stress = 0.85 \( f'_c \)
Concrete cylinder strength

Concrete strain at maximum stress \( f'_c \)

\[ \varepsilon_0 = \frac{2 f'_c}{E_{c1}} \text{ in/in} \]

\( E_{c1} \) = Initial modulus of elasticity

\( E_c \) = Modulus of elasticity at stress \( f_c \)

\( \varepsilon_u \) = Ultimate compressive strain

\[ = -0.0038 \text{ in/in} \]

Kent and Park \((45,51)\), proposed a stress-strain relationship for unconfined or confined concrete, which assumes that the ascending part of the curve is represented by a second degree parabola and that the confining steel has no effect on its shape, nor on the strain at maximum stress. It is also assumed that the maximum stress reached by the concrete is the cylinder strength \( f'_c \). The descending branch of the proposed curve is modeled by straight lines with different slopes for confined and unconfined concretes, as shown in Fig. 2.2.

The relationship can be represented by the following expressions:

For \( 0 \geq \varepsilon \geq \varepsilon_0 \)

\[ f_c = f'_c \left[ 2 \varepsilon /\varepsilon_0 - (\varepsilon /\varepsilon_0)^2 \right] \]

For \( \varepsilon_0 > \varepsilon \geq \varepsilon_{20c} \)

\[ f_c = f'_c \left[ 1 - Z (\varepsilon - \varepsilon_0) \right] \]

\[ Z = 0.5 / (\varepsilon_{50u} - \varepsilon_0) \quad \text{unconfined} \]

\[ Z = 0.5 / (\varepsilon_{50c} - \varepsilon_0) \quad \text{confined} \]

For \( \varepsilon_{20c} > \varepsilon \)

\[ f_c = 0.2 f'_c \]
Where

\[ Z = \text{Slope of the descending branch of the stress-strain relationship.} \]

\[ \varepsilon_{50u} = \text{Strain at } 0.5 \frac{f_c'}{f_c} \text{ on the descending branch for unconfined concrete.} \]

\[ \varepsilon_{50c} = \text{Strain at } 0.5 \frac{f_c'}{f_c} \text{ on the descending branch for confined concrete.} \]

The strain \( \varepsilon_0 \) was assumed to be equal to -0.002 in/in, \( \varepsilon_{50u} \) and \( \varepsilon_{50c} \) were found from the analysis of test results from various investigators, e.g. Soliman and Yu (52), Roy and Sozen (53), Bertero and Felippa (54), to be as follows:

\[
\varepsilon_{50u} = -\left( 3 + 0.002 \left| \frac{f_c'}{f_c} \right| \right) \left( \frac{1000}{\left| \frac{f_c'}{f_c} \right|} \right) \text{ in/in}
\]

\[
\varepsilon_{50c} = \varepsilon_{50u} - 0.75 \rho \left( \frac{b}{s} \right)^t \text{ in/in}
\]

Where

\[ \rho = \text{Volumetric ratio of lateral reinforcement to bound concrete} \]

\[ = 2 \left( \frac{b + d}{b d s} \right) A_s \]

\[ b = \text{Width of confined core} \]

\[ d = \text{Depth of confined core} \]

\[ A_s = \text{Cross section of one leg of lateral reinforcement} \]

\[ s = \text{Longitudinal spacing of lateral reinforcement} \]

Lee (46), in his study of inelastic behavior of reinforced concrete members, assumed that both ascending and descending branches of the curve could be represented by a single parabolic curve as:

\[
f_c = f_c^u \left[ 2 \varepsilon/\varepsilon_0 - \left( \varepsilon/\varepsilon_0 \right)^2 \right], \quad 0 < \varepsilon/\varepsilon_0 < 2
\]
Fig 2.1 - Hognestad's stress-strain curve for concrete in compression.

Fig 2.2 - Kent and Park stress-strain relationship for confined and unconfined concrete in compression.
Kriz and Lee \(^{(55)}\), represented the relationship by a quadratic equation as:

\[
f_c^2 + a \varepsilon^2 + b f_c \varepsilon + c f_c + d \varepsilon = 0
\]

Where the constants \(a\), \(b\), \(c\), and \(d\) were empirical and estimated from uniaxial compression test data by solving four simultaneous equations. Those equations were obtained from substituting the values of the stresses correspondent to four specified strain points.

The CEB \((55)\) recommended an expression representing the compressive stress-strain relationship of normal weight concrete as:

\[
f_c / f_c' = (k x - x^2) / [1 + (k - 2) x]
\]

\[
k = E_{ci} \varepsilon_0 / f_c'
\]

Where

\[
x = \varepsilon / \varepsilon_0
\]

\[
\varepsilon_0 = \text{Strain at maximum stress } f_c'
\]

\[
E_{ci} = \text{Initial modulus of elasticity}
\]

\[
e = 10450 \left( f_c' + 8 \right)^{\frac{1}{2}}, \text{ MPa}
\]

The values of \(E_0\) are found to range from -0.002 to -0.0025 in/in. The value of \(E_0\) of -0.0022 in/in is recommended for the stress-strain curve. The values of the ultimate strain \(\varepsilon_u\) are found to range from -0.0035 to -0.007 in/in, and the values of the stress at ultimate strain to range from 0.75 \(f_c'\) to 0.25 \(f_c'\).
The ACI (56) recommends the analysis of uncracked sections, under service loads, by the linear elastic theory assuming a constant relationship between strains and stresses as:

\[ E_c = \frac{f'_c}{\varepsilon_c} = \frac{W^3}{2.33} \left( \frac{f'_c}{|f'_c|} \right)^{\frac{1}{2}} \]

Where

- \( f'_c \) = Concrete cylinder strength in psi
- \( W \) = Concrete weight, ranging from 90 to 155 lb/cu.ft

For normal weight concrete the modulus of elasticity \( E_c \) may be approximated by:

\[ E_c = 57000 \left( \frac{f'_c}{|f'_c|} \right)^{\frac{1}{2}} \]

For structural members, under the combined action of bending and axial load, it suggested the use of a rectangular stress block, within the compression zone, for the calculation of their ultimate capacity. This stress block extends, with constant compressive stress \( f''_c \), from the extreme fiber to a depth "a" into the compression zone, such that

\[ f''_c = 0.85 f'_c \\
\begin{align*}
a &= \alpha c \\
\alpha &= 0.85, \quad f'_c \leq 4000 \text{ psi} \\
&= 0.85 - (0.05/1000)(f'_c - 4000) \geq 0.65, \quad f'_c \geq 4000 \text{ psi} 
\end{align*} \]

Where

- \( f'_c \) = Concrete cylinder strength
$f_c'' = \text{Maximum compressive strength}$

$c = \text{Depth of the compression zone}$

$a = \text{Depth of the rectangular stress block}$

$a = \text{Ratio } a/c$

The resultant force from this rectangular stress block is assumed to coincide with the resultant force from the actual concrete stress distribution in the compression zone.

From the existing experimental data and the stress-strain relationships proposed by various investigators, some of them previously mentioned, the following characteristics of the stress-strain relationship for a normal weight concrete specimen in compression are observed:

a) Compressive stress increases as compressive strain increases, at a decreasing rate, up to the maximum compressive stress, $f_c''$, of the concrete, where the slope of the stress-strain curve is equal to zero.

b) The maximum compressive stress, $f_c''$, is taken as a fraction of the maximum compressive strength of the specimen $f_c'$, ranging from 0.85 $f_c'$ to $f_c'$.

c) Most investigators gave the strain at the maximum compressive stress $\varepsilon_o$ independent of the strength and the modulus of elasticity of concrete. $\varepsilon_o$ ranges from -0.002 to -0.0025 in/in.

d) For compressive strains greater than $\varepsilon_o$, compressive stresses decrease as compressive strains increase. The descending branch of the stress-strain curve can be adequately represented by a straight line.
until the stress drops to a certain level. Concrete can sustain that stress level at relatively large strains, property referred to as ductility. The slope of the descending branch depends on the strength of the concrete and the degree of lateral confinement.

Several expressions have been proposed to estimate the modulus of elasticity for concrete, $E_c$. Sargin (45) listed the following expressions:

**GRAF**  
$$E_c = 4.92 \left| f'_c \right| \cdot 10^6 / (1970 + \left| f'_c \right|)$$

**ROS**  
$$E_c = 7.82 \left| f'_c \right| \cdot 10^6 / (2133 + \left| f'_c \right|)$$

**JENSEN**  
$$E_c = 8.00 \left| f'_c \right| \cdot 10^6 / (2000 + \left| f'_c \right|)$$

**HOGNESTAD**  
$$E_c = 1\ 800\ 000 + 460 \left| f'_c \right|$$

**WALKER**  
$$E_c = 66\ 000 \left( \left| f'_c \right| \right)^{1/2}$$

**PAUW**  
$$E_c = 33\ W^{3/2} \left( \left| f'_c \right| \right)^{1/2}$$

**SAENZ**  
$$E_c = \left( \left| f'_c \right| \right)^{1/2} \cdot 10^5 / [1 + 0.006 \left( \left| f'_c \right| \right)^{1/2}]$$

where, $E_c$ and $f'_c$ are in psi, and $W$ is in pcf.

Pauw's expression for $E_c$ is somewhat more general than the others and was adopted by the ACI (56).

A number of expressions have also been proposed to estimate the ultimate strain of concrete under compression. Some of them are:

**HOGNESTAD**  
$$\varepsilon_u = 0.0038$$

**BRANDTZAEG**  
$$\varepsilon_u = \left( 6.88 + 0.77 \left| f'_c \right| / 1000 \right) \cdot 10^{-3}$$

**SALIGER**  
$$\varepsilon_u = \left( 1.75 \left| f'_c \right| \right) \cdot 10^{-5}$$
JENSEN  \[ \varepsilon_u = \frac{f'_c}{(1 - \beta) E_c} \]

ROS  \[ \varepsilon_u = \left( 3.5 - \frac{2860}{f'_c} \right) \cdot 10^{-3} \]

CHANBAND  \[ \varepsilon_u = 0.0036 \]

where \( E_c \) and \( f'_c \) are in psi.

The ACI (56) has adopted a more conservative value of \( \varepsilon_u = 0.003 \) in/in.

As it can be seen from this brief review, several different shapes of stress-strain curves have been proposed for the behavior of concrete under compression. From the existing experimental data, it has been observed that none of the proposed stress-strain relationships can fit accurately the actual concrete stress-strain curve for all different loading conditions.

Hognestad's (44) stress-strain relationship (a parabolic ascending part, up to a maximum stress of \( f''_c = 0.85 f'_c \) and a straight line descending part, up to an ultimate strain of \(-0.0038\) in/in and \( f_c = 0.85 f''_c \)) has been adopted by a number of investigators with the result of the analysis in fairly good agreement with experimental data.

The present study does not propose a new stress-strain relationship for concrete under compression, nor does it intend to compare the effects of the use of different relationships on the behavior of the structures studied. However, two different stress-strain relationships for concrete under compression have been adopted and can be used alternatively for performing the analyses.

For a more refined analysis, although somewhat more time consuming,
the Hognestad's stress-strain relationship is chosen. The initial modulus of elasticity for concrete can be approximated by either: the Hognestad's proposed expression

\[ E_c = 1800000 + 460 |f'_c| \]

or by the ACI's adopted expression

\[ E_c = 33 W^{3/2} (|f'_c|)^{1/2} \]

when experimental data for this particular property are not available.

For a somewhat less time consuming analysis, or for a preliminary, supposedly less refined, analysis a bilinear stress-strain relationship can be adopted, as shown in Fig. 2.3, and with the following equations:

For \( 0 > \varepsilon > \varepsilon_0 \)

\[ f_c = E_{ci} \varepsilon \]

For \( \varepsilon_0 > \varepsilon > \varepsilon_u \)

\[ f_c = f'_c \]

where

\( f_c \) = Concrete stress at strain \( \varepsilon \)

\( f'_c \) = Maximum concrete cylinder strength

\( E_{ci} \) = Initial modulus of elasticity

\[ = 33 W^{3/2} (|f'_c|)^{1/2} \] for as obtained experimentally

\( \varepsilon_0 = f'_c / E_{ci} \)

\( \varepsilon_u = -0.003 \) for as obtained experimentally

The use of any other stress-strain relationship, including the option of some curve fitting in available experimental data, is possible to be incorporated in the proposed analysis, without any major
Fig 2.3 - Bilinear stress-strain relationship for concrete in compression.

Fig 2.4 - Complete stress-strain relationship for concrete under tension and compression, for loading, unloading and reloading.
programming effort.

For a more accurate modeling of the behavior of concrete in a structure subjected to bending and axial load, the complete stress-strain relationship is needed. The complementary part of the stress-strain curves outlined previously, is referred to as the tensile stress-strain relationship of concrete.

2.1.1.2 Tensile Behavior

Tensile strength of concrete is usually neglected in conventional reinforced concrete theory. Its magnitude is so small that the stress induced by shrinkage in reinforced concrete members may exceed the tensile strength and cracks may develop even before the member is loaded. It does seem logical to neglect its effect in reinforced concrete members. However, in a prestressed concrete member, concrete stress can be controlled in such a way that under service load conditions it rarely exceeds the tensile strength of concrete. Because the stiffness of the member after it is cracked is greatly reduced, to neglect the tensile strength of concrete would greatly affect the prediction of the response of a prestressed beam prior to cracking.

Since the tensile strength of concrete is relatively small, compared to the compressive strength, and stresses in a prestressed concrete beam are mostly in compression, the tensile stress-strain relationship for concrete may be adequately represented as a straight line at a slope equal to the initial modulus of elasticity for concrete $E_{ci}$ up to the modulus of rupture $f_r$ as:

For $0 > \varepsilon > \varepsilon_r$

$$f_t = E_{ci} \varepsilon$$
For $\varepsilon_r > \varepsilon$

$$f_t = 0$$

where

- $f_t = \text{Concrete tensile stress at a strain}$
- $E_{ci} = \text{Initial modulus of elasticity}$
- $\varepsilon_r = \text{Concrete strain corresponding to the modulus of rupture} = \frac{f_r}{E_{ci}}$
- $f_r = \text{Modulus of rupture of concrete}$

The modulus of rupture is obtained generally by bending tests of short prismatic plain concrete beams, or by splitting tension test of concrete cylinders. The latter is also known as Brazilian test.

The accuracy of the estimate of the modulus of rupture is important, for it directly influences the cracking moment of the sections. The current trend is to express the modulus of rupture as a power function of the maximum cylinder strength as:

$$f_r = a \left| f'_c \right|^b$$

The ACI (56), recommends the value of $a$ as 7.5 for normal weight concrete, 0.85 x 7.5 for sand-lightweight concrete, and 0.75 x 7.5 for all-lightweight concrete, and the value of $b$ as 1/2. However, tensile stress ranging from $6.0 \left(\left|f'_c\right|\right)^{1/2}$ to $12.0 \left(\left|f'_c\right|\right)^{1/2}$ is allowed at service load for a prestressed concrete member in flexure. The deflection must be checked by a rational analysis for those higher stress levels which may produce cracking.
Some investigators (57), based on tests on post-tensioned composite bridge girders, stated that the use of $f_T$ equal to 7.5 ($|f_c'|$) gave good indication of the positive moment cracking potential and 6.0 ($|f_c'|$)$^2$ worked well for the negative cracking moment and cracking at the junctions between the precast girders and the cast-in-place splice concrete. For the present study, where available experimental data are lacking, the ACI's recommendation would be used for the prediction of the concrete modulus of rupture.

2.1.1.3 Cyclic Loading

The previously mentioned stress-strain relationships for concrete were proposed to model the behavior of the material under a monotonic increase of stress with small strain increments. At any point inside a prestressed or reinforced concrete member; however, there may be a superposition of tensile and compressive strains at different times during the loading history. This may be a consequence of not only the different loading conditions but also the possible redistribution of stresses as well as the effects of time.

After an intensive investigation on concrete cylinders tested under cyclic loading, Sinha, et al (58) have observed that the action of repeated compressive loading produces a pronounced hysteresis effect in the stress-strain curve of concrete. Karsan and Jirsa (59) by testing prismatic concrete specimens, have also observed that the envelope curve of a specimen tested under cyclic loading and unloading coincides with the stress-strain curve for a specimen under monotonic loading to failure. The stress-strain path reached the envelope curve regardless of the strain accumulated prior to a particular loading cycle. Both investigators have developed a series of analytical expressions for the
prediction of intermediate unloading and reloading curves obtained from the hysteresis effect. From the experimental data, however, it can be observed that during unloading and reloading at relatively low levels of strain, the stress-strain curves follow a line which is approximately parallel to the initial tangent to the envelope stress-strain curve. Such a simplification seemed reasonable and has been adopted for this study.

The assumed stress-strain relationship, then, follows a Hognestad's or a bilinear stress-strain curve under monotonically increasing load. When unloading occurs, a straight line parallel to the initial tangent is followed. For reloading, this same line is tracked until reaching the envelope curve which is then followed for further loading. A complete picture of the assumed stress-strain relationship for concrete is shown in Fig. 2.4.

2.1.2 Types of Strain

Total strain in concrete under sustained load at any time may be considered as consisting of two types: instantaneous strain and time-dependent strain. Different definitions of the strain types may be used by different investigators. In this study, the following definitions will be used.

Instantaneous strain - The instantaneous, elastic or inelastic strain, is the strain that occurs during the changing of load. The stress-strain relationship follows a previously defined stress-strain curve. In the event of the changing of the stress-strain relationship,
with time, the stress-strain curve at the time of consideration will be followed.

Time-dependent strain - The time-dependent strain is the one that occurs in excess of the instantaneous strain. It can be further subdivided into two components: creep or creep recovery, the increase or decrease in strain with the time under sustained load or removal of load, and shrinkage strain, the change in strain with time due to the change in moisture content of concrete.

Creep or creep recovery and shrinkage occur simultaneously and are not independent of one another, thereby limiting the application of the principle of superposition. Shrinkage tends to increase the magnitude of creep and vice-versa (60,61). However, for convenience and simplicity, they are usually assumed to be independent and additive.

Shrinkage is regarded as the portion of time-dependent strain occurring in a specimen due to the change in moisture content of concrete, if the specimen is not loaded. Creep or creep recovery is regarded as the portion of time-dependent strain in excess of shrinkage, caused by sustained load or removal of sustained load.

It is rather difficult, if not impossible, to experimentally obtain the effect of creep alone in a continuously loaded concrete specimen. It is necessary to assure that no shrinkage occurs during the process which can only be accomplished by keeping in the specimen the same original moisture content. By loading the specimen, however, this moisture content may be varied and some load induced shrinkage (61) may occur, increasing the strain supposedly attributed to creep. On the
other hand, there is no such interaction problem in pure shrinkage tests, in which the specimen is free of load.

Commonly, two separate tests are conducted, one for pure shrinkage and one for combined creep and shrinkage, and the results are linearly subtracted, attributing the difference to pure creep. The interaction between creep and shrinkage is therefore neglected and the resultant creep coefficients may be overestimated.

2.1.3 Time-dependent Properties
2.1.3.1 Creep Under Constant Stress

Creep of concrete under constant sustained stress follows a specific pattern. Experimental evidence indicates that the creep strain occurring over a given period of time is proportional to the applied stress, up to some limiting stress level. Researchers are in conflict with respect to the stress level at which the linearity between creep and the applied stress ceases. Some indicate loss of linearity at stresses as low as 0.2 $f'_c$, others suggest a value of 0.5 $f'_c$ and even 0.9 $f'_c$. Rate of creep decreases with time providing that stress is not high enough to cause progressive internal cracking, which leads to time failure. Creep tends to approach a finite limiting value. Although some experimental data show that creep deformation continues over periods as long as thirty years, the rate of creep at later ages is so small that it would not cause any significant deformation.

Attempts have been made to represent the creep-time response of concrete with mathematical functions so that the amount of creep under
sustained load may be estimated without performing long term creep measurements. Extensive references may be found in the report by Corley (62), ACI committee 209 (63) and the book by Neville (60).

Four different types of creep functions have been proposed by various investigators: power functions, logarithmic functions, exponential functions and hyperbolic functions.

The ACI Committee 209 (63), based on the work by Branson and Christiason (64), represented the creep-time relationship by a hyperbolic function as:

$$C_t = C_u \frac{t^b}{(a + t^b)}$$

The empirical constants, $a$ and $b$, and the ultimate creep coefficient $C_u$ are functions of material properties, geometry, ambient conditions and the time of loading. The values of $a$ are found to range from 6 to 30, $b$ ranges from 0.4 to 0.8 and $C_u$ from 1.3 to 4.15. To estimate creep a standard creep equation is recommended,

$$C_{t,t'} = C_u (t - t')^{0.6} / [10 + (t - t')^{0.6}]$$

The equation is for concrete with 4 in or less slump, in 40 percent ambient relative humidity, with minimum thickness of 6 in or less, with loading age of 7 days. Concrete is moist cured or steamed cured for 1 to 3 days. The suggested average value of $C_u$ is 2.35. In the above expressions $t$ and $t'$ are expressed in days and stand for age of concrete and age of concrete when loaded, respectively.
Correction factors are suggested for concrete under different conditions, as listed below.

For moist cured concrete -
\[
CC_{LA} = 1.25 t' - 0.118, \quad t' > 7 \text{ days}
\]

For steamed cured concrete -
\[
CC_{LA} = 1.13 t' - 0.118, \quad t' > 7 \text{ days}
\]

where \( CC_{LA} \) is the creep correction factor for age of concrete at loading \( t' \) days. Ambient relative humidity greater than 40 percent -
\[
CC_{H} = 1.27 - 0.0067 H, \quad H > 40\%
\]

where \( CC_{H} \) is the creep correction factor for ambient relative humidity \( H \) percent. Minimum thickness of the member -
\[
CC_T = 1.14 - 0.023 T, \quad \text{for} \:< 1 \text{ year}
\]
\[
CC_T = 1.10 - 0.017 T, \quad \text{for ultimate}
\]

where \( CC_T \) is the creep correction factor for member thickness \( T \) in in.

Slump test, \( S \) in in -
\[
CC_S = 0.82 + 0.067 S
\]

where \( CC_S \) is the creep correction factor for consistency of concrete.

Percent of fine aggregate by weight -
\[
CC_F = 0.88 + 0.24 F
\]

where \( CC_F \) is the creep correction factor for percent of fine aggregate \( F \) of concrete. Air content, \( A \), in percent -
\[
CC_A = 1.0, \quad A < 6\%
\]
\[
CC_A = 0.46 + 0.09 A, \quad A > 9\%
\]

where \( CC_A \) is the creep correction factor for air content \( A \) of concrete.
The PCI Committee on Prestress Losses (23), recommended a method in determining prestress losses using a series of curves. The prestress loss due to creep may be converted into the equivalent creep strain by dividing the loss by the modulus of elasticity of the prestressing steel (28.10^6 psi). Creep may then be written as follows:

\[(\varepsilon_c)_{t,t'} = (c_c)_u \cdot CC_T \cdot CC_{LA} \cdot (g_c)_t \cdot \varepsilon_c\]

where

- \(CC_T\) = Correction factor for size and shape of member, a function of volume to exposed surface ratio.
- \(CC_{LA}\) = Correction factor for age of loading and length of curing.
- \((g_c)_t\) = Function representing creep with time evaluated at time \(t\).

For the curves representing the factors \(CC_T\), \(CC_{LA}\) and the function \((g_c)_t\), refer to reference (23).

The ultimate creep strain per unit stress, \((c_c)_u\) may be estimated as follows:

**Normal weight concrete, moist cured not exceeding 7 days** -

\[(c_c)_u = 3.455 \cdot 10^{-6} - 7.273 \cdot 10^{-13} \cdot E_c > 4.00 \cdot 10^{-6}\]

**Normal weight concrete, accelerated curing** -

\[(c_c)_u = 2.291 \cdot 10^{-6} - 7.273 \cdot 10^{-13} \cdot E_c > 4.00 \cdot 10^{-6}\]

**Lightweight concrete, moist cured not exceeding 7 days** -

\[(c_c)_u = 2.764 \cdot 10^{-6} - 7.273 \cdot 10^{-13} \cdot E_c > 4.00 \cdot 10^{-6}\]

**Lightweight concrete, accelerated curing** -

\[(c_c)_u = 2.291 \cdot 10^{-6} - 7.273 \cdot 10^{-13} \cdot E_c > 4.00 \cdot 10^{-6}\]
where \( (c_ε)_u \) is in in/in/psi.

The most complete creep prediction procedures are those recommended by the ACI Committee 209 (63), the PCI Committee on Prestress Losses (23), and the CEB (55). The latter two are not listed here.

In practice, creep-time response of a specimen can be conveniently estimated using one of these procedures with minimal data. Because of the scattering of the experimental data, as suggested by the range of the empirical constants for the function recommended by the ACI Committee 209, these procedures may not give enough accuracy. It may be preferable, although costly, to perform a short term test simulating the actual atmospheric conditions, and use the creep-time curve in the form of one of the recommended methods as a basis for estimating the creep-time response.

The expression in the form recommended by the ACI Committee 209, has been adopted by many investigators (21, 28, 64, 65) with satisfactory results. In this study, the measured data on creep response will be used when available. In the absence of the long-term measurements, the ACI Committee 209 recommendations will be adopted.

2.1.3.2 Creep Under Variable Stress

The creep functions previously mentioned, represent the experimental data of concrete under a state of constant stress. In reality, under a variable arrangement of loading, stress in every part of the structure is changing, gradually or abruptly, with time. Although attempts have
been made, with considerable success, to use a single creep function to predict creep under gradually varying sustained stress, rather poor results were obtained in the case of abruptly changing stresses. Several techniques have been proposed to handle this situation.

It is assumed that there is a creep function representing creep-time relation of concrete under a constant stress, and assuming that creep is proportional to the applied stress, several methods are available to estimate the amount of creep under time-dependent stress. They may be divided into three broad categories: a) Creep depends on the stress at the time of consideration only. b) Rate of creep depends on the stress at time of consideration, and c) Rate of creep depends on the complete history of the stress application. The following are the most commonly used methods.

2.1.3.2.1 The Effective Modulus Method

This is probably the simplest and most widely used method. It uses an elastic approach, taking into account creep effects, exclusive of shrinkage, by using the effective modulus instead of the normal modulus of elasticity. The effective modulus is a function of time and can be defined as:

\[
( \text{E}_{\text{eff}} )_t = \left( \frac{\text{E}_c}{1 + C_t} \right)
\]

where:
- \(( \text{E}_{\text{eff}} )_t\) = Effective modulus of elasticity of concrete at time \( t \)
- \(( \text{E}_c )_t\) = Modulus of elasticity of concrete at time \( t \)
- \( C_t\) = Creep coefficient at time \( t \).
The effective modulus method is of the first category, creep at any time depends on stress at that instant only. Stress history is disregarded. Upon the removal of stress complete creep recovery is assumed, which is rarely the case for concrete. The method will overestimate creep under gradually increasing stress and underestimate creep under gradually decreasing stress (66). Satisfactory results will be obtained for creep under approximately constant stress.

2.1.3.2.2 The Rate of Creep Method

Assuming that rate of creep at any age of concrete is independent of the time at which concrete is loaded, creep may be estimated as:

\[
(\varepsilon_c)_{tn} = \sum_{i=0}^{n} (f_c)_i (\Delta\varepsilon_c)_i
\]

where

- \( n \) = number of time increments in the interval of time
- \( t_n \) = time at the end of time increment \( n \)
- \( (f_c)_i \) = concrete stress within time increment \( i \)
- \( (\Delta\varepsilon_c)_i = (\varepsilon_c)_{t_i} - (\varepsilon_c)_{t_{i-1}} \), specific creep increment in the time increment \( i \)

The method takes into account, to some extent, the history of the applied stress. Since the rate of creep in concrete decreases with the age of concrete, the method will underestimate the amount of creep due to stress applied at later ages. The rate of creep method, will underestimate creep under gradually increasing stresses and vice-versa. No creep recovery will be predicted upon the removal of load. Under constant stress, the method will give adequate accuracy.
2.1.3.2.3 The Superposition Method

The superposition method is of the third category, rate of creep depends on the complete history of the applied stress. According to Ross (66), the superposition hypothesis may be stated as: The strain produced in concrete at any time $t$ by stress increment applied at time $t' < t$ is independent of the effects of any stresses applied either earlier or later than the time $t'$. The stress increments may be either positive or negative, assuming that creep in tension and compression are equal for equal stresses, which is approximately true for concrete. For the principle of superposition, it is not easy to write a general equation, as done for the rate of creep method, but a numerical solution for any case is not a complicated matter.

Several investigators have compared the accuracy of the creep predictions, using different methods of analysis. Ross (66) have compared, in their original forms, the effective modulus method, the rate of creep method, and the superposition method with the measurements of creep under gradually increasing and decreasing stresses, and under severely variable stresses. The effective modulus method gave the poorest results. The rate of creep method gave results comparable to the superposition method. The first one underestimated creep under gradually increasing stress and vice-versa, the latter gave errors in the opposite sense. For severely variable stresses, both methods estimated creep approximately the same magnitude as the experimental data. However, the lack of creep recovery in the rate of creep method, upon removal of the stress, led to the error in the general shape of the
strain-time response curve. The trend of the strain-time response curve predicted by the superposition method agreed well with the data.

In the present study, both the rate of creep and the superposition methods are utilized, since the effective modulus method does not completely suit the needs of the problems analyzed.

2.1.3.3 Shrinkage

As in the creep-time relationship, shrinkage increases with time at a decreasing rate, providing that there is no change in ambient conditions. Shrinkage also tends to approach a finite value. While the final value of shrinkage of concrete depends on many factors, i.e. the composition of concrete, time of exposure to the atmosphere conditions, size and shape of the specimen, etc, the shape of the shrinkage-time curves for various concretes under different conditions are remarkably similar. It is then convenient to express the shrinkage-time relationship as a product of the ultimate shrinkage and a function of time, as done for the case of creep.

Although shrinkage has been noticed and studied from as earlier as the beginning of the century (67), Ross (66), was probably the first one to express the shrinkage-time relationship as a mathematical function. Since then, several investigators have proposed many different models and mathematical functions to represent the shrinkage-time relationship of concrete, and used with satisfactory results.

Based on the work by Branson and Christiason (64), the ACI Committee 209 (63) recommended the following expression to predict
For normal, sand-lightweight and all-lightweight concretes, both moist and steamed cured, and using type I and type III cements, the constants a and b, were found to range from 20 to 150 and from 0.9 to 1.1 respectively. The ultimate shrinkage \( (\varepsilon_{sh})_u \) was found to range from \(-415.10^6 \text{ in/in}\) to \(-1070.10^6 \text{ in/in}\).

Standard shrinkage equations are suggested. The equations are for concrete with 4 in or less slump, placed in ambient relative humidity of 40 percent with minimum member thickness of 6 in or less.

Shrinkage of concrete after 7 days of moist curing -
\[
(\varepsilon_{sh})_t = \frac{(\varepsilon_{sh})_u}{(35 + t)}
\]

Shrinkage of concrete after 1 to 3 days of steam curing -
\[
(\varepsilon_{sh})_t = \frac{(\varepsilon_{sh})_u}{(55 + t)}
\]

The suggested average value for \( (\varepsilon_{sh})_u \) is \(-800.10^6 \text{ in/in}\) for moist cured concrete and \(-730.10 \text{ in/in}\) for steam cured concrete. The following corrections are used for concrete with other conditions:

Shrinkage correction factor for ambient humidity greater than 40 percent
\[
CS_h = 1.4 - 0.01 H \quad , \quad 40 < H < 80
\]
\[
CS_h = 3.0 - 0.03 H \quad , \quad 80 < H < 100
\]

Shrinkage correction factor for member thickness -
\[
CS_t = 1.23 - 0.038 T \quad , \quad \text{for 1 yr. of drying}
\]
\[ \text{CS}_t = 1.17 - 0.029 T \] for ultimate value

Shrinkage correction factor for consistency of concrete -

\[ \text{CS}_s = 0.89 - 0.041 S \] where \( S = \text{slump in in} \)

Shrinkage correction factor for cement content of concrete -

\[ \text{CS}_b = 0.75 + 0.034 B \]

where \( B \) is the number of 94 lb sacks of cement per cubic yard.

Shrinkage correction factor for aggregate content of concrete -

\[ \text{CS}_r = 0.3 + 0.014 F \] , \( F < 50 \)
\[ \text{CS}_r = 0.9 + 0.002 F \] , \( F > 50 \)

where \( F \) is the percent of fine aggregate by weight.

Shrinkage correction factor for air content -

\[ \text{CS}_a = 0.95 + 0.008 A \]

where \( A \) is the air content in percent.

For shrinkage of concrete moist cured one day, a correction factor of 1.2 is used. A linear interpolation may be used between 1.2 at 1 day and 1.0 at 7 days.

The PCI Committee on Prestress Losses (23) recommended equations to calculate prestress losses due to shrinkage. By dividing the values of the prestress loss due to shrinkage by the modulus of elasticity of the prestressing steel, (28.10^6 psi), those values may be written in terms of the equivalent shrinkage strain as:

\[ (\varepsilon_{sh})_t = (\varepsilon_{sh})_u \cdot \text{CS}_t \cdot (\varepsilon_s)_t \]

where

\[ (\varepsilon_{sh})_t = \text{Shrinkage strain at time } t \]
Basic value for ultimate shrinkage

\[ (\varepsilon_{sh})_u = \text{Basic value for ultimate shrinkage} \]

Correction factor for shape and size of member

\[ C_{S_t} = \text{Correction factor for shape and size of member} \]

Function representing the development of shrinkage with time, evaluated at time \( t \)

\[ (g_g)_t = \text{Function representing the development of shrinkage with time, evaluated at time } t \]

The basic value for ultimate shrinkage \((\varepsilon_{sh})_u\) may be written as:

For normal weight concrete:

\[ (\varepsilon_{sh})_u = -9.643 \times 10^{-6} + 1.071 \times 10^{-10} E_c < 4.29 \times 10^{-6} \]

For lightweight concrete:

\[ (\varepsilon_{sh})_u = -1.464 \times 10^{-6} + 3.571 \times 10^{-10} E_c < 4.29 \times 10^{-6} \]

The values are in in/in. The correction factor for the shape and size of the member \( C_{S_t} \) and the shrinkage-time function, \( g_g \), can be found in tables and curves shown in the PCI publication (23).

The expressions in the form proposed by the ACI Committee 209, have been used by several investigators \((49, 64, 65)\) with satisfactory results. In the present study, the measured values of ultimate shrinkage, from the comparable specimens, will be used when available. In the absence of such experimental data, the ACI committee 209 recommendations will be used.

2.1.3.4 Aging

Concrete under normal ambient conditions gains strength with age because of further hydration of the cement. A study of fifty year properties of concrete by Washa and Wendt (68), showed the increase in
strength of concrete to reach a peak value at the ages of about 10 to 50 years, depending on the storage conditions and the type of cement used, and showed strength decrease thereafter. The average increase in compression strength at the age of fifty years is about 10 to 40 percent of the strength of the comparable specimens at the age of 28 days. Although the increase in strength is usually neglected in design, for more accuracy, the effect will be included in the present study.

Compressive strength tests of concrete cylinders are usually made at the age of 7 or 28 days. Several investigators have attempted, almost all of them with experimental data, to relate the strength of concrete at a later age to the strength at a standard early age. The relationship depends on many factors, such as water-cement ratio, mix proportion, qualities of cement, curing conditions, and cross-sectional shape. For a particular mix, age strength relationship of concrete may be expressed as a function of the age of concrete and curing conditions. Several empirical expressions have been proposed by many investigators. In this study, the ACI Committee 209 recommendations (63) will be adopted.

Based on the work by Branson and Christiason (64), the ACI Committee 209 recommends the use of a hyperbolic function for the prediction of the aging effect on the strength of concrete

\[
( f'_c )_t = \frac{ ( f'_c )_{28} }{ t } \left( a + b t \right)
\]

From the measurement of some 88 specimens, of normal weight, sand-lightweight, and all-lightweight concrete, using both moist curing and steam curing, and type I and type III cements, the values of the constant a are found to range from 0.5 to 9.25, and the constant b from
The following values have been suggested for different types of cement and curing conditions.

Moist cured concrete and type I cement -
\[( f'_c) _t = ( f'_c )_{28} t / ( 4.00 + 0.85 t ) \]

Moist cured concrete and type III cement -
\[( f'_c) _t = ( f'_c )_{28} t / ( 2.30 + 0.92 t ) \]

Steamed cured concrete and type I cement -
\[( f'_c) _t = ( f'_c )_{28} t / ( 1.00 + 0.95 t ) \]

Steamed cured concrete and type III cement -
\[( f'_c) _t = ( f'_c )_{28} t / ( 0.70 + 0.98 t ) \]

The stress-strain curves of concrete, as previously discussed, can be expressed as a function of the cylinder compressive strength of concrete, \( f'_c \). For consistency, every key value should be estimated from the strength of concrete at the time of consideration. Without redefining the general shape of the curve, a stress at any strain, at different ages, may be found from modified curves taking into account the aging factor \( ( f'_c )_t / ( f'_c )_{28} \).

The factors are applied in such a way that the initial and instantaneous modulus, and the stress and maximum compressive strength of the modified curve will be the same as those of the actual curve at that age. The strain limits, however, will not be modified in the new curves.
2.2 Reinforcement Steel

2.2.1 Constitutive Relationship for Loading and Unloading

Instantaneous stress-strain relationship of steel is usually obtained from uniaxial tensile tests of a sample taken from the steel of interest. Unlike concrete, the tensile stress-strain curve of steel can be obtained up to the ultimate strain of the material, with relative ease. The stress-strain curve is commonly assumed to have symmetry about the origin, i.e. the stress-strain response in compression is the same as in tension. Since there is no aging effect in steel, the stress-strain relationship can be used to represent the instantaneous tensile and compressive response without any modification.

As commonly used in design and analysis the stress-strain relationship of mild steel may be approximated by a perfect elasto-plastic, bilinear curve, as shown in Fig. 2.5, with results presenting a very acceptable degree of accuracy.

For the case of unloading and reloading of the reinforcement steel, it is well known that this material presents the effect of hysteresis, although much less pronounced then in the case of concrete. However, an usual simplification is to assume that the unloading and reloading curves follow straight lines, parallel to the initial loading curve, inscribed within the envelope, monotonically obtained stress-strain relationship (58,69).

In the present study, both simplifications have been assumed, and the adopted stress-strain relationship for loading, unloading and reloading is sketched in Fig. 2.5.
2.3 Prestressing Steel

2.3.1 Constitutive Relationship for Loading and Unloading

As will be seen in a later chapter, the approach used in this study requires stresses in the member to be obtained from the strains. The problem is less complicated in the case of steel, since there is no change in strain of steel due to aging and moisture content. The total strain in steel may be considered as the sum of instantaneous and relaxation strains. Relaxation will be defined as the time-dependent loss of stress of steel under relatively constant strain. Relaxation strain will be defined as the reduction in strain that gives the reduction in stress equal to relaxation, utilizing the instantaneous stress-strain relationship of the material.

As with the case of mild steel, the instantaneous stress-strain relationship for prestressing steel is obtained from uniaxial tensile tests. This material, however, presents a somewhat different stress-strain curve, in the sense that there is no well-defined yield point and it shows a comparatively smaller ductility range.

A simplified tri-linear stress-strain curve seems to be a reasonable approximation for the stress-strain relationship under monotonic tensile stressing of prestressing steel, and it has been adopted in this study. As the case of concrete and reinforcing steel, the stress-strain relationships for unloading and reloading have been assumed as straight lines parallel to the initial loading curve. The assumed and simplified stress-strain relationship for the prestressing steel, in tension, for loading, unloading and reloading, is shown in Fig. 2.6.
Fig 2.5 - Elasto-plastic stress-strain relationship for reinforcing steel.

Fig 2.6 - Tri-linear stress-strain relationship for prestressing steel.
2.3.2 Relaxation Under Constant Strain

For prestressed concrete members, the relaxation of steel is of great concern. Initially the stress in prestressing steel is as high as 0.8 of its ultimate strength. At this level, relaxation over a long period of time may be as high as 15 percent of its initial stress (70). Although relaxation does not decrease the flexural strength of a bonded prestressed member, it affects the servicibility of the member.

Relaxation is a function of type of steel, initial stress-strength ratio, and temperature. It is also affected to some extent by rate of loading. High temperature may increase long-term relaxation several times, but under ordinary ambient conditions the temperature effect is negligible. The effect of rate of loading may also be neglected, since it has been shown that the rate of loading has an effect at the initial stress-strength ratio of about 0.8 to 0.94 (70), well above the stress level that would occur under service load.

Many papers have been published reporting relaxation test results and some investigators have attempted to describe relaxation as mathematical functions, like power functions, quadratic functions and logarithmic functions.

The PCI Committee on Prestress Losses (23) recommended the relaxation expression from Magura, et al (71), with the distinction between stress-relieved and low-relaxation steels. The following expressions are recommended:

For stress relieved steel -

\[(f_r)_{t,t_1} = (f_s)_t - (f_s)_{t_1}\]
\[(f_s)_t [(\log 24t - \log 24t_1)/10][(f_s)_t/f_{sy} - 0.55] \]

\[(f_s)_t / f_{sy} > 0.60 \quad , \quad f_{sy} = 0.85 f_{su} \]

For low-relaxation steel -

\[(f_r)_{t,t_1} = (f_s)_t - (f_s)_{t_1} \]
\[(f_s)_t [(\log 24t - \log 24t_1)/45][(f_s)_t/f_{sy} - 0.55] \]

\[(f_s)_t / f_{sy} > 0.60 \quad , \quad f_{sy} = 0.90 f_{su} \]

where

\[(f_s)_t = \text{Stress in prestressing steel at time t in days} \]
\[(f_r)_{t,t_1} = \text{Relaxation stress of prestressing steel between times t and t_1 in days} \]
\[f_{sy} = \text{Specified yield stress of steel} \]
\[f_{su} = \text{Ultimate strength of steel} \]

In this study the above expressions are used.

2.3.3 Relaxation Under Variable Strain

Strain variation in prestressing steel is not as severe as stress variations in concrete and reinforcing steel. In an ordinary prestressed concrete member, the change in strain of prestressing steel under service load rarely exceeds 20 percent of the initial strain. Consequently, not much attention had been paid to relaxation under variable strain. Most of the relaxation data were obtained under constant strain, and the relaxation functions previously mentioned were based on this type of data. Under varying strain, however, Ghali and Trevino (16) have shown that the relaxation in prestressed concrete steel is of smaller magnitude than the intrinsic relaxation which occurs
when a tendon is stretched with constant strain, and it is assumed \( ^{23} \) that a method equivalent to the rate of creep method is adequate to handle the situation of variable strain. It was decided then, in the present study, to use a method equivalent to the rate of creep method in determining the relaxation of prestressing steel under variable strain.

2.4 Elastomeric Bearing Pads

Bearing pads are widely used in all kinds of precast structures. Their main purpose is to distribute vertical loads over the bearing area and to reduce force build-up at the connections by permitting some small displacements and rotations. In the case of bridge girders, especially for accommodating the temperature variation, their use has proven beneficial and often may be necessary for satisfactory performance.

Several different materials and compositions have been used for the manufacturing of structural bearing pads such as AASHTO-grade chloroprene pads, made of pure neoprene and recently also mixed with reinforcing fibers; cotton-duck fabric reinforced pads, used for high compressive stress; tempered hardboard pads, used in hollow core slabs; TFE (trade name Teflon) coated pads, often used for large horizontal movements. For bridge girders, chloroprene pads laminated with alternated layers of bonded steel or fiberglass are widely used.

The design of such pads is influenced by many factors \( ^{18} \), like material used, hardness characteristics, shear modulus, ambient temperature and relative humidity variation, amount of pressure to be transmitted and overall characteristics of the structure where the pad
is to be used. Not many publications are found presenting experimental data from tests on bearing pads \((72)\), and few concern design recommendations for such connections \((18,72,73)\).

The PCI \((18)\) based on the work from Iverson, et al \((73)\), presents some recommendations for the selection and design of elastomeric bearing pads. Refering to Fig. 2.7, the compressive stress \(f\) which depends on the hardness characteristic of the material and ambient temperature, is limited to a maximum of 1000 psi.

\[
 f = \frac{V}{b\ w} < 1000 \text{ psi}
\]

The maximum allowed compressive stress, for 15 percent strain, is obtained according to the shape factor \(S\),

\[
 S = \frac{w\ b}{2\ t\ (w + b)}
\]

and the maximum unfactored shear force \(N\) is obtained from

\[
 N = \Delta\ w\ b\ G_t / t
\]

where

- \(f\) = Unfactored compressive stress, in psi
- \(V\) = Unfactored vertical reaction, in lb
- \(N\) = Unfactored horizontal reaction, in lb
- \(W\) = Dimension parallel to the beam span, in in
- \(b\) = Dimension perpendicular to the beam span, in in
- \(t\) = Total thickness of the pad, in in
- \(G_t\) = Long-term shear modulus, in psi
  \(= G / 2\)
- \(G\) = Shear modulus
Fig 2.7 - Bearing pads.

Fig 2.8 - Linear shear-deformation relationship for bearing pads.
\[ \Delta = \text{Shear deformation, in in} \]

The shear force-deformation relationship \((N - \Delta)\) presented by those pads, as shown in the work by Vinje (72), is nonlinear. Assuming that they are properly designed and working within the service load range of the structure, a linear relationship may be assumed, neglecting the nonlinear effect on the value of the horizontal reaction \(N\). In the present study, a linear force-deformation relationship as shown in Fig. 2.8 is assumed for modeling the support bearing pads, and their stiffness coefficient is obtained as:

\[ N = K_b \Delta \]

\[ K_b = G_t A / t \]

where

- \(N\) = Unfactored horizontal reaction, in lb
- \(\Delta\) = Maximum expected shear deformation in in
- \(A\) = Contact area of the bearing pad, in \(in^2\)
- \(t\) = Total pad thickness, in in
- \(G_t\) = Long-term shear modulus, in psi
- \(K_b\) = Shear stiffness of the bearing pad, in lb.in\(^3\)

Where experimental data for the bearing shear stiffness is available, this value will be used. In the absence of such data, the value obtained as shown above will be assumed.
3. METHOD OF ANALYSIS

3.1 Introduction

A general composite beam may have a variable cross-section along its length. Its constituent members may be simply supported over one span or continuous over two or more spans. Continuity may be fully obtained by casting a continuous girder and deck or by connecting individual one-span girders to one another through diaphragms over the intermediate supports. Girders may be, otherwise, indirectly connected to its companion by casting a continuous deck slab. Loads and restraints may be applied in any of the three directions, i.e. the member axial direction, the transverse direction and the rotational direction. Applied loads may be concentrated or linearly distributed. The beam may be of reinforced concrete, prestressed concrete or steel. The amount of reinforcing steel in various layers and the magnitude and location of the prestressing force may also vary along the length. Response analysis of such a structure may encounter a considerable amount of complexity when nonlinear and time-dependent effects are also introduced and analytical closed-form solutions are not possible. A numerical solution, however, seems to be a suitable approach.

Many numerical solutions for the analysis of beams are found in the literature. Some, using the finite element technique, are adequate for reinforced concrete members (28, 29, 30, 31, 32, 33) and others based on the discrete element method are applied to prestressed concrete members (34, 35, 36, 37, 38). However, none was found broad enough as to suit the needs of the problem under consideration, as discussed in Chapter 1.
In this chapter a method of analysis based on the finite element technique is presented. Two elements are formulated and used for modeling beams and deck-continuity among the beams. Beam elements may be used for the analysis of either steel, reinforced concrete, prestressed concrete or composite steel-concrete members. Connection elements, however, are used only for modeling reinforced concrete deck slabs. Both instantaneous and time-dependent responses of a beam may be obtained by the proposed solution. Linear and nonlinear ranges of behavior are captured in one single analysis where loading and restraints are predefined according to the actual stages of construction.

3.2 Beam Element

3.2.1 Conventional Beam Element

The formulation of a conventional beam element is well known and can be found in many books on finite element methods. For purpose of illustration, however, it is shown in this section and used for comparison with the formulation of the isoparametric beam element adopted in this thesis.

Let us for the time being consider a two-noded three degrees-of-freedom per node beam element as shown in Fig. 3.1, having a cross-section with a vertical plane of symmetry. The behavior of such an element relies on the three basic assumptions that follow:

(a) The direct strains in the transverse direction y are negligibly small.
Fig 3.1 - Conventional beam element.

Fig 3.2 - Isoparametric beam element.
(b) A plane cross-section normal to the longitudinal axis of the beam remains plane after the beam deforms. Such is true when the transverse shear strain is constant through the thickness of the beam.

(c) A normal to the longitudinal axis of the beam, in the vertical plane of symmetry, remains normal to the longitudinal axis after deformation. It is thus implied that not only assumption (b) holds but that the transverse shear deformation is negligibly small, $\gamma_{xy}=0$. Assumption (a) above leads to the following.

$$\frac{\partial w}{\partial y} = 0$$

Accordingly the transverse displacement $w$ is a function of the coordinate $x$ alone. From assumptions (b) and (c) one have

$$\frac{\partial u}{\partial y} = -\frac{dw}{dx}$$

3.2

Since $w$ is not a function of $y$, eq. 3.2 may be integrated to give.

$$u = -\left(\frac{dw}{dx}\right)y + u_o(x)$$

3.3

in which $u_o(x)$ is the longitudinal displacement in the direction of the reference axis $x$. The longitudinal strain $\varepsilon_x$ is obtained by taking the derivative of the longitudinal displacement $u$ in the equation above, leading to

$$\varepsilon_x = -\left(\frac{d^2w}{dx^2}\right)y + \frac{du_o}{dx}$$

3.4

or

$$\varepsilon_x = xy + \varepsilon_{xo}$$

$$\chi = -\frac{d^2w}{dx^2}$$

$$\varepsilon_{xo} = \frac{du_o}{dx}$$
The beam is in a state of plane stress. For a linear, elastic, isotropic material the stress-strain relationship is given by

\[
\begin{pmatrix}
\sigma_x \\
\sigma_y
\end{pmatrix} = \frac{E}{1 - \nu^2} \begin{pmatrix}
1 & \nu \\
\nu & 1
\end{pmatrix} \begin{pmatrix}
\varepsilon_x \\
\varepsilon_y
\end{pmatrix}
\]

It is known that the direct stress in the transverse direction \( \sigma_y \), is negligibly small. Therefore, we assume \( \sigma_y = 0 \) which with eq. 3.5 gives

\[
\varepsilon_y = -\nu \varepsilon_x
\]

and

\[
\sigma_x = E \varepsilon_x
\]

In developing the strain-displacement relationship above we have assumed \( \varepsilon_y = 0 \), which is inconsistent with the assumption \( \sigma_y = 0 \) made in the stress-strain relationship. This apparent inconsistency can be explained as follows. Neither \( \varepsilon_y \) nor \( \sigma_y \) are exactly equal to zero, but both are quite small. In using these assumptions, however, we should avoid using the condition \( \varepsilon_y = 0 \) for evaluating \( \sigma_y = \frac{\nu E \varepsilon_x}{1 - \nu^2} \), and using the condition \( \sigma_y = 0 \) for evaluating \( \varepsilon_y = -\varepsilon_x \). A further justification of the above assumptions is that they lead to a beam theory which adequately agrees with the observed behavior.

According to eqs. 3.4 and 3.7, the longitudinal stress \( \sigma_x \) is written as

\[
\sigma_x = E \chi y + E \varepsilon_{x0}
\]

and a normal force \( F \), in the longitudinal direction, can be obtained by integrating the above stresses over the element cross-section as
\[ F = \int \sigma_x \, dA = \int \int \varepsilon_y \, dA + \varepsilon \int \varepsilon \, dA \]  \hspace{1cm} (3.9)

Taking the reference axis of the above integration as the cross-section neutral axis and assuming that no axial force \( F \) is applied to the element, eq. 3.10 follows.

\[ \varepsilon_{x_0} = \frac{du_0}{dx} = 0 \]  \hspace{1cm} (3.10)

Assuming no rigid body motion is present, displacement \( u_0 = 0 \). From assumption (a) it is concluded that \( w = w(x) \). In the conventional beam element the transversal displacement field is represented by a cubic polynomial in \( x \).

\[ w = a + bx + cx^2 + dx^3 = (x) \{a\} \]  \hspace{1cm} (3.11)

\[ (x) = (1 \ x \ x^2 \ x^3), \ \{a\}^T = (a \ b \ c \ d) \]

The nodal displacements at nodes 1 and 2, vertical displacements \( w_1 \) and \( w_2 \) and rotations \( \theta_1 \) and \( \theta_2 \) respectively, may be expressed in terms of the coefficients of the assumed polynomial and the length of the element as

\[ \{d\} = [A]\{a\} \hspace{1cm} (3.12) \]

or

\[
\begin{align*}
\begin{bmatrix}
w_1 \\
\theta_1 \\
w_2 \\
\theta_2
\end{bmatrix}
&= 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
1 & 1 & 1^2 & 1^3 \\
0 & 1 & 2! & 3!
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d
\end{bmatrix}
\end{align*}
\]
In other words, the coefficients $a$, $b$, $c$ and $d$ of the polynomial defining the vertical displacement at any point within the element may be obtained in terms of the nodal displacements as

$$a = w_1$$

$$b = \theta_1$$

$$c = \left[ 3 \left( w_2 - w_1 \right) - 2 \left( \theta_1 + \theta_2 \right) \right] / l^2$$

$$d = \left[ -2 \left( w_2 - w_1 \right) + 1 \left( \theta_1 + \theta_2 \right) \right] / l^3$$

For obtaining the vertical displacements $w$ in terms of the nodal displacements, a set of shape functions $N$ are defined as

$$w = (N) \{d\}, \quad N = (x) [A]^{-1}$$

or

$$w = \frac{1}{l^3} \begin{bmatrix}
1^3 - 3 1 x^2 + 2 x^3 \\
1^3 x - 2 1 2 x^2 + 1 x^3 \\
3 1 x^2 - 2 x^3 \\
-1^2 x^2 + 1 x^3
\end{bmatrix}^T \begin{bmatrix}
w_1 \\
\theta_1 \\
w_2 \\
\theta_2
\end{bmatrix}$$

Rotations $\theta$, defined as the incremental rate of the vertical displacements $w$ with respect to the horizontal coordinate $x$, are obtained by differentiating the shape functions $(N)$ in eq. 3.14 as

$$\theta = \frac{d}{dx} (N) \{d\} = (N') \{d\}$$

or
Curvatures $\chi$ are obtained as the second derivative of the vertical displacements $w$, or the incremental rate of the rotations $\theta$ with respect to the axial coordinate $x$ as

$$\chi = \frac{d^2w}{dx^2} = -\frac{M}{EI}$$

or

$$\chi = -\frac{d^2}{dx^2} (N) \{d\} = (N'') \{d\} = (B) \{d\}$$

Normal strains $\varepsilon_x$, which vary linearly through the depth of the element according to eqs. 3.7 and 3.10, may be obtained from the curvature $\chi$ as

$$\varepsilon_x = \chi y$$

or

$$\varepsilon_x = (B) \{d\} y$$
Observing the linear stress-strain relationship assumed, shown in eq. 3.7, the normal stresses $\sigma_x$ are derived from the normal strains $\varepsilon_x$ as

$$\sigma_x = E ( B ) \{ d \} y$$  \hspace{1cm} 3.18

By applying to an element a set of arbitrary virtual displacements $\{ \delta d \}$, normal strains $\delta \varepsilon_x$ are therefore developed and may be expressed as

$$\delta \varepsilon_x = ( B ) \{ \delta d \} y$$  \hspace{1cm} 3.19

The internal work done by the stresses $\sigma_x$ may then be obtained as

$$\delta W_I = \int_v \sigma_x \delta \varepsilon_x \, dv$$

or

$$\delta W = \int_v \sigma_x ( B ) \{ \delta d \} y \, dv$$  \hspace{1cm} 3.20

Under the action of the external nodal forces $\{ f \}$ and the applier virtual displacements $\{ \delta d \}$, some external work is performed and may be expressed as

$$\delta W_e = \{ f \} \{ \delta d \}$$

or

$$\delta W_e = ( q_1 \, \bar{m}_1 \, q_2 \, \bar{m}_2 )^T \left\{ \begin{array}{c} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{array} \right\}$$  \hspace{1cm} 3.21

As an equilibrium requirement, both internal and external work must be the same. Since $\{ \delta d \}$ is arbitrary eqs. 3.20 and 3.21 give
\[ \{ f \} = \int_{V} c_{x} (B) y \, dv \quad 3.22 \]

By substituting eq. 3.18 into eq. 3.22

\[ \{ f \} = \int_{V} E (B) \{ d \} (B) y^{2} \, dv \]

or

\[ \{ f \} = [ \int_{V} E (B)^{T} (B) y^{2} \, dv ] \{ d \} \quad 3.23 \]

The element stiffness matrix \([k]\), defined as the relation between applied forces \(\{f\}\) and the nodal displacements \(\{d\}\) is then expressed as

\[ [k] = \int_{V} E (B)^{T} (B) y^{2} \, dv \quad 3.24 \]

or

\[ [k] = \int EI (B)^{T} (B) \, dx \]

When stiffness \(EI\) is constant throughout the element the stiffness matrix may be expressed as

\[ [k] = EI \int_{1} (B)^{T} (B) \, dx \quad 3.25 \]

The coefficients of the \((4 \times 4)\) stiffness matrix are obtained by substituting \((B)\), as shown in eq. 3.16, into eq. 3.25 and performing the linear integration as shown for the first coefficient \(k_{11}\)

\[ k_{11} = EI \int_{0}^{1} \frac{1}{12} \left(- 6 1 + 12 x \right)^{2} \, dx = \frac{12 EI}{3} \quad 3.26 \]
By applying an equivalent procedure, the remaining coefficients may be obtained and the final stiffness matrix is shown as

\[
[k] = \begin{bmatrix}
12 & 61 & -12 & 61 \\
61 & 41^2 & -61 & 21^2 \\
-12 & -61 & 12 & -61 \\
61 & 21^2 & -61 & 41^2
\end{bmatrix} \frac{EI}{1^3} \tag{3.27}
\]

The derivation shown above models the behavior of a two-noded conventional beam element with two degrees-of-freedom per node, as shown in Fig. 3.1. Its formulation considers a beam element without any axial force and deformations. It is customary to include a horizontal degree-of-freedom at the two nodes if the axial forces are present. If the reference axis coincides with the neutral axis of the beam, as previously assumed, it can be shown that the axial and bending stiffness terms are mutually uncoupled. In order to apply support conditions and axial forces, which may not be at the neutral axis, a linear transformation may be used.

\[
u(\text{at } y) = \nu(\text{at neutral axis}) - y^\theta \tag{3.28}
\]

This, however, presents a problem as pointed out by Gupta and Ma (74). The axial displacement along the reference axis varies linearly with the x coordinate. On the other hand, the transverse displacement is cubic in x and consequently the rotational displacement is quadratic. According to eq. 3.28, u for any nonzero value of y is therefore quadratic. This creates an inconsistency, and renders the element unfit for cases when the transformation required in eq. 3.28 needs to be
applied. Therefore, the conventional beam element has been discarded for use in the present study.

3.2.2 Isoparametric Beam Element

Since their first appearance, in the sixties (75), isoparametric beam elements have shown remarkable versatility, they have proven effective in two- and three-dimensional structural analyses and non structural applications as well. They are named isoparametric for their property of expressing the displacement field and the coordinate transformation through the same (ISO) set of interpolation functions (parameters), commonly known as shape functions. Their general formulation can be found in most finite element books (76,77,78). Here, we will restrict ourselves to presenting the formulation of the one-dimensional beam element adopted.

As in the conventional beam element, here too, we assume that the direct strains in the transverse direction y are negligibly small and that the plane cross-section normal to the longitudinal axis of the beam remains plane after the beam deforms (\( \gamma_{xy} \) constant along y). Unlike the conventional beam element, normal to the longitudinal axis of the beam, in the vertical plane of symmetry, are not assumed to remain normal after deformation. Further, the direct stress in the normal direction is assumed to be negligibly small, giving \( \sigma_x = E \varepsilon_x \), as in the conventional beam element.

Referring to Fig. 3.2, taking \( u_i, w_i \) and \( \theta_i, i = 1,2 \), as the nodal degrees-of-freedom in the x direction, y direction and rotational
direction respectively, the displacements at any point within the element can be expressed in terms of the shape functions as

\[ u(y) = \sum_i N_i u_i - y \sum_i N_i \theta_i \]

\[ w = \sum_i N_i w_i \]

\[ \theta = \frac{\partial u}{\partial y} = -\sum_i N_i \theta_i \]

The nodal coordinates can as well define the global coordinates at a point in the element, through the use of the same shape functions as

\[ x = \sum_i N_i x_i \]

The interpolation or shape functions are defined in terms of the local coordinate \( \zeta \) as

\[
(N) = \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} \frac{1 - \zeta}{2} \\ \frac{1 + \zeta}{2} \end{pmatrix}
\]

The strain field is obtained as the first derivative of the displacements and also expressed in terms of the shape functions appear as

\[ \varepsilon_x = \frac{\partial u}{\partial x} = \sum_i \frac{\partial N_i}{\partial x} u_i - y \sum_i \frac{\partial N_i}{\partial x} \theta_i \]

\[ \varepsilon_y = \frac{\partial w}{\partial y} = \sum_i \frac{\partial N_i}{\partial y} w_i = 0 \]

\[ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x} = -\sum_i N_i \theta_i + \sum_i \frac{\partial N_i}{\partial x} w_i \]
which in matrix notation leads to the definition of the \([B]\) matrix as follows

\[
\begin{bmatrix}
\varepsilon_x \\
y_{xy}
\end{bmatrix} = \Sigma_i \begin{bmatrix}
\frac{\partial N_i}{\partial x} & 0 & -y \frac{\partial N_i}{\partial x} \\
0 & \frac{\partial N_i}{\partial x} & -N_i
\end{bmatrix} \begin{bmatrix}
u_i \\
w_i \\
\theta_i
\end{bmatrix}
\] 

or

\[
\{ \varepsilon \} = [B] \{ d \}
\]

from which \(\varsigma\) is deleted because it is assumed to be zero. The nodal displacement vector consists of nodal degrees-of-freedom and is given by

\[
\{ d \}^T = (u_1 w_1 \theta_1 u_2 w_2 \theta_2)
\]

The shape functions are expressed in terms of the local coordinate \(\varsigma\), so the derivation chain rule is invoked

\[
\frac{\partial N_i}{\partial x} = \frac{\partial N_i}{\partial \xi} \frac{\partial \xi}{\partial x}
\]

\[
\frac{\partial \xi}{\partial \xi} = \Sigma_i \frac{\partial N_i}{\partial \xi} x_i = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} \end{bmatrix} \begin{bmatrix} x_1 \\
x_2 \end{bmatrix}
\]

and the scale factor between the two coordinate systems is defined as

\[
\frac{\partial \xi}{\partial x} = \frac{2}{\gamma}
\]

The \([B]\) matrix is then obtained as
According to eq. 3.37 the transverse shear strain $\gamma_{xy}$ varies linearly with $\zeta$ and therefore with $x$. The direct strain $\varepsilon_x$ does not vary with $\zeta$ or $x$, which would be caused by a state of constant moment and axial force. The shear strain distribution is inconsistent with the moment distribution and is known to render the element stiffness matrix too stiff in shear (79). The spurious shear strains and stresses have been called parasitic and the effect on the element stiffness matrix shear-locking. It can be shown that eq. 3.33 evaluates the shear strain accurately at $\zeta = 0$, which points to the remedy of the problem: setting $\zeta = 0$ in eq. 3.37. Therefore the matrix $[B]$ becomes

$$
[ B ] = \begin{bmatrix}
-\frac{1}{Y} & 0 & \frac{1}{Y} & 0 & -\frac{Y}{Y} \\
0 & -\frac{1}{Y} & -\frac{1-\xi}{2} & 0 & \frac{1}{2}
\end{bmatrix}
$$

3.37

As stated earlier, a uniaxial stress-strain relationship is used. Therefore, the stress-displacement relationship may be defined as

$$
\{ \sigma \} = [ E ] \{ \varepsilon \} = [ E ] [ B ] \{ d \}
$$

where

$$
[ E ] = \begin{bmatrix} E & 0 \\ 0 & G \end{bmatrix}, \quad G = \frac{E}{2(1+\nu)}
$$

3.39

Normal strains $\{ \delta \varepsilon \}$ corresponding to a virtual displacement vector $\{ \delta d \}$ are given by
\[ \{ \delta \epsilon \} = [B] \{ \delta \mathbf{d} \} \]  

and the internal work done by the stresses may be obtained as

\[ \delta W_i = \int_V \{ \sigma \} \{ \delta \epsilon \} \, dv \]  

or by substituting eqs. 3.39 and 3.40 into eq. 3.41 as

\[ \delta W_i = \int_V \{ E \} \{ B \} \{ d \} \{ B \} \{ \delta d \} \, dv \]  

Under the action of the external nodal forces \( \{ f \} \), where

\[ \{ f \} = (f_1 \ q_1 \ m_1 \ f_2 \ q_2 \ m_2)^T \]  

and the applied virtual displacements \( \{ \delta d \} \), some external work is performed by those nodal forces and may be written as

\[ \delta W_e = \{ f \}^T \{ \delta d \} \]  

By equating 3.41 and 3.44, seeking equilibrium between internal and external works, the nodal forces may be expressed as

\[ \{ f \} = \int_V \{ E \} \{ B \} \{ d \} \{ B \} \, dv \]  

or

\[ \{ f \} = \int_V \{ B \}^T \{ E \} \{ B \} \, dv \{ d \} \]  

which define the element stiffness matrix \([k]\) as

\[ [k] = \int_V \{ B \}^T \{ E \} \{ B \} \, dv \]  

The coefficients of the \([k]\) matrix are obtained by performing the integration in eq. 3.46 as shown for the first coefficient \(k_{11}\)

\[ k_{11} = \int_V (-\frac{1}{T})^2 E \, dv = \frac{1}{T^2} \int_1^T EA \, dx = \frac{EA}{T} \]
By applying a similar procedure to the coefficients the complete matrix is obtained as

\[
[k] = \begin{bmatrix}
\frac{EA}{T} & 0 & -\frac{ES}{T} & -\frac{EA}{T} & 0 & \frac{ES}{T} \\
\frac{GA}{T} & \frac{GA}{2} & 0 & -\frac{GA}{T} & \frac{GA}{2} \\
\frac{EI + GA}{T^2} & \frac{ES}{T} & -\frac{GA}{2} & -\frac{EI + GA1}{T^2} \\
\frac{EA}{T} & 0 & -\frac{ES}{T} & \frac{GA}{T} & -\frac{GA}{2} \\
\frac{EI + GA1}{T} & 0 & 0 & \frac{EI + GA1}{T^2}
\end{bmatrix}
\]

where

\[
EA = \int E \, dA, \quad ES = \int Ey \, dA \quad \text{and} \quad EI = \int E \, y^2 \, dA
\]

In the development of the isoparametric beam element it is not necessary to take the reference x-axis along the neutral axis. Therefore, the integral for ES is not necessarily zero.

### 3.2.3 Initial Strains

When studying the effects of prestressing and temperature variation in bridge beams, these effects are applied by specifying initial strains to an element. Such procedure is rather simple and may be shown as follows.
Let us assume that an initial strain vector \( \{ \epsilon_0 \} \) is specified to a beam element as shown in Fig. 3.2. According to eq. 3.39, the final state of stress may be written as
\[
\{ \sigma \} = \{ E \} \{ \epsilon - \epsilon_0 \}
\] 3.49
where \( \{ \epsilon \} \) is the current strain vector of the system exclusive of the effects contained in \( \{ \epsilon_0 \} \). Equating 3.39 and 3.45 the vector of nodal forces may be written as
\[
\{ f \} = \int_\nu [ B ]^T \{ \sigma \} \, dv
\] 3.50
and substituting 3.49 into 3.50 as
\[
\{ f \} = \int_\nu [ B ]^T \{ E \} \{ \epsilon \} \, dv - \int_\nu [ B ]^T \{ E \} \{ \epsilon_0 \} \, dv
\] 3.51
The first term in eq. 3.51 may be identified with eq. 3.45 whereas the latter term may, accordingly, be written as a vector of nodal forces like
\[
\{ f \} = [ k ] \{ d \} - \{ f_0 \}, \quad \{ f_0 \} = \int_\nu [ B ]^T \{ E \} \{ \epsilon_0 \} \, dv
\]
or
\[
\{ f \} + \{ f_0 \} = [ k ] \{ d \}
\] 3.52
The vector of nodal forces \( \{ f_0 \} \) represents the effect of the specified strains \( \{ \epsilon_0 \} \) on the element. Thus, according to eq. 3.52, it can be added to the current vector of nodal forces \( \{ f \} \).

3.2.4 Prestressing Effect

As mentioned earlier in this chapter the precast or cast-in-place girders of a bridge structure may be of prestressed concrete, among other methods of construction. Prestressing forces are taken into
account by specifying initial prestressing strains in the tendons. The tendons initial strains $\varepsilon_{po}$ are defined as the strains due to prestressing forces before releasing of the forces, i.e. the prestressing strain at the zero deflection state of the member. Since the initial prestressing force in a tendon is usually known, the strain that produces that stress level can be obtained from the stress level and the instantaneous stress-strain relationship of the tendon steel as shown in Section 2.3.1 as

$$\varepsilon_{po} = - \frac{N_{po}}{A_p E_p}$$  \hspace{1cm} (3.53)

where

$N_{po}$ = Initial prestressing force, including friction loss, if any.
$A_p$ = Cross-sectional area of the tendons.
$E_p$ = Modulus of elasticity of the prestressing steel.

After transfer of the prestressing forces and throughout the life of the member the cable forces vary continually in time and along the length of the cable as well. To introduce this variational effect into the analysis a procedure has been devised so as to introduce the constant presence of the prestressing tendons into the mathematical formulation of the elements that model the member. In such a way any variation in the elements displacement field is automatically imposed upon the tendons, and vice-versa. The variable tendon force may be expressed as

$$N_p = E_p A_p (\varepsilon_{pd} - \varepsilon_{po})$$  \hspace{1cm} (3.54)

where $\varepsilon_{pd}$ is the current strain at the level of the prestressing steel due to effects other than prestressing.
By assuming that:

i) the tendon profiles can be represented by their centroidal lines, 

ii) tendons may be approximated as straight within the length of an element, 

iii) the tendon force may be taken as constant within each element, 

iv) complete bond exists between the tendons and the concrete, 

the following relations between displacements at the element nodal points and displacements at the extremities of the tendon can be drawn, according to Fig. 3.3.

\[ \begin{align*} 
  u_{pi} &= u_i - \theta_i y_{pi} \\
  w_{pi} &= w_i, \quad i = 1, 2 
\end{align*} \]  \hspace{1cm} 3.55

The axial displacements of the cable, at points 1 and 2, may also be expressed in terms of the nodal displacements by

\[ \Delta_i = (u_i - \theta_i y_{pi}) \cos \alpha - w_{pi} \sin \alpha \]  \hspace{1cm} 3.56

The relationship between the tendon strains and the nodal displacements is

\[ \varepsilon_{pd} = \frac{1}{l_p} (\Delta_2 - \Delta_1) = (B_p) \{d\}, \quad l_p = 1 / \cos \alpha \]  \hspace{1cm} 3.57

where

\[ (B_p) = \frac{1}{l_p} \begin{bmatrix} 
  - \cos \alpha \\
  \sin \alpha \\
  y_{p1} \cos \alpha \\
  \cos \alpha \\
  - \sin \alpha \\
  - y_{p2} \cos \alpha 
\end{bmatrix}^T \]

According to Section 3.2.3, the vector of nodal forces due to
Fig 3.3 - Prestressing cable model.

Fig 3.4 - Prestressing force variation due to friction.
$[ k_p ] = \frac{E_p A_p}{L} = \begin{bmatrix}
\cos^3\alpha & -\sin\alpha\cos^2\alpha & -y_{p1}\cos^3\alpha & -\cos^3\alpha & \sin\alpha\cos^2\alpha & y_{p2}\cos^3\alpha \\
\sin^2\alpha\cos\alpha & y_{p1}\sin\alpha\cos^2\alpha & \sin\alpha\cos^2\alpha & -\sin^2\alpha\cos\alpha & -y_{p2}\sin\alpha\cos^2\alpha \\
y_{p1}^2\cos^3\alpha & y_{p1}\cos^3\alpha & -y_{p1}\sin\alpha\cos^2\alpha & -y_{p1}y_{p2}\cos^3\alpha \\
\cos^3\alpha & -\sin\alpha\cos^2\alpha & -y_{p2}\cos^3\alpha \\
\sin^2\alpha\cos\alpha & y_{p2}\sin\alpha\cos^2\alpha \\
y_{p2}^2\cos^3\alpha
\end{bmatrix}$
prestressing may be expressed as

\[
\{ f_p \} = \{ f_{pd} \} + \{ f_{po} \} = [ k_p ] \{ d \}
\]

where

\[\{ f_{pd} \} = \text{Nodal forces due to variation of the prestressing force } N_p \text{ caused by deformation of the member.}\]

\[\{ f_{po} \} = \text{Nodal forces due to initial prestressing force } N_{po} \text{ and further losses from relaxation.}\]

\[
= \int_V (B_p)^T E_p \epsilon_{po} \, dv
\]

\[
= - l_p (B_p)^T N_{po}
\]

\[k_p = \text{Stiffness matrix of the tendon as shown in page 74}\]

\[
= \int_V (B_p)^T E_p (B_p) \, dv
\]

\[
= A_p E_p l_p (B_p)^T (B_p)
\]

Initially, the vector \(\{ f_{po} \}\) is used to perform an analysis of the beam system to evaluate the initial strains \(\{ \epsilon_{po} \}\) in the beam element. Subsequently, any changes in the value of \(N_p\) are automatically accounted for in the analyses of the assembled beam-prestressing tendon system. In pre-tensioned beams all analyses are performed by including the tendon stiffness matrix \([k_p]\) into the beam element stiffness matrix \([k]\). Therefore, losses due to elastic shortening in the member are obtained. On the other hand, in post-tensioned members the above mentioned stiffness matrices are superposed only after the first analysis has been performed, i.e. after the effects of initial prestressing have been obtained.
The post-tensioned tendons, due to the curvature of the profile and also due to out-of-plane deflection or wobble, present a variation of the prestressing force along its length, even before this force is transferred to the concrete member. This loss of the prestressing force, due to friction, is a function of the stress in the tendon, its profile, wobble and the coefficients of friction between the tendons and the surrounding materials.

The PCI Committee on Prestress Losses (23) recommends an expression to estimate the variation of the prestressing force, for a single end force, as follows:

\[ N_{pl} = N_{po} e^{-(\mu\phi + kx)} \]  

where

- \( N_{pl} \) = steel force at a point
- \( N_{po} \) = steel force at the jacking end
- \( e \) = base of the Neperian logarithm
- \( \mu \) = curvature friction coefficient
- \( k \) = wobble friction coefficient
- \( \phi \) = total angular change of prestressing tendon profile, in radians, from jacking end to the point of interest
- \( x \) = length along the prestressing tendon from jacking end to the point of interest

In the numerical solution, once the continuous cable profile is discretized into straight segments within the length of each element, the application of eq. 3.59 must be adapted for the calculation of the end forces applied at each nodal point. Referring to Fig. 3.4, assume a string of elemental cable segments, i, j and k, for which the jacking
end is positioned somewhere before segment i. And further assume that the change in the angular direction between two consecutive nodal points, I and J, can be expressed by the average of the changes of the angular directions at nodes I and J, weighted by the length of the adjacent elements as

$$\phi_j = \frac{\phi_{ij}}{l_j} + \frac{\phi_{jk}}{l_j + l_k}$$

where

$$\phi_{ij} = |\alpha_i - \alpha_j|, \quad \phi_{jk} = |\alpha_j - \alpha_k|$$

The prestressing force at the center of the element $N_j$ can then be obtained as the average of the forces at the nodal points I and J, $N^1_j$ and $N^2_j$ respectively as

$$N_j = \frac{1}{2} (N^1_j + N^2_j) = \frac{1}{2} N^2_j [1 + e^{-\left(\mu \phi_j + k l_j\right)}]$$

where $N^2_j$ has been previously obtained when calculating the force in element i, since equilibrium must be maintained at the nodes and consequently $N^2_j = N^1_i$.

### 3.2.5 Temperature Effect

Bridges are usually exposed to the variable climatic conditions. Depending on its geographic location, temperature variation may be quite extensive not only due to seasonal changes but also within a daily cycle. Numerous random factors contribute to temperature distribution on a bridge, namely: the surrounding air temperature, the solar energy striking a surface, convection caused by the wind and various forms of
precipitations. As those conditions change continuously temperature distribution varies with position and time.

Hoffman, et al (80), studying temperature effects on a experimental segmental concrete bridge, have shown that the vertical temperature gradient among the cross-sections is highly nonlinear although little variation has been noticed in the longitudinal and transversal directions. This highly unpredictable phenomenon, according to some investigators (80,81,82), is somewhat neglected in today's design codes, though it is frequently the main cause of structural distress if inadequate provisions are considered in design. In jointless long span bridge construction this effect is of paramount importance.

In the present study thermal effects are analyzed considering a given temperature distribution over the composite cross-section, though stationary in time and nonvariable along the length of the member. A nonlinear temperature gradient is approximated by a bilinear variation as shown in Fig. 3.5, with the input values of the temperatures at top, deck-girder interface and bottom, T1, T2 and T3 respectively.

The inclusion of temperature effects in a finite element solution is straight forward and its formulation can be found in most books (76,77). Temperature variation at any given point $\Delta T$ induces a thermal strain $\varepsilon_T$, which may result in a stress-free strain or cause an additional stress, provided that free deformation is prevented. Assuming as the coefficient of thermal expansion of the material, the final strains may be expressed as

$$\varepsilon_T = \alpha \Delta T$$

3.63
Fig 3.5 - Temperature variation in the cross-section.

Fig 3.6 - Layered cross-section and stress and strain distributions.
These strains are treated as initial strains as described in Section 3.2.3.

3.2.6 Layered Section

To simplify the discussion, stiffness of the isoparametric beam element has been derived in Section 3.2.2 as if the element consists of one material having constant modulus of elasticity. In reality the beam cross-section consists of two distinct materials, concrete and steel. (Stiffness of the prestressing tendons has been evaluated separately in Section 3.2.4.) Further, the modulus of concrete may vary due to variation in stresses and strains along the y-direction within an element. To account for the variation in material and the modulus along depth, a numerical approach is used to perform integration on the section of an element. The element is divided into several layers, each consisting of one material. Further, depth of each layer is kept sufficiently small such that the stresses within the layer may be assumed constant. Therefore, each layer has a distinct modulus. According to Fig. 3.6, integrals in eq. 3.48 are then replaced by

\[
\begin{align*}
\bar{EA} &= \int E \, dA = \varepsilon_i E_i A_i \\
\bar{ES} &= \int E y \, dA = \varepsilon_i E_i y_i A_i \\
\bar{EI} &= \int E y^2 \, dA = \varepsilon_i E_i y_i^2 A_i
\end{align*}
\]

3.64
3.2.7 Coordinate Transformation

Beams rarely have their supports located at the level of their neutral axis, as commonly assumed in basic structural analysis. Supports are generally provided at the bottom face of the beams and sometimes even below the bottom flange, as in the case of steel rocker bearings. For accurately modeling the actual supporting conditions of a general beam the adopted elements have their nodes located at a variable position within the vertical planes containing their extreme sections, as shown in Fig. 3.2. The coefficients of the stiffness matrix in eq. 3.46 may be obtained by performing the necessary integrations with respect to the section neutral axis, to the bottom surface as shown in eq. 3.48 or to any other horizontal axis chosen. It is found reasonable to obtain the matrix \([k]\) with respect to the bottom surface of the element, as a general rule. Should the nodes be located at a variable position \(y\) from this surface a linear transformation is performed as shown below.

Assuming that \(\{d\}\) are the displacements of a node positioned at the bottom surface of the element and \(\{ar{d}\}\) are the displacements of a point positioned at a general location \(y\) from this surface, the following relations can be drawn.

\[
\begin{align*}
  u &= \bar{u} - y\bar{\theta} \\
  w &= \bar{w} \\
  \theta &= \bar{\theta}
\end{align*}
\]

or for the element
which leads to the definition of the transformation matrix \([A]\) as

\[
\{ \mathbf{d} \} = [A] \{ \mathbf{\tilde{d}} \}
\]

3.67

Using the principle of virtual work one can show that

\[
\{ \mathbf{f} \} = [A]^T \{ \mathbf{f} \}
\]

3.68

Regardless where the nodes are located the equilibrium equation can still be written as

\[
\{ \mathbf{f} \} = [k] \{ \mathbf{d} \}
\]

3.69

or

\[
\{ \mathbf{f} \} = [\tilde{k}] \{ \mathbf{\tilde{d}} \}
\]

where \([\tilde{k}]\) is the transformed element stiffness matrix for the case where the nodes are shifted to a general position away from the bottom surface. By substituting eqs. 3.67 and 3.68 into eq. 3.69, the new transformed element stiffness matrix is obtained as

\[
[\tilde{k}] = [A]^T [k] [A]
\]

3.70

where \([k]\) is the element stiffness matrix as defined in eq. 3.48 and \([A]\) is the transformation matrix defined in eq. 3.68.
3.2.8 Bearing Supports

Bearing supports are widely used in bridge beams, as explained in Section 2.4. In the present study bearing supports are modeled as uniaxial spring-like elements attached to the respective nodal points as shown in Fig. 3.2. As such, their axial stiffnesses \([kb]\) are calculated with the procedure outlined in Section 2.4 and added to the corresponding degrees-of-freedom of the element stiffness matrix, as shown in the following section.

3.2.9 Element Equilibrium Equation

The elements formulated previously are subjected to nodal forces from the externally applied loading and due to initially prescribed strains. Displacements at the nodal points are related to these nodal forces through the following equilibrium equation.

\[
\{ f \} + \{ f_0 \} = [k_e] \{ d \}
\]

where

- \(\{ f \} = \) Vector of externally applied nodal forces.
- \(\{ f_0 \} = \) Force vector generated from the initial strains due to prestressing, temperature variation and time-depended effects.
- \([k_e] = \) Stiffness matrix of the composed element as shown in Fig. 3.2 obtained by superposition of the stiffness matrices of the isoparametric beam element, prestressing tendon and bearing supports, from eqs. 3.48 and 3.58 and Section 3.2.8 respectively.
\[ [k_e] = [k] + [k_p] + [k_b] \]

3.3 Connection Element

A connection element is used to model the single deck connection between two adjacent girders or the deck connection to the end abutment, if any, when no joints are provided in these regions. These connections are very short in length when compared to the dimensions of the cross-section of the beam. They are found to vary in the range of 2 to 6 inches, whether a diaphragm is provided or a gap exists between the two girders \((9,39)\). The deck has a very small moment of inertia when compared to that of the composite section, especially for the case of concrete girders. It seems reasonable to model the deck connection by a spring like element, which has only axial stiffness and is located at the centroidal line of the deck, away from the centroidal line of the composite section. Although the flexural stiffness of the deck is being disregarded at the connection, the predominant effect of the presence of the deck is accounted for by the rotational restriction provided by the spring, in its offset position from the centroid of the composite section.

The formulation of the connection element is remarkably simpler than the one presented for the beam element and is obtained as follows. Referring to Fig. 3.7, consider that \(u_1\), \(w_1\), \(\theta_1\), and \(u_2\), \(w_2\), and \(\theta_2\), are the displacements of the nodal points 1 and 2, respectively, which are shared by the connection element and the two adjacent beam elements. Consider also that \(u^*_1\) and \(u^*_2\) are the two horizontal displacements at the centroidal points A and B, at the opposite faces of
Fig 3.7 - Connection element.

Fig 3.8 - Cracking stress and position.
the connection element. These two sets of displacements can be related to one another by the transformation matrix \([T]\) as

\[
\begin{pmatrix}
  u_1^* \\
  u_2^*
\end{pmatrix} = \begin{bmatrix}
  1 & 0 & -\left(h_1 + y_1\right) & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & -\left(h_2 + y_2\right)
\end{bmatrix} \begin{pmatrix}
  u_1 \\
  w_1 \\
  \theta_1 \\
  u_2 \\
  w_2 \\
  \theta_2
\end{pmatrix}
\]

3.72

or

\[
\{ d^* \} = [ T ] \{ d \}
\]

The same transformation matrix can be used to relate the nodal forces \([f]\) to the horizontal normal forces \([f^*]\) applied at the centroidal points A and B as

\[
\{ f \} = [ T ] \{ f^* \}
\]

3.73

The element equilibrium equation is written as

\[
\{ f^* \} = [ k_c^* ] \{ d^* \}
\]

3.74

where the axial stiffness matrix \([k_c^*]\) is defined by

\[
[k_c^*] = \frac{EA}{L} \begin{bmatrix}
  -1 & 0 \\
  0 & -1
\end{bmatrix}
\]

3.75

By equating 3.72, 3.73 and 3.74 the element equilibrium equation relating nodal forces and displacements is given by

\[
\{ f \} = [ k_c ] \{ d \}
\]

3.76
where the matrix \([k_c]\) is obtained as

\[
[k_c] = [T]^T [k_c^*] [T]
\]

As for the beam element the contribution of the bearing stiffness \([k_b]\) at the nodal points is considered by superposing this value to the element stiffness matrix. The final connection element stiffness matrix can be written as

\[
[k_c] = \begin{bmatrix}
\frac{EA}{L} + k_b & 0 & -\frac{EA}{L} (h_1 + y_1) & -\frac{EA}{L} & 0 & \frac{EA}{L} (h_2 + y_2) \\
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{EA}{L} (h_1 + y_1)^2 & \frac{EA}{L} (h_1 + y_1) & 0 & -\frac{EA}{L} (h_1 + y_1)x & \frac{EA}{L} (h_2 + y_2) \\
\frac{EA}{L} + k_b & 0 & -\frac{EA}{L} (h_2 + y_2) & 0 & 0 & \frac{EA}{L} (h_2 + y_2)^2
\end{bmatrix}
\]

where

\[
\frac{EA}{L} = \text{Axial stiffness of the element considering concrete and steel reinforcements.}
\]

\(h_1, h_2 = \text{Distance from the CGC of the deck section to the bottom}\)
surface of the adjacent beam elements, at nodes 1 and 2.  
\[ y_1, y_2 = \text{Distances from the nodal points 1 and 2 to the bottom surface of the adjacent elements.} \]

3.4 Global Problem

Using the finite elements formulated in previous sections a member is modeled as several discrete elements connecting end-to-end at nodal points. It is idealized as a straight line passing through these nodal points, each of which having three degrees-of-freedom. The global stiffness matrix of the system is obtained by appropriate assembly of the stiffness matrices of the individual beam elements as shown in eq. 3.70, and connection elements if any, as shown in eq. 3.78. Both elements share a single nodal point and therefore their stiffness matrices are superposed at the common degrees-of-freedom.

The elements are used to represent material and cross-section properties of the member. Loads and restraints in any of the three directions are lumped and applied only at the nodal points. The global equilibrium equation of the system is written as

\[ \{ F \} + \{ F_0 \} = [ K ] [ D ] \]

where

\[ F = \text{Assembled vector of applied loads.} \]
\[ F_0 = \text{Force vector generated from initial strains.} \]
\[ [K] = \text{Global stiffness matrix.} \]
\[ D = \text{Vector of nodal displacements.} \]
Under the action of loads, a solution for displacements is performed. Displacements are used for the calculation of strains and, in turn, for obtaining stresses. Element resisting forces, obtained by integrating stresses over the cross-section are eventually transformed back into nodal forces to seek equilibrium of the system. Moments, axial and shear forces are constant within the length of each element and thus their distributions along the length of the member are linearly approximated. The more elements are used for modeling the member, the more accurate is the approximation. Reactions are calculated at the nodal supports and correspond to the resisting forces shared by the elements connected at these nodal points.

Response analysis of a member may include both linear and nonlinear ranges of behavior. Nonlinearity is introduced by the assumed nonlinear properties of the materials as well as due to cracking of the sections as explained in following section. The solution for displacements is performed by an incremental method using the tangent stiffness matrix of the system, either by increments of load or displacement as described in Section 3.8. Time-dependent response analysis is discussed in Section 3.9.

3.5 Loading

A structural member behaves according to the imposed loading and constraints. In the case of composite precast construction these loadings and constraints are very well defined qualitatively, quantitatively, and in time. A numerical analysis, therefore, has to properly model each and all of these conditions in sequence and carrying
the current state of the member from one loading stage to the next. In this study, the following seven different loadings are considered:

Prestressing
Girders Dead Load
Deck Dead Load
Live Loads
Support Displacements
Temperature
Time-Dependent Effects

Each of the above loadings is applied upon the structure at a predefined time or time interval and subdivided into a specified number of steps whether of load, displacement or time. Prestressing and temperature are applied as equivalent nodal loads as explained in sections 3.2.4 and 3.2.5 respectively. Support displacements will be discussed in Section 3.6 and time-dependent effects in Section 3.9.

Dead loads are linearly distributed along the member and are divided into two categories: girder dead load and deck dead load. The latter includes not only the weight of the deck slab but also of any permanent component like diaphragms, wearing course, cubes and rails, etc. Both dead loads, often applied at different ages of the structure, are modeled as equivalent nodal loads.

Live loads represent any non permanent external action applied to the members due to traffic, wind, earthquakes, etc. Here, however, only vertically and statically applied loads are assumed. Live loads are usually concentrated or distributed within limited regions. In this study the inclusion of any distributed or concentrated live load is
accepted. In the analysis, however, they are also modeled by equivalent nodal loads. Within the solution, the response under the action of such loads may be obtained by using the Method of Load Increments as well as by using the Displacement Increment Method, both described in following sections.

In many structural analyses, it is essential to evaluate the strength of a member, i.e. the member response to the ultimate load level. The Displacement Increment Method is suitable for the purpose, according to the reasons mentioned in Sections 3.8.1 and 3.8.2. Appendices A and C show in more detail how to input live and dead loads into the program.

3.6 Support Displacements

Structures are often very sensitive to support displacements. A simple beam would rarely be affected by any small prescribed displacement at the supports. A continuous beam, however, may drastically change its response behavior should its supporting conditions be altered. Vertical displacements at a support of a beam may occur due to a settlement of the supporting structure or an intentional adjustment of one or more of the supporting points.

Two different procedures have been commonly used for the solution of a structural systems under support displacements, and they are briefly outlined as follows. Consider the equilibrium equation of a system, as shown in eq. 3.80, for which some of the degrees-of-freedom have prescribed displacements \( D_1 \). Then the remaining displacements \( D_2 \)
and the nodal loads associated with the prescribed degrees-of-freedom \( \{ F_2 \} \) are unknowns. The system stiffness matrix is then partitioned accordingly as

\[
\begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\begin{bmatrix}
D_1 \\
D_2
\end{bmatrix}
= 
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix}
\]  

3.80

and the unknown displacements and loads can be obtained by solving the following set of equations, respectively.

\[
\begin{bmatrix}
K_{22}
\end{bmatrix}
\begin{bmatrix}
D_2
\end{bmatrix}
= 
\begin{bmatrix}
F_2
\end{bmatrix}
- 
\begin{bmatrix}
K_{21}
\end{bmatrix}
\begin{bmatrix}
D_1
\end{bmatrix}
\]  

3.81

The above procedure, though mathematically correct, increases enormously the computational work involved in solving the system. Not only the stiffness matrix has to be partitioned, as in eq. 3.80, but also there is one more set of multiplications to be performed, as in eq. 3.81, a process that might be repeated numerous times for a nonlinear system. However, a more economical and as precise procedure may be used to solve the same problem, regarding the level of accuracy of the problem studied and acknowledging the possibility of numerical weaknesses.

Assume, for convenience of illustration, the same structural system solved by the above procedure, for which one or more degrees-of-freedom have prescribed displacements, \( D_i, D_j, \ldots \), where \( i, j, \ldots \), represent supporting nodal points. Eq. 3.80 may be written as
Where the stiffness matrix $[K]$ does not need to be partitioned and is used as for the solution of any other loading condition. Furthermore, assume that the coefficients of the stiffness matrix, $K_{ii}, K_{jj}, \ldots$, are altered so that they become extremely greater than the other coefficients of the same lines of the matrix, i.e. the remaining coefficients of the equilibrium equations of the nodes in question as

$$K_{ii} = \beta K_{ii}, \quad i = 1, 2 \quad 3.83$$

where $\beta$ is a convenient large number as $10^{12}$ for instance, and in the load vector $\{F\}$ the corresponding elements, $F_{ii}, F_{jj}, \ldots$, are also altered accordingly so that

$$F_i = \beta K_{ii} D_i, \quad i = 1, 2 \quad 3.84$$

Then, the system of equations 3.82 may be usually solved by any process being employed in the numerical procedure and applied to the remaining loading cases. This procedure, which is more economical than the one described previously has been successfully used by some investigators (83) and is applied in this study.
3.7 Cracking Model

In the analysis of concrete structures the low tensile strength of concrete and the ensuing cracking that results therefrom is of paramount importance. Cracking effect in finite element analysis of reinforced concrete structures was first introduced by Ngo and Scordelis (29) through the use of a predefined discrete crack system. Nilson (30) proposed a model for progressive crack propagation by separating elements at each side of a node and, consequently, having to deal with a continuous change in the topology of the mathematical model.

A second approach was later introduced by Rashid (84) who in studying the behavior of prestressed concrete pressure vessels, proposed the so-called smeared cracking model. In such a model, rather than representing a single crack, many finely spaced cracks perpendicular to the principal strain direction are assumed to occur. This approach has been used by a majority of investigators (33) who acknowledged its main features of permitting an automatic generation of cracks without the redefinition of the finite element topology and the complete generality in possible crack directions.

In the present study, regarding the class of structures analyzed and the type of analysis proposed, the smeared cracking model has been adopted due to its proven efficiency and easy application. Cracking is considered through a strain criteria. It is assumed to happen when the principal strain at a point $\varepsilon_1$ reaches the concrete rupture strain $\varepsilon_r$. Cracks are expected to be oriented perpendicularly to the direction of the principal stress as shown in Fig. 3.8. Considering that normal strains in the vertical direction are disregarded in the formulation
Fig 3.9 - Stress distributions on the cross-section.

Fig 3.10 - Crack formation and closing.
of the element, as explained in Sections 3.2.1 and 3.2.2, the principal strain at a point may be obtained as

\[ \varepsilon_1 = \frac{1}{2} [ \varepsilon_x + (\varepsilon_x^2 + \gamma_{xy})^{1/2} ] , \quad \varepsilon_x = \frac{\sigma_x}{E} , \quad \gamma_{xy} = \frac{\tau_{xy}}{G} \]  

3.85

and its orientation \( \alpha \) by

\[ \alpha = \frac{1}{2} \tan^{-1} \left( \frac{\gamma_{xy}}{\varepsilon_x} \right) \]  

3.86

where \( \varepsilon_x \) and \( \gamma_{xy} \) are the normal strain in the axial direction and the shear strain, respectively. From eq. 3.37 and 3.38 it is observed that the normal strain \( \varepsilon_x \) varies linearly through the depth of the element whereas the shear strain \( \gamma_{xy} \) is constant. The assumption of constant \( \gamma_{xy} \) in the element cross-section is reasonable for modeling the global behavior of the member under a bending mode of deformation. However, shear strains do vary along the depth of the beam, according to the distributions of shear stresses and material properties.

The local distribution of shear stresses may be obtained by equating the equilibrium of an infinitesimal beam element as shown in Fig. 3.9. At a general position \( y \) from the section neutral axis the normal strain developed under the applied forces may be written as

\[ \varepsilon_x = \frac{N}{EA} + \frac{M}{EI} y = \varepsilon_{xo} + xy = \begin{bmatrix} \varepsilon_{xo} \\ x \end{bmatrix} \begin{bmatrix} 1 \\ y \end{bmatrix} \]  

3.87

and the corresponding normal stress as

\[ \sigma_x = E \varepsilon_{xo} + E x y \]

\[ = E \begin{bmatrix} \varepsilon_{xo} \\ x \end{bmatrix} \begin{bmatrix} 1 \\ y \end{bmatrix} \]  

3.88
The axial force \( N \) and bending moment \( M \) may be obtained by integrating the above stresses as

\[
N = \int \sigma_x \, dA = \varepsilon_{x_0} EA + x ES
\]

\[
M = - \int \sigma_x y \, dA = - \varepsilon_{x_0} ES - x ET
\]

where

\[
\begin{align*}
EA &= \int y_b \, E \, dA, \\
ES &= \int y_b \, E \, y \, dA \\
ET &= \int y_b \, E \, y^2 \, dA
\end{align*}
\]

Equation 3.89 may also be written in matrix form as

\[
\begin{bmatrix}
N \\
M
\end{bmatrix}
\begin{bmatrix}
EA & ES \\
ES & ET
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{x_0} \\
x
\end{bmatrix}
\]

Equation 3.89 may also be written in matrix form as

\[
\begin{bmatrix}
N \\
M
\end{bmatrix}
\begin{bmatrix}
EA & ES \\
ES & ET
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{x_0} \\
x
\end{bmatrix}
\]

Substituting eq. 3.92 into eq. 3.88 and proceeding with a matrix multiplication, the normal stress may be written as

\[
\sigma_x = \frac{E}{ES^2 - EA ET} \left[ \frac{(-ET + ES \, y) \, N + (-ES + EA \, y) \, M}{ES} \right]
\]

According to Fig. 3.9 the equilibrium between normal and tangential stresses may be equated as

\[
\tau_{xy} \, b \, dx = - \int y_b \, d\sigma_x \, dA
\]

and therefore the shear stress is given by
\[ \tau_{xy} = -\frac{1}{b} \int_{y_b}^{y} \frac{\sigma x}{dx} \, dA \]  

By assuming the reference axis \( y \) for the integrals in eq. 3.90 as the section centroidal axis as shown in Fig. 3.9, \( E_S = 0 \). Taking the derivative of eq. 3.93 and substituting into eq. 3.95 the shear strain is obtained as

\[ \tau_{xy} = \frac{1}{b} \int_{y_b}^{y} \left( \frac{E}{EA} \right) \left( -\frac{dN}{dx} + \frac{y}{EA} \frac{dM}{dx} \right) \, dA \]  

Normal force \( N \) is constant through the element and the variation of the bending moment \( M \) with \( x \) is represented by the total shear force \( Q \). The integration in eq. 3.96 leads to eq. 3.97 and using the expression in eq. 3.90 becomes

\[ \tau_{xy} = \frac{Q}{b} \left( \frac{ES}{EI} \right) \]  

Should material properties \( E \) be constant through the depth, eq. 3.97 would become the common expression for the distribution of shear stresses

\[ \tau_{xy} = \frac{QS}{bI} \]  

where \( S \) stands for the first moment of area of the area \( A \) in Fig. 3.9 and \( I \) stands for the moment of inertia of the section. Both \( S \) and \( I \) are calculated with respect to the centroidal axis of the section. The above expression generally leads to a parabolic shear stress distribution. In the present study, material properties do vary throughout the sections as a consequence of the nonlinear stress-strain relationships assumed
for the materials. Therefore, the shear stress and consequent shear strain distributions must be obtained from eq. 3.97, where the quantities $\bar{E}S_y$ and $\bar{E}I$ carry the effects of material nonlinearity. For computing the principal strain at a general point or layer, as in eq. 3.85, the shear strain $\gamma_{xy}$ is then obtained as

$$\gamma_{xy} = \frac{T_{xy}}{G} = \frac{Q}{bG} \left( \frac{\bar{E}S_y}{\bar{E}I} \right)$$

3.99

where $G$ is the shear modulus of the material at the point of interest, taken as $G = E/2(1+v)$.

The above procedure accounts for the variation of the material properties not only with the different materials that may comprise the beam but also within the same material. Once a crack is formed one or more layers of an element are considered cracked, their axial stiffnesses are disregarded by setting the modulus of elasticity to zero. The analysis then proceeds and crack propagation is obtained by applying the same procedure to the consecutive layers.

In a structural member, however, even when monotonic loading is applied it is possible for some stress redistribution to take place causing some regions of the member to unload. This phenomenon may be due to intrinsic nonlinear stress-strain characteristics of the materials, or the cracking process, or the time-dependent effects that exist, and may cause some cracks to close. If that should happen a strain quantity $\varepsilon_p$ is observed, see Fig. 3.10, and crack closing is obtained whenever this value is surpassed when concrete is able to pick up compressive stress normal to the closed crack.
Fig 3.11 - Load Increment Method.

PRECAST GIRDERS - SIMPLY SUPPORTED

Connection elem. Deck-slab

Beam elem. Girder

PRECAST GIRDERS AND CAST-IN-PLACE DECK-SLAB - COMPOSITE BEAM

Fig 3.12 - Sequence of construction of a two-span composite beam.
3.8 Nonlinear Analysis

3.8.1 The Load Increment Method

The Load Increment Method for nonlinear analysis uses an iterative procedure in which the loads applied to the system are successively corrected until an equilibrium position is obtained. It is a variation of the direct stiffness technique where the incremental stiffness matrix based on the current position of the system is used, instead of a constant stiffness matrix, and equivalent to the modified Newton-Raphson Method.

For illustration of the method, consider a single degree-of-freedom system as shown in Fig. 3.11. Assume that the load-deformation relationship of the system can be represented by the function

\[ P = f(u) \]

and further assume that the current state of the system is at the load level \( p \) and at the deformation \( u_0 \). It is required to determine the deformation at the new load level \( p_1 \), \( u_1 \). A new step starts with a predefined load increment \( \Delta p_0 \). By assuming that the load-deformation relationship is linear at that deformation level and, therefore, using the tangent stiffness at that level as the stiffness of the system a deformation increment is calculated as \( \Delta u_0 \). The current deformed state is then obtained as

\[ u_1 = u_0 + \Delta u_0 \]

and the resisting internal forces at this deformation level as

\[ p_1 = f(u_1) \]
For a nonlinear system $p_i$ will rarely be equal to $p_1$ and the difference

$$\Delta p_i = p_1 - p_i$$

is the equilibrium error or residual load vector. The residual vector is then compared with the specified tolerance limit. If this limit is exceeded the residual load vector is used as the additional load applied to the system, at the trial deformation level of $u_i$. This process is repeated until the equilibrium error is within the specified limits. The solution is then assumed to be converged for the load level $p_1$. The same process is followed for the next load step, if any, when the tangent stiffness matrix is updated. The solution will then converge when an equilibrium state of the system, under the applied load, is obtained.

Under some circumstances such as the case where the new load level exceeds the maximum capacity of the system, or the new load is less than the capacity but with some unusual shape of the load-deformation relationship, the solution may diverge. An unusual shape of the load-deformation relationship in the case of structural members may be the nearly horizontal plateau observed at the level of their maximum capacity.

3.8.2 The Displacement Increment Method

The nonlinear response of a structural member, within some limiting points, can be predicted by using various numerical techniques like the Newton-Raphson Method and its modified version, the Incremental Stiffness Method, the first and second order Self Correcting Method,
etc. When these methods are used in terms of the incremental load vectors convergence may become difficult to achieve, whenever the sought track has some particular shape as mentioned in the previous section. In the analysis of concrete structures the load-deflection curves may frequently present sudden drops due to instantaneous reduction in stiffness, as a consequence of cracks. They are also frequently characterized by possessing a flat plateau which may follow large increments in displacements, an important phenomenon for the measure of ductility. The displacement increment technique, however, has proven very effective in capturing the complete load-deformation curve including these particular drops and plateau. A few different versions of the method are found in the literature (85,86,87) and some investigators have reported using it with satisfactory results (88).

For illustration purpose, assume the equilibrium equation of a structural system relating displacements and applied loads as

\[
[K]\{D\} = \{F\} 
\]

If an incremental procedure is adopted, eq. 3.104 may be written as

\[
[K]\{\Delta D\} = \Delta \alpha \{F\} 
\]

where \([K]\) is in general a symmetric banded matrix of order \(n\), here taken as the tangent matrix evaluated at the beginning of a new step, \((\Delta D)\) is the vector of \(n\) unknown displacements, \(\{F\}\) the known load vector with \(n\) components and \(\Delta \alpha\) an unknown factor applied to the load vector.

For applying the Displacement Increment Method to the system in question a single degree-of-freedom is conveniently chosen. A specified displacement \(\Delta D_2\) is incrementally applied to the nodal point until the
system reaches equilibrium at a desired load level. The displacement vector in eq. 3.105 may then be partitioned into two components, where \( \{ \Delta D_1 \} \) is the unknown incremental displacement vector for the remaining degrees-of-freedom of the system, as in eq. 3.106. The stiffness matrix of the system and the applied load vector from eq. 3.105 may then be partitioned accordingly

\[
\begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta D_1 \\
\Delta D_2
\end{bmatrix}
= \Delta \alpha
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix}
\]

Eq. 3.106 can also be written as

\[
\begin{bmatrix}
K_{11} \\
K_{21}
\end{bmatrix}
\begin{bmatrix}
\Delta D_1 \\
\Delta D_1
\end{bmatrix}
= \Delta \alpha
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix}
- \begin{bmatrix}
K_{12} \\
K_{22}
\end{bmatrix}
\Delta D_2
\]

A direct method of solving eqs. 3.107 and 3.108 would lead to the use of a non-symmetric matrix, which is not very effective. Nevertheless, by further partitioning the unknown displacement vector as

\[
\{ \Delta D_1 \} = \Delta \alpha \{ \Delta D_1 \}^0 + \{ \Delta D_1 \}^1
\]

eq. 3.107 can be solved in two simple steps as

\[
\begin{bmatrix}
K_{11}
\end{bmatrix}
\{ \Delta D_1 \}^0 = \{ F_1 \}
\]

and

\[
\begin{bmatrix}
K_{11}
\end{bmatrix}
\{ \Delta D_1 \}^1 = - \begin{bmatrix}
K_{12}
\end{bmatrix}
\Delta D_2
\]

where the matrix \([K_{11}]\), already rearranged for the solution procedure in eq. 3.110 is also used in the subsequent equation, saving some computational effort in repeating the process.
The unknown factor $\Delta \alpha$ can then be obtained by equating 3.109, 3.110 and 3.111 as

$$\Delta \alpha = \frac{(K_{21}) \{ \Delta D_1 \}^I - K_{22} \Delta D_2}{F_2 - (K_{21}) \{ \Delta D_1 \}^O}$$ \hspace{1cm} 3.112

At this point, equilibrium of the system should be checked as observed for the Load Increment Method. The resisting load vector can be obtained from the current deformed state of the system and, if equilibrium is not achieved, further corrections in the displacement vector should be made. The process followed to this point is the first iteration of a step. For the second and subsequent iterations $(\Delta D_2) = 0$ and in its place there is a residual load vector $(\Delta F_1)$ and eq. 3.107 and 3.108 may be written as

$$[K_{11}] \{ \Delta D_1 \} = \Delta \alpha \{ F_1 \} + \{ \Delta F_1 \}$$ \hspace{1cm} 3.113

and

$$(K_{21}) \{ \Delta D_1 \} = \Delta \alpha F_2 + \Delta F_2$$ \hspace{1cm} 3.114

The solution of the above equations, similar to that for eqs. 3.110 and 3.111, can be expressed as

$$[K_{11}] \{ \Delta D_1 \}^O = \{ F_1 \}$$ \hspace{1cm} 3.115

and

$$[K_{11}] \{ \Delta D_1 \}^I = \{ \Delta F_1 \}$$ \hspace{1cm} 3.116

where once more the same matrix $[K_{11}]$ is used. $[K_{11}]$ needs to be updated only at the beginning of a new incremental step, if any. Eq. 3.112 is now replaced by
For each iteration the current load vector is obtained as \((\Sigma \Delta \alpha) \{ F \}\), where the \(\Delta \alpha\)'s are to be added for all the previous iterations and steps. Until the residual load vector is sufficiently small when convergence is obtained, the iterations are to be continued.

3.9 Time-Dependent Response Analysis

Section 3.8 describes the procedures utilized for obtaining the instantaneous response of a general composite bridge beam. Concrete members, however, when loaded at different ages may respond quite differently due to the highly nonlinear time-dependent properties of the materials involved, such as creep, shrinkage and aging of concrete, and relaxation of prestressing steel.

Creep is defined as the time-dependent deformations of concrete which is the function of stress levels and the time at which the stresses are applied. Shrinkage represents the time-dependent deformation of concrete that is independent of stress levels and a function of time only. Relaxation is defined as the time-dependent stress decrements of steel under constant strain. The aging of concrete is reflected in the change of the stress-strain curve of the material with time. The stress-strain curve intrinsic shape is assumed to be the same but stresses at a same strain level will be a function of time, as outlined in Section 2.1.3.4.
Time-dependent response analysis may be obtained by subdividing the interval of time under consideration into several small time increments, within which the previously described instantaneous solution is used. In each time increment, or step, it is assumed that material properties and the response of each fiber of the beam follow the functions of time and stress level history up to the start of the time increment. The equilibrium position of the member can be found and is used as the basis for the analysis of the response in the next time step. Starting from the beginning of the time interval the whole range of the time-dependent response can be captured.

Time at which loads are applied is also considered. Loads may be applied any time after the erection time of the beam and they will be accumulated. Some portions of the cross-section of the member may be added later, i.e. the composite beam action is taken into account as described in the following section. External support restraints are generally independent of time and applied at the beginning of the solution. However, as seen in Chapter 5, some changes in the supporting conditions can be accommodated within the solution.

As previously mentioned, in determining the instantaneous response of a beam, the beam is subdivided into several elements connected end-to-end at nodal points. Each of the elements is further subdivided vertically into layers. A layer is able to carry strains, observed at their centroids, assumed constant throughout their length and cross-section. Loads and restraints are discretized and applied only at the nodal points.
For the time-dependent response analysis it is assumed that the strain in each layer consists of time-dependent strain parts in addition to instantaneous strain parts. Time dependent strains of concrete consist of shrinkage strains and creep strains, and those of prestressing steel are relaxation strains. Shrinkage strains are the function of time alone thus they can be found immediately at any given time increment, assuming the shrinkage function described in Section 3.1.3.3. Creep strains under constant stress may be represented by creep functions. For creep under variable stress, as in a beam, two methods of predicting creep strains under variable stresses may be used. The methods, outlined in Section 2.1.3.2, are the Rate of Creep Method and the Superposition Method. The Rate of Creep Method uses the strain at the beginning of the time increment as the basis for finding the creep increment strain. For the Superposition Method the stress history up to the current time step is used in the calculation. Relaxation strains are treated the same way as the creep strains. But, instead of determining the creep strains, relaxation stresses are calculated from the stress history and only a method similar to the rate of creep method is used. A method similar to the superposition method is invalid since relaxation is not a linear function of stress level.

The so obtained time-dependent strains at any incremental time step are then related to stresses and eventually transformed into nodal forces. The load vector is subsequently applied to the model and an age-corrected instantaneous solution is performed, seeking the current equilibrium position of the member as in the procedure detailed in Section 3.8.1. This process is repeatedly applied for each increment of time and the time-dependent response of the member is obtained.
Composite Beam Action

Bridge beams are usually constructed as composite beams (9,10,18,39,89,90) where the girders are of precast concrete or steel and the deck slab is cast-in-place on top of the girders at some later date as shown in Fig. 3.12. At the time of casting the deck slab the beams may be shored, i.e. the precast girders do not carry slab loads at the time of casting, or unshored, that is the precast girders carry slab and formwork loads at the time of casting of the slab. After the slab has gained sufficient strength, forming the composite beam, both act together under the additional loads.

For assuring perfect bond between the girders and the deck slab shear connectors are often provided at that interface (10,91). Experimental data show that adequate composite action is obtained when those connections are properly designed (11,92).

In this analysis the following technique is used for modeling the composite behavior at some specified age of the member. As described before, the stiffnesses of the elements are obtained by integrating the individual stiffnesses of the various layers that compose each element. When any new loading is applied, a companion variable ICOMP is assigned a value 0 or 1, indicating that the stiffness integration should be performed within the girder section or extended to the deck portion, respectively. In the early phases of the construction, before the deck is cast, the beam elements represent the behavior of the girders alone. Under the action of deck dead load the variable ICOMP may have its value set to 1 or 0 representing shored or unshored construction, respectively. After the deck has attained enough strength and
additional loading is applied, ICOMP remains equal to 1. Composite
action is then obtained and the connection elements, if any, link the
various spans.

3.11 Program Flow Diagram

With the procedures described in previous sections a method of
analysis is developed for both instantaneous and time-dependent
responses of a general composite bridge beam. The method can be
summarized by the flow diagram in Fig. 3.13 using the subroutines shown
in Fig. 3.14. The steps include reading and printing of the input data.
A series of time increments is specified and it is assumed that loads
and parts of the cross-section are added at different times but only at
the beginning of a given time interval. For each time increment an
equilibrium position of the member is estimated for the changes in time-
dependent strains in each fiber. Additional loads are discretized as
nodal point loads and a new equilibrium position of the beam is
estimated under the new loading condition. Steps may be omitted if they
are not applicable to the time increment being considered and solution
output may be printed if needed. The process is repeated for all
loading cases and time increments over the interval of interest.
Fig. 3.13  Program's Flow Diagram
Fig. 3.14 Subroutines for phases in Fig. 3.13
4. VALIDATION OF THE ANALYTICAL MODEL

4.1 Introduction

The proposed program is intended for the analysis of instantaneous and time-dependent responses of non-composite or composite beams, built in steel, reinforced or prestressed concrete, and simple or continuous over two or more spans. Even though its main purpose is aimed at the analysis of beams with continuous deck slabs, as detailed in Chapters 1, 3 and 5, it is also applicable to the analyses of simple and fully-continuous beams. The purpose of this chapter is to validate the analytical model by studying a group of beam members for which analytical and experimental results are available in the literature.

In the following sections, eighteen different beams are analyzed and presented as ten separated problems. Problems 1 to 5, cover the instantaneous response and strength of non-composite and composite beams, in steel, reinforced and prestressed concrete. The remaining five problems cover the time-dependent response of similar beams. Material properties, i.e. stress-strain-time relationships, are based on the assumed models described in Chapter 2. Where actual material properties are known, characteristic constants for the models are adjusted to fit the actual data for each particular case.

4.2 Instantaneous Response and Strength of Non-Composite Beams

4.2.1 Steel I-Beam

A simply supported W18x50 steel I-beam, under a uniformly distributed load over a span of 15 ft, is analyzed for its response up
to the ultimate strength (93). One half of the beam is modeled by 9 beam elements, subdivided vertically into 30 horizontal layers. The load-deflection curve obtained by the use of the Displacement Increment Method with a specified displacement increment of 0.04 in. at the mid section is shown in Fig. 4.1 along with the assumed material properties. The curve compared very closely with analytical results obtained by the elastic theory and the plastic moment presented in Ref. (93).

The analytically obtained plastic moment is 5050 in-lb, whereas the numerically obtained is 5056 in-lb, for an ultimate deflection of 2.9 in, when a plastic hinge is formed at the mid-span.

4.2.2 Reinforced Concrete Rectangular Beam.

In a joint program carried out by the Instituto de Materiales y Modelos Estructuralles (IMME) at the Universidad Central de Venezuela, Caracas, Venezuela and the Laboratory for Structural Models at MIT, a total of 132 beams at five geometric scales and two reinforcement ratios were tested. Included in the test program were four sets of reinforced concrete beams, having two scale factors with maximum aggregate size of 1 in. and 3/8 in., and eighteen sets of reinforced mortar beams, with five scale factors and maximum size of aggregate of 0.033 in., regardless of scale. From the test results presented by Litle and Paparont (94), and the numerical results by Lazaro and Richards (31), a set of six similar beams, M7.1 to M12.1, with the scale factor of one and reinforcement ratio of 1% is selected for comparison with the predictions by the author's computer model.
Fig 4.1 - Load-deflection curve for beam Example 4.2.1.

Elastic th.

\[ P_u \]

\[ P_y \]

\( f_y = 50 \text{ ksi} \)

\( E_S = 29000 \text{ ksi} \)

Failure mechanism

Numeric

Deflections (in)
The beams were simply supported and loaded in their third points, and with the cross-section and material properties shown in Fig. 4.2. In the numerical analysis, which led to the load-deflection curves presented in Fig. 4.3, one half of the beam is modeled by 14 elements, subdivided into 30 layers, and the Displacement Increment Method is used, assuming both the Hognestad's and the bi-linear stress-strain relationships for concrete.

A comparison of the results obtained experimentally, analytically and numerically, is also presented in Fig. 4.2. In the post-cracking range the observed results indicate that the beam is stiffer than the numerically predicted behavior due to the absence of the tension stiffening effect in the analytical model. The ultimate mid-span deflection based on bi-linear stress-strain relationship is 33% more than that based on Hognestad stress-strain curve. No observed ultimate deflection is presented in the published results (94).

4.2.3 Prestressed Concrete I-Beam

Keyder (95), reported the testing of two simply supported non-composite pre-tensioned prestressed concrete beams, one of which was also numerically analyzed by Chang (36). The beam was 14 ft long and loaded symmetrically at the two points 4 ft 6 in from each support. It contained only five prestressed strands as reinforcement and the total initial pre-tensioning force was 70 Kips. (14 Kips./strand).

The cross-section of the beam and the material properties are shown in Fig. 4.4. In modeling the beam, one half of the span is analyzed by
Section and material properties

\[ A_s = 0.394 \text{ in}^2 \quad \rho = 0.98\% \]
\[ f'_c = 5070 \text{ psi} \quad f_y = 49 \text{ ksi} \]

Assumed concrete properties

\[ E_{ci} = 57000 \sqrt{f'_c} = 4060000 \text{ psi} \]
\[ f_t = 7.5 \sqrt{f'_c} = 534 \text{ psi} \]

Observed and proposed results

<table>
<thead>
<tr>
<th></th>
<th>( P_{cr} )</th>
<th>( P_y )</th>
<th>( P_u )</th>
<th>( M_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>---</td>
<td>---</td>
<td>6050</td>
<td>132000</td>
</tr>
<tr>
<td>ACI's</td>
<td>1808</td>
<td>5510</td>
<td>5803</td>
<td>130000</td>
</tr>
<tr>
<td>Num.bln.</td>
<td>1740</td>
<td>5550</td>
<td>5820</td>
<td>131000</td>
</tr>
<tr>
<td>Num.Hogn.</td>
<td>1716</td>
<td>5490</td>
<td>5730</td>
<td>129000</td>
</tr>
<tr>
<td>(lb)</td>
<td>(lb)</td>
<td>(lb)</td>
<td>(lb)</td>
<td>(lb/in)</td>
</tr>
</tbody>
</table>

Fig 4.2 - Properties and results for beam Example 4.2.2.
Fig 4.3 - Load-deflection curve for beam Example 4.2.2.
Fig 4.4 - Pre-tensioned beam for Example 4.2.3.
Fig 4.5 - Load-deflection curve for beam Example 4.2.3.
using 11 elements, subdivided into 24 layers. The Displacement Increment Method is used for live load using both the Hognestad and bi-linear stress-strain relationships. The load-deflection curves are shown in Fig. 4.5.

The ultimate loads obtained numerically, by using the two different stress-strain relationships, are respectively 3.6 and 6.5 percent lower than the 30.8 kips. reported by Keyder. In the post-cracking range, the beam appears to be stiffer as predicted by the computer model, which is believed to be due mainly to the sharp tri-linear modeling of the stress-strain relationship for the prestressing steel.

4.3 Instantaneous Response and Strength of Composite Beams

4.3.1 Prestressed Concrete Double Tee Beam

A standard double tee section taken from the PCI Design Handbook (18), with a simple span of 36 ft was selected to test the ability of the program in analyzing the composite beam action. This section was also analyzed by Lo (37), using a modified form of the program developed by Chang (36), as well as by the elastic theory, the effective modulus method and the ACI method for ultimate moment. Since no measurement of the actual behavior of the beam is available, Lo's results will be used to compare with the results obtained from the computer analysis.

The beam is pre-tensioned by two 7-wire strands, 270 K grade, with initial prestressing stress $f_{si}$ of 189 ksi. At transfer the prestressing stress is assumed to be 0.9 $f_{si}$ and equal to 170 ksi. Details of the draped cable profile, the cross-section and material
properties assumed in the analysis are given in Fig. 4.6. The beam is assumed to be pre-tensioned before the topping is cast. When the topping is cast, its weight is supported by the beam alone. Live load is uniformly distributed over the top surface and applied after the composite action is obtained.

In the proposed analysis a symmetric quadrant of the beam is modeled by 18 beam elements, subdivided into 20 and 5 layers for the beam and topping, respectively. Prestressing and dead loads are applied by using the Load Increment Method, and live load by the Displacement Increment Method, with specified displacement increments of 0.01 in. Both the Hognestad and bi-linear stress-strain relationships for concrete are assumed, and the results are presented in Fig. 4.7.

The load-deflection responses from the computer analysis and from the elastic theory are in fairly good agreement up to the cracking load of the beam. A stiffer beam is predicted by Lo's analysis before the composite action is formed. However, no change in the stiffness of the section is indicated by Lo's results after the development of composite action. Evidently, he has shown the camber for a stiffer section and thus his curve begins with less initial upward deflection prior to downward load application. A slightly higher cracking load is predicted by the computer analysis, as compared to Lo's results and that of the elastic theory, which is probably due to approximation in modeling the section.

The responses to applied load after the composite action is developed, as represented by both analyses are in good agreement. However, the Effective Modulus of Inertia Method predicts a much less
Fig 4.6 - Pre-tensioned composite beam for Example 4.3.1.
Fig 4.7 - Load-deflection curve for beam Example 4.3.1.

Total load (psf) vs. Deflections (in)

- Lo's analysis
- Num.bilinear
- Num.Hognestad
- I_{eff} method
- Elastic theory

ACI's P_u = 210 psf

- Composite beam action starts - topping dead load
- Girder dead load
- Pre-tensioning
stiff beam after cracking and does not show the ductility of the beam in the range where the prestressing steel yields. The ultimate loads as presented by the different methods are: 227 psf. by Lo's analysis, 225 and 221 psf. by the computed analyses, using a bi-linear and Hognestad's stress-strain relationships, respectively, and 210 psf. by the ACI Method, which are within about 8 percent of one another.

4.3.2 Two-Span Prestressed Concrete I-Beam

From the group of three pre-tensioned composite and continuous beams tested by Wong (38), Beam BS-3 is chosen for validation of the author's program in modeling prestressed continuous beams. The beam was also analyzed previously by Chang (36).

The beam, as shown in Fig. 4.8, was composed of two standard I-girders, each 15 ft long, and connected by a cast-in-place deck slab with a rectangular diaphragm, positioned over the intermediate support. The girders were pre-tensioned, after that the deck and diaphragm were cast making a continuous beam for live load. Both deck and girders were reinforced with steel bars with yield strength of 60 and 85 ksi, respectively.

The assumed material properties are shown in Fig. 4.8. The computer analysis was performed by modeling one of the spans with 17 beam elements, subdivided into 20 and 10 layers for the beam and deck, respectively. Loading was applied as two concentrated loads, at the center of each span, and the analysis was carried out by the Displacement Increment Method, with specified displacement increments of
0.01 in. Analysis for the effect of prestressing, however, was obtained by using the Load Increment Method.

The general shape of the load-deflection curve, shown in Fig. 4.9 for the mid-span deflection, is in good agreement with the observed results, although the beam seems to be somewhat stiffer up to near the ultimate range based on the computed results. This stiffer response may be attributed to the early occurrence of cracking in the deck slab near the intermediate support. This cracking is not evidenced in the analytical curve by the sudden drops characteristic of cracking development. Flexural cracks due to positive moment, evidenced in the analytical curve by the first drop at a load level of 54 kips., are however in good agreement with the observed results. The ultimate loads obtained by using a bi-linear and Hognestad stress-strain relationships are 117 and 115 kips. respectively, which are within 7 percent of the observed ultimate load of 123.5 kips.

4.4 Time-Dependent Response of Non-Composite Beams

4.4.1 Reinforced Concrete Rectangular Beam

Bakoss, et al (28) have reported the testing for time-dependent effects of four reinforced concrete beams, performed at the N.S.W. Institute of Technology, Sidney, Australia. Two similar one-span beams, designated 1B1 and 1B2, cast with one of the two concrete mixes used, Mix A, are selected for checking the time-dependent response analysis by the author's method. The beams were singly reinforced with two 12 mm bars, at a reinforcement ratio of 0.017, as shown on Fig. 4.10. They were cast and moist cured for 14 days, remaining in a climate-controlled
Fig 4.8 - Pre-tensioned composite two-span continuous beam for Example 4.3.2.

- $f'_c = 6720$ psi (girder)
- $E_{ci} = 3300$ ksi ("")
- $f'_c = 4410$ psi (deck)
- $E_{ci} = 2700$ ksi ("")
- $A_s = 0.88$ in$^2$
- $A'_s = 1.80$ in$^2$
- $A_{ps} = 0.40$ in$^2$
- $\varepsilon_{si} = 0.00622$
First observed flexural crack for positive moment

- Observed
- Num.bilinear
- Num.Hognestad

First observed flexural crack for negative moment

From pre-tensioning

Fig 4.9 - Load-deflection curve for beam Example 4.3.2.
environment for the next 14 days, when they were tested under load at 28 days.

As shown in Fig. 4.10, the beams were loaded in their third points, by two concentrated loads of 2.6 KN (585 lb), 45 percent greater than their cracking load capacity. The resulting mid-span bending moment was 62 percent of the calculated moment capacity of the sections. From that time on, the beams were observed for their post-cracking time-dependent deflection up to 500 days.

The effects of creep and shrinkage were observed in a companion beam, with the same cracking pattern as the one under test, but free of loading. The ultimate coefficients for creep and shrinkage, as obtained according to four different codes, vary from 1.4 to 2.8 and 0.0002 to 0.0007, at ages of 600 and 800 days, respectively. The coefficients obtained by following the ACI specifications (56) were 1.5 and 0.0007, which were also used in the numerical analysis.

The beam was modeled by using 9 elements, subdivided into 12 layers, and the time-dependent response under the effects of creep, shrinkage and aging of the concrete was obtained by following the procedure outlined in Section 3.9. Fig. 4.10 shows the comparison of the time-dependent responses obtained experimentally and numerically, by using both assumed stress-strain relationships for concrete. Very good agreement is found concerning the general shape of the response up to an age of 500 days.
Fig 4.10 - Time-deflection curve, loading arrangement, and cross-section and material properties for beam Example 4.4.1.
4.4.2 Two-Span Reinforced Concrete Rectangular Beam

In the same work by Bakoss, et al (28), two two-span continuous beams were also tested for their time-dependent response under concentrated sustained loads. These beams had similar characteristics as the ones discussed above although they were cast with a different concrete mix, Mix B, and reinforced for both positive and negative moments as shown in Fig. 4.11. Two concentrated loads of 6 KN (1350 lb) were applied at the center of each span and the resulting maximum negative moment was 63 percent of the calculated moment capacity of the section, or 47 percent greater than the cracking moment capacity.

The assumed material properties, as listed in Fig. 4.11, were used for the analysis of the post-cracking time-dependent response of the beam, modeled by 12 elements, subdivided into 12 layers, for one span. The observed and computed responses are in good agreement, as seen in Fig. 4.11.

4.4.3 Prestressed Concrete Rectangular Beams

An investigation of the time-dependent responses of non-composite, simply supported and pre-tensioned concrete beams, prior to cracking, was conducted by Zundelevich, et al (65) at the University of Hawaii, in conjunction with the State of Hawaii Department of Transportation, Highway Division. The beams were made of normal and lightweight concretes manufactured with Hawaiian aggregates. Three sets of specimens, made from basalt, cinderlite, and vulcanite aggregates, were used. Each set contained seven beams, three for studying camber, three
P = 1350 lb = 1.47 \text{P}_{cr}

Observed

Num.bilinear

Num.Hognestad

First cracking

Days under load

\[ \Delta_{cr} \]

\[ \frac{P}{2} \]

\[ \frac{P}{2} \]

\[
\begin{array}{c}
137.8'' (3500) \\
3.94'' (100) \\
0.79'' (20) \\
4.33'' (110) \\
0.79'' (20)
\end{array}
\]

\[
\begin{array}{c}
f'_{c} = 4350 \text{ psi} (30 \text{ N/mm}^2) \\
f_{t} = 710 \text{ psi} (4.9 \text{ N/mm}^2) \\
A_{s} = A'_{s} = 2\phi 12 \text{ mm} = 0.35 \text{ in}^2 \\
C_{u} = 2.1, \varepsilon_{shu} = 0.0007
\end{array}
\]

Fig 4.11 - Time-deflection curve, loading arrangement, and cross-section and material properties for beam Example 4.4.2.
PRESTRESSING STEEL

\[ f'_c = 5900 \text{ psi} \]
\[ E_{ci} = 4400 \text{ ksi} \]
\[ E_{ps} = 28000 \text{ ksi} \]
\[ f_{su} = 270000 \text{ psi} \]
\[ C_u = 3.3 \]
\[ \varepsilon_{shu} = 0.00105 \]

Fig 4.12 - Pre-tensioned beam for Example 4.4.3.
Fig 4.13 - Camber and deflection-time curves for beam Example 4.4.3.
for studying deflection and the other as a shrinkage specimen. The basalt specimens, designated as BEAM1 series, are selected for analysis by the computer program in handling creep recovery under partial unloading of a beam.

The beams were 4 in. by 6 in. in cross-section, and simply supported over their 15 ft span. They were prestressed at the age of seven days by two straight 7-wire strands at an eccentricity of 1.25 in.. The details of the beam and material properties are shown in Fig. 4.12. The camber specimens were observed for camber growth with the beams carrying only their own weight. For the deflection specimens, the beams were loaded at their third points by two 750 lb loads per beam at 21 days after prestressing.

For the computer analysis, one half of the beam was modeled by 9 elements, subdivided into 12 layers. The pre-tensioning effect is obtained by the Load Increment Method, live load application obtained by the Displacement Increment Method and the time-dependent responses by using the Rate of Creep Method and the Superposition Method, as explained in Chapters 2 and 3.

The camber and deflection obtained by assuming both the Hognestad and bi-linear stress-strain relationships for concrete are compared with the observed results in Fig. 4.13. The calculated instantaneous deflections, either for prestressing or for live load, are in good agreement with the observed results. So are the general shapes of the time-dependent responses. Using the Hognestad stress-strain relationship seems to overestimate the camber growth and underestimate the deflection under a creep recovery process. The bi-linear stress-strain relationship
gives errors in the opposite sense for deflection, but seems to predict closely the observed camber growth due to prestressing.

4.5 Time-Dependent Response of Composite Beams

4.5.1 Reinforced Concrete Rectangular Beams

Kripanarayanan and Branson (96) reported studies on the time-dependent behavior of composite reinforced concrete beams, resulted from the age differential between the casting of the beams and the casting of the deck slab. Three simply supported beams, as shown in Fig. 4.14, were cast and tested under the influence of dead load only. One of the beams was non-composite and served as the control specimen for the other two, of which their decks were cast at the age of one and seven weeks respectively after positioning of the beams.

The beams were made of sand-lightweight concrete with a 28-day cylinder strength of 3950 psi, and the decks of normal weight concrete with a 28-day cylinder strength of 4225 psi. All beams were singly reinforced with three #4 bars, having a reinforcement ratio of 0.01667, the deck slabs were not reinforced. The assumed ultimate creep and shrinkage coefficients were 4.0 and 0.0006 for the beams, and 4.0 and 0.0004 for the slabs, respectively.

The beams are modeled for half span by 6 beam elements, subdivided into 12 and 6 layers, for the web and flange respectively. Dead load is applied by the Load Increment Method and the time-dependent response obtained by using the Rate of Creep and Superposition Methods.
Fig 4.14 - Composite beams for Example 4.5.1.
Fig 4.15 - Time-deflection curves for beams Example 4.5.1.
Shown in Fig. 4.15 are the time-dependent responses for the three beams, as obtained from the analysis and compared with the observed data. A very close agreement is obtained by the Hognestad and a bilinear stress-strain relationships. In addition, close agreement is obtained also between calculated and experimental results. For the third beam, however, the one with the deck cast at seven weeks, the calculated results seem to overestimate the deflection response after about 20 days from the casting of the slab, predicting a final deflection some 10 percent greater than the measured value.

4.5.2 Prestressed Concrete Rectangular Beams

In studying the time-dependent response of composite prestressed concrete beams, Rao and Dilger (21) reported the testing of six one-span simply supported beams, having different reinforcement arrangements and subjected to superimposed sustained live loads.

The beams, with the cross-sections shown in Fig 4.16, were numbered 1 to 6. Beams 1 and 2 were pre-tensioned but did not have any mild steel reinforcement. Beams 3 and 4 had an additional deck reinforcement, and beams 5 and 6 also had additional reinforcement in the web. All six beams were 12 ft. long and pre-tensioned by two straight 7-wire strands with an initial total prestressing force of 65.8 kips. The prestressing force was transferred to the concrete seven days after casting. The deck was cast forty one days after the casting of the beam, by using shored construction. Live load was applied on the composite member twelve days after the deck was cast. Two 5.8 kips. concentrated loads were applied at the third points of beams 2, 4 and 6.
The beams are modeled by 12 beam elements, for one half span, subdivided into 12 and 5 layers for the web and top flange, respectively. Prestressing force, dead and live loads are applied by using the Load Increment Method. The assumed material properties are shown in Fig. 4.16.

Since the strands were stressed eight days prior to transferring of the forces to the beams, the initial prestressing force used in the analysis is reduced from the initial force of 65.8 kips to account for the effects of relaxation. The assumed initial prestressing force, obtained by using the recommended PCI equations for loss due to relaxation in stress-relieved steel, given in Section 2.3.2, is 60.5 kips, 8 percent smaller than the previous value.

From the time of prestressing to one hundred and forty days after casting of the beam section, measurements were taken in the six beams for camber, deflection and curvature of the mid-span section. Figs. 4.17 to 4.20, show a comparison of the observed and calculated results, for curvature in beams 1 and 2, and camber and deflection for beams 1 to 6, respectively.

The use of the Hognestad stress-strain relationship for the web and deck concrete, shows a consistent overestimate of camber and accordingly underestimate of deflections, as compared to similar results by using the bi-linear stress-strain relationship. The particular shape of the camber- and deflection-time curves for each of the three sets of beams, vary slightly depending on the amount and position of the mild steel reinforcement. The general trend of the time-dependent responses for all sets of beams are, however, in agreement with the measured results.
Fig 4.16 - Pre-tensioned beam for Example 4.5.2.
Fig 4.17 - Time-curvature curves for beams 1&2, Example 4.5.2.
Fig 4.18 - Time-deflection curves for beams 1&2, Example 4.5.2.

A - Prestressing
B - Deck casting
C - Live load

- Observed beam 1
- Observed beam 2
- Num.bilinear
- Num.Hognestad

Beam age (days)
Fig 4.19 - Time-deflection curves for beams 3&4, Example 4.5.2.
Fig 4.20 - Time-deflection curves for beams 5&6, Example 4.5.2.

- △ Observed beam 5
- ○ Observed beam 6
- — Num.bilinear
- —- Num.Hognestad

A - Prestressing
B - Deck casting
C - Live load
5. APPLICATION OF THE ANALYTICAL MODEL TO JOINTLESS BRIDGE BEAMS

5.1 Introduction

Jointless bridge beams may be considered as a solution for the persistent problems caused by the insertion of joints in multi-span bridges. The elimination of joints may be achieved in several ways:

(a) By using a fully-continuous beam, for deck and girders, (FCB).
(b) By casting a continuous deck and a connecting segment or diaphragm between two simple beams over each intermediate support, to develop continuity for carrying live load, (FCLL).
(c) By casting a fully-continuous deck slab over the simple span non-continuous girders, (FCD).

Fully-continuous beam has generally established an excellent performance record. However, it is not as widely used as beams of simple span. Beams made continuous for live load have proven suitable for precast concrete construction and show remarkable performance as far as continuity is concerned when compared to fully-continuous beams. Nevertheless, they may not be adequate for the case of steel girders which are usually constructed as simple span beams with joints being formed at the deck.

A simpler and thus more economical option is to connect simply supported girders through a fully-continuous deck (FCD), with no continuity between the adjacent girders. Being also suitable for precast construction and complying with the joint-free idea, they appear
to be an alternative solution to one of the severe bridge maintenance problems. Beams with continuous deck provide a simple and efficient solution not only for the construction of new bridges but also for the rehabilitation of old ones, by re-decking the still serviceable simply supported girders.

For beams in which continuity is obtained only through the deck, conventional analytical techniques are inadequate since they are based on the concept of full continuity of the beams. The numerical solution developed in this study, nevertheless, can adequately handle such unconventional cases and, due to its versatility, investigate all the different situations described previously.

In the following sections, two commonly used structures for mid-size span bridges over highway intersections will be analyzed, and comparisons will be made for different situations of loading, continuity and supporting conditions. In Section 5.2, a 100 ft two-span bridge with steel girders will be considered and in Section 5.3, a 264 ft four-span bridge with prestressed girders of equal and unequal spans will be analyzed.

5.2 Analysis of a Jointless-Deck on Two-span Beam With Steel Girders
5.2.1 General Behavior

Shown in Fig. 5.1 is a two-span bridge with two fifty feet W33x118 steel girders supporting a 7 ft by 7 in reinforced concrete deck slab (93). Three different construction details are given in Fig. 5.2, namely: fully-continuous, deck-continuous and non-continuous. The
$A_{SD} = (28)#5 @ 3"$

$W33 \times 118$

Fig 5.1 - Cross-section.

Fig 5.2 - Two-span composite beam with steel girders.
Fig 5.3 - Horizontal movements for different supporting conditions.
performance of these three types of design will be analyzed and compared.

The steel girders are placed into position over the supports, and the concrete deck is cast, either by shored or unshored construction. Supports are provided at the bottom flange of the respective girders, and assumed as hinges, preventing any horizontal movement, or as bearing pads, allowing for some horizontal displacements. Bearing pads are designed according to the procedures outlined in Section 2.4.

When hinges are used, it is assumed that free horizontal movement of the supports is possible only under the action of girder dead load. Horizontal movement of the supports is considered to be restricted for all other superimposed loads on the girders. This supporting condition allows for a more realistic modeling of the structure, since horizontal reactions at the hinges, if any, only appear under the action of live load, or any loading condition following the setting of the hinged supports. The existence of horizontal reactions in the restrictive supports, as well as the force built-up in the deck connection are taken into consideration in the analysis. Should the supports be located at the centroidal plane of the sections, as commonly assumed in basic structural analysis no restricting forces would be generated even if the horizontal movements are prevented.

The deck connection over the intermediate supports is, under any applied loading, subjected to bending and axial force. As shown in Fig. 5.3, depending on the type and arrangement of the supporting conditions, tension or compression may occur at the deck connection, producing a different behavior of the whole structure for each supporting condition.
A numerical analysis is performed by using the Load Increment Method for girders subjected to its own dead load and deck dead load. After composite action is established the beams are analyzed for an increasing uniformly distributed live load up to failure, by using the Displacement Increment Method. Both spans are divided into 12 equal elements, subdivided into 18 and 12 layers for the girders and deck respectively. A connection element is provided when there is deck continuity.

Figure 5.4 shows the load-deflection responses for the deck-continuous beam under different support arrangements, and comparison is made with the extreme cases of a fully-continuous beam and a non-continuous beam. It is assumed that the beams are unshored during the deck construction, and bearing pads are provided at all non-hinged supports. It is interesting to observe that the different beams present a remarkably different behavior, depending on the boundary conditions. Nevertheless, all cases fall within the upper and lower bounds, i.e. fully-continuous and non-continuous cases. The beams in cases 3, 4 and 5, for which a tensile force occurs in the deck connection, behave quite similarly to a non-continuous beam but with slightly less deflections and much less ductility. The maximum load capacity would be governed by yield of the reinforcing steel (assumed at a limiting strain of 1%) in the deck connection.

A compressive force is encountered at the deck connection for the beam in case 2, which yields a much lower moment carrying capacity and ductility, allowing failure to occur by crushing of concrete at the bottom face of the deck. Under loading on one span only, however, as shown in Fig. 5.5, a tensile force is transmitted through the
Fig 5.4 - Load-deflection responses under full span loading for various support arrangements. Unshored deck construction.
Fig 5.5 - Load-deflection responses under one-span loading. Deflections of both spans for all cases in Fig 5.4.
Fig 5.6 - Horizontal reaction build-up for cases in Figs 5.4 and 5.5.
connection, thus enhancing its performance in deflections and strength.

The beam Case 1, having hinges at the intermediate supports, which assure a thorough tensile behavior of the deck connection, performs remarkably better than the beams of all other cases, under either symmetric or unsymmetric loading. Deflections at the service load range are comparable to the case of full continuity, as seen in Figs. 5.4 and 5.5. It is also true for the case of shored construction as shown in Fig. 5.8. Failure is obtained by extensive yielding of the reinforcing steel, at the connection, and its ultimate load capacity is well above that of a non-continuous beam.

The horizontal reactions supported by the hinges for the situation of full span loading in Case 1 are well above those for the remaining cases, as shown in Fig. 5.6, but comparable to the other cases for one span loading. However the reactions are coupled over one single support, thus allowing for the use of some local resisting device. For the beams in Cases 2 and 3, even though the reactions are of smaller magnitude they must be supported by the end abutments or by a single intermediate hinge.

From the observations made to this point, it is concluded that:

a) Cases 4 and 5 appear to be a good alternative for the case of non continuity. Their service load range behavior is better, as far as deflections are concerned and, more importantly, they do eliminate the undesirable joint. No horizontal reactions are created at the supports. After yielding of the deck connection reinforcement steel, the beams will behave as if a joint were provided at that location, thus gaining
all the ductility capacity provided by the non-continuous beams.

b) If hinges are used they should be placed in pairs over the intermediate supports as in Case 1. The behavior of the beams is enhanced remarkably in deflections, strength and ductility. It constitutes a valid, more economical alternative for a fully continuous structure.

5.2.2 Deck-Continuity Over Hinged Supports

Among the different beams analyzed previously the beam supported as in Case 1, with deck-continuity over hinged supports, presented an interesting behavior. Its response under uniformly distributed load is the closest to the response of a fully-continuous beam under the same loading conditions. Further analyses are made to compare the responses of both FCB and FCD under this and other situations of loading. Some conclusions are described next.

(a) Linear Response

From Figs. 5.7 and 5.8 it is observed that both beams present a very similar stiffness within service load range. Linearity extends up to a load level approximately eight times the total dead load, for either shored or unshored construction.

(b) First Cracking

First cracking of the deck slab occurs at a load level about four times the total dead load. Cracking is first developed in a region near the internal supports, as shown by point 2 in Fig. 5.7. Cracking of the
CHARACTERISTIC POINTS

1 - Deck connection cracks.
2 - Deck cracks near support.
3 - Connection steel yields.
4 - Girder yields at support.
5 - Girder yields at mid-span.
6 - Deck cracking at mid-span.
7 - Deck steel yields at 1% strain.
8 - Concrete crushing.

Fig 5.7 - Load-deflection responses for the FCB and FCD beams, both under unshored deck construction and loaded at both spans.
Fig 5.8 - Load-deflection responses for beams under full span loading and with shored deck construction.
deck connection in the FCD beam follows closely, at five times the total dead load (point 1).

(c) Yielding of the Deck Reinforcement

Yielding of the deck reinforcement in the FCD beam (point 3) occurs at a load level eight times the total dead load, when the beam response becomes nonlinear. However, in the FCB deck reinforcement yield happens at a much higher load, after yielding of the steel girders (points 4 and 5).

Force build-up in the deck connection of the FCD beam is virtually linear before yielding of the reinforcement. After that a constant force is maintained until failure, see Fig. 5.9. The development of stresses in the deck reinforcing steel of both beams is shown in Fig. 5.10. It can be observed that under service load conditions, when cracks are expected to be controlled, steel stresses remain in the elastic range therefore allowing cracks to close under unloading of the structure.

(d) Load Capacity

Failure of the FCD beam occurs by extensive yielding of the deck reinforcement whereas in the FCB crushing of the deck concrete is observed after a considerable mid-span deflection, and at a higher load as shown in Fig. 5.7. However, the load capacity of a deck-continuous beam may be enhanced by increasing the amount of deck reinforcement at the connection. Fig. 5.11 shows a comparison of their responses for different reinforcement ratios of 1.5, 2.0, 2.5 and 3.0 percent respectively. The same amount of deck reinforcement is maintained for
Fig 5.9 - Force build-up at the deck connection.
Beam case 1: Full span loading, shored and unshored deck construction.
Refer to Fig. 5.7.

Full span loading and unshored construction.

Fig 5.10 - Stresses in the deck reinforcing steel for...
Fig 5.11 - Load-deflection responses for continuous deck beams, Case 1, with different reinforcement ratios at the connection. Unshored construction and full span loading.
Fig 5.12 - Responses of a deck-continuous beam with different widths of the deck connection. Unshored deck construction and full span loading.
the spans. It is observed that the beam response comes closer and closer to the response of the FCB. But for reinforcement ratios of 3.0 percent and more the deck becomes over-reinforced causing failure to occur by crushing of concrete with much less ductility.

(e) Ductility Capacity

Ductility and strength of a FCD beam are associated with the width of the concrete deck connection. The shorter is the gap between adjacent girders the higher is the strain associated with the same amount of rotation of the end faces of the girders, thus allowing failure to occur prematurely. However, their responses under service loads are not differentiated by this variable, as shown in Fig. 5.12.

A wider deck connection could be achieved by separating the end faces of the adjacent girders, but a more robust support would be required. Yet, another solution could be to provide some small region near the gap to be unbonded, i.e. the deck-girder interface at both sides of the end faces of the girders could be left unbonded when casting the deck slab. The analytical model developed herein is not able to handle such a situation. However, should the unbonded length be kept small it is expected that the results of this analysis may be adequate.

(f) Sustained Loading

The development of mid-span deflections over a period of two years under a sustained live load is shown in Fig. 5.13. During this period, shrinkage and creep of the deck concrete are expected to cause the initial deflections to increase. It is once more observed that a FCD
beam behaves similarly to a FCB, unlike the non-continuous one. A non-continuous beam has a 70 percent increase in deflection whereas the other two beams present an initial increment of about 50 percent, in a six months period, decreasing to a steady 30 percent after two years. It is worth noting that such responses correspond to a post-cracking behavior of the decks, as seen in Fig. 5.7.

(g) Temperature Effect

Under the action of the same service live load the beams are also analyzed for the effects of two temperature variation gradients. A constant temperature variation through the cross-sections is assumed to model a seasonal temperature change. For a daily oscillation of temperature a linearly decreasing gradient is used, as seen in Fig. 5.14. No significant changes in deflections nor in cracking development are noted for any of the beams analyzed.

(h) Compressive Effect

An amazingly similar response is presented by the three beams when all supports are hinged, i.e. when no horizontal movement is permitted after dead load is applied. In this situation horizontal reactions are created at the hinges. The compressive effect provided by these reactions overwhelmingly dictates a tremendously stiff response of the beams. Their load capacity are nearly doubled and their load-deflection responses become barely distinguishable, as shown in Fig. 5.15.

In this situation it is undoubtly clear the superiority of a FCD beam over the other two beams. Such constructional scheme will avoid the use of expansion joints and yet permit the use of precast elements
POST-CRACKING BEHAVIOR UNDER SUSTAINED LIVE LOAD OF 4.2 k/ft.

Fig 5.13 - Development of mid-span deflections with time. (Refer to Fig 5.4.)

Deflections (in)

Days under load

WITH JOINTS

FCD

FCB

0 28 100 200 300 400 500 600
Fig 5.14 - Development of mid-span deflections for temperature variation. (Refer to Fig 5.4)
Fig 5.15 - Load-deflection response for beams loaded on full span and hinged at all supports (a). Horizontal reactions at the exterior hinges (b).
therefore lowering the cost associated with construction and maintenance.

The results presented up to this point predict the behavior of a particular two-span beam composed of steel girders and a reinforced concrete deck slab. A good performance is observed by the deck-continuous beam as compared to the upper and lower bound cases of full and no continuity. It is expected that similar conclusions may be extended to beams with more than two spans.

5.3 Analysis of a Jointless-Deck on Four-Span Beam With Precast Prestressed Concrete Girders.

5.3.1 Properties

A four-span bridge beam, composed of four pre-tensioned concrete girders and a 6 ft by 6 in reinforced concrete deck slab, is analyzed to study the performance of a joint-free fully-continuous deck system. Comparison is made to beams with no continuity and full continuity for live load only. Layout, cross-sections and material properties are shown in Figs. 5.16, 5.17 and 5.18. Two situations are considered:

(a) The beams have four equal spans, each with a 66 ft precast pre-tensioned AASHTO TYPE IV girder.

(b) The beams have four unequal spans, with two exterior spans of a 24 ft precast pre-tensioned AASHTO TYPE II girder and two interior spans being the same as in case (a).

All girders are pre-tensioned by straight 7-wire strands 270 K grade
AASHTO
TYPE IV

CROSS-SECTION AT FOUR SPANS

Girders

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$</td>
<td>789 in$^2$</td>
</tr>
<tr>
<td>$I_c$</td>
<td>260741 in$^4$</td>
</tr>
<tr>
<td>$W_t$</td>
<td>822 plf</td>
</tr>
<tr>
<td>$E_c$</td>
<td>$4.5 \times 10^6$ psi</td>
</tr>
<tr>
<td>$f_c$</td>
<td>6000 psi</td>
</tr>
</tbody>
</table>

Deck

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s$</td>
<td>432 in$^2$</td>
</tr>
<tr>
<td>$W_t$</td>
<td>450 plf</td>
</tr>
<tr>
<td>$E_c$</td>
<td>$3.5 \times 10^6$ psi</td>
</tr>
<tr>
<td>$f_c$</td>
<td>3500 psi</td>
</tr>
</tbody>
</table>

Reinforcement

<table>
<thead>
<tr>
<th>Component</th>
<th>Span</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>deck</td>
<td>1.5%</td>
<td></td>
</tr>
<tr>
<td>girder</td>
<td></td>
<td>2.6%</td>
</tr>
</tbody>
</table>

Prestressing

- (32) 7 wire strands
- $f_{pu}$ = 270 ksi
- $f_{pi}$ = 0.7 $f_{pu}$
- $A_{ps}$ = 2.72 in$^2$
- $e$ = 21.73 in

Fig 5.16 - Cross-section and material properties for the beam with four equal spans.
Fig 5.17 - Layout and reinforcement arrangement for both prestressed concrete composite beams.
CROSS-SECTION AT SHORTER SPANS

\[ Ag = 369 \text{ in}^2 \]
\[ Ig = 50797 \text{ in}^4 \]
\[ Wt = 384 \text{ plf} \]

Reinforcement

<table>
<thead>
<tr>
<th>Spans</th>
<th>Supports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deck</td>
<td>1.5%</td>
</tr>
<tr>
<td>Girders</td>
<td>( A_s' = 1.2 \text{ in}^2 )</td>
</tr>
</tbody>
</table>

Prestressing

- \( f_{pu} = 270 \text{ ksi} \)
- \( f_{pi} = 0.7 f_{pu} \)
- \( A_{ps} = 1.19 \text{ in}^2 \)
- \( e = 12 \text{ in} \)

Fig 5.18 - Cross-section and material properties for the two shorter spans.
low-relaxation steel at an initial stress $f_1 = 0.7$ fpu. Prestressing is designed for zero stress at top girders under dead load. Reinforcement of 1.5% is also provided in the deck for the entire span. Additional deck reinforcement of 1.1% is also provided over the intermediate support. For the beams with full continuity for live load additional girder reinforcement is provided for negative and positive moments at the supports and the gap between adjacent girders is filled with concrete to assure perfect girder continuity. Unshored construction is assumed and bearing pads are provided for all supports. If hinges are used, they are effective only after dead load is applied.

5.3.2 Modeling

The analysis is performed by modeling the beams by 12 and 11 elements for the short and long spans respectively. All elements are subdivided into 20 layers for the girders and 10 for the deck. For the deck-continuous beams three connection elements are provided, one at the junction of each two adjacent girders. Prestressing and dead loads are applied by using the Load Increment Method and the Displacement Increment Method is used for applying live load up to failure. Various situations of loading, supports, temperature and time effects are studied and the results lead the observations that follow.

5.3.3 Results

(a) Equal Spans and Loading
\[ f_{ps} = 0.85 \, f_{pu} \]

FOUR SIMPLE SPANS

Concrete crushing

Hinged for live load

Bearing pads

\[ f_{ps} = 0.98 \, f_{pu} \]

Composite action

Pre-tensioning

DDL

GDL

Fig 5.19 - Load-deflection responses under full span loading.
Fig 5.20 - Load-deflection responses under full span loading for the beams made continuous for live load.
Fig 5.21 - Load-deflection responses under full span loading.
Figures 5.19, 5.20 and 5.21 show the load-deflection responses for the three cases of non-continuity, full continuity for live load and deck continuity only, respectively. Beams are loaded on the entire span and two situations of supports are considered: bearing pads and hinges for live load. Deflections are measured at center of the exterior spans for all beams.

When supported by bearing pads, the beams show a rather ductile behavior, especially for the non-continuous cases failing by crushing of concrete at mid-span. Their stiffnesses are greatly increased when hinges are considered but failure is sudden, by concrete crushing at the hinges with no ductility plateau whatsoever.

(b) Equal Spans and Unequal Loading

Comparison is shown for the three continuity cases, in Figs. 5.22, 5.23 and 5.24, where beams are supported by bearing pads and loaded at different spans: fully-loaded, exterior and interior, respectively. The most critical loading condition appears to be the case where all spans are fully loaded. Other conditions of loading, not shown, seem to have less effect in deflections. The same applies to the hinged beams shown in Figs. 5.25, 5.26 and 5.27.

From the observed results conclusions that can be drawn are not much different from the ones outlined in Section 5.2 for steel girders. The deck-continuous beam behaves quite similarly to a non-continuous beam, even though slightly improved when supported by bearing pads. When hinges are used, its behavior becomes comparable to the behavior of a fully-continuous beam.
Fig 5.22 - Load-deflection responses under full span loading.
Fig 5.23 - Load-deflection responses for exterior spans loading.
Fig. 5.24 - Load-deflection responses for interior spans loading.
Fig 5.25 - Load-deflection responses under full span loading.
Fig 5.26 - Load-deflection responses for exterior spans loading.
Fig 5.27 - Load-deflection responses for interior spans loading.
(c) Support Conditions

The use of hinged supports improves remarkably the stiffness and thus the response of the beams under service load conditions. However, attention should be paid to the very large reactions created at these hinges as shown in Figs. 5.25, 5.26 and 5.27. It may not be possible in design to accommodate such large reactions with hinges, particularly with precast concrete girders. If it is feasible to design for the large reactions developed in the hinges, the use of deck-continuous beams would be more economical than beams made continuous for live load.

On the other hand, the use of bearing pad supports is much less expensive. Deck-continuous beams with bearing pad supports present an improvement over non-continuous beams. Their responses under service loads are slightly enhanced and joints are avoided.

(d) Time and Temperature Effects

Both a deck-continuous beam and a jointed non-continuous beam are analyzed for their responses under time effects. The beams are left unloaded for a period of one year under the effects of aging, shrinkage and creep of concrete and relaxation of the prestressing steel. After one year the beams are loaded to failure. Their long-term strength is compared to the instantaneous strength when loaded at 28 days as shown in Figs. 5.28 and 5.29. No reduction in strength is found as a consequence of time effects. Both beams once more behave quite similarly.

Under the effects of a temperature gradient no perceptible difference is found in their responses as seen in Fig. 5.30. Camber is
Fig 5.28 - Load-deflection responses under full span loading at ages of 28 days and one year.
Fig 5.29 - Load-deflection responses under full span loading at ages of 28 days and one year.
increased when temperature varies across the depth of the beams, but under a constant temperature gradient no other effects are encountered.

(e) Unequal Spans

The use of different spans and smaller girders, see Fig. 5.17, by no means change the above conclusions. A similar behavior is found under different loading conditions as shown in Figs. 5.31, 5.32 and 5.33.

In conclusion, one may suggest that continuous-deck beams constitute a valid alternative solution for jointless bridge beams, either with steel or with precast concrete girders, for new construction or for the replacement of bridge decks. This new design concept shows promise as a method for achieving a jointless bridge deck.
Fig 5.30 - Development of mid-span deflections under prestressing, dead load and temperature.
Fig 5.31 - Load-deflection responses under full span loading.
Fig 5.32 - Load-deflection responses for interior spans loading.
Fig 5.33 - Load-deflection responses for exterior spans loading.
6. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary

The use of jointless construction is viewed as a new solution for the design of composite bridge beams. Expansion joints often become such a source of problems in maintaining bridges that their complete elimination seems preferable to some bridge designers. Among the few design solutions for a joint-free beam, the use of a fully-continuous deck slab over non-continuous girders appears a promising alternative. This solution is feasible not only for the construction of new bridges but also for the rehabilitation of old bridges requiring deck replacement.

The analysis of such beams is more complex than that of fully-continuous or non-continuous beams. However, numerical methods for structural analysis can be employed efficiently for either fully-continuous, non-continuous as well as deck-continuous beams. Instantaneous and time-dependent responses can be obtained for both linear and nonlinear ranges.

Many analytical and numerical solutions have been proposed by various investigators. Yet, none was found broad enough to completely suit the needs of the problem under consideration. It is the purpose of this study to propose a new numerical solution for the analysis of linear and nonlinear, instantaneous and time-dependent responses and strength of such beams. A finite element computer program has been developed for the analysis of both composite and non-composite, fully-
continuous, non-continuous and deck-continuous bridge beams. Girders may be cast-in-place or precast, in steel, reinforced or prestressed concrete topped by a reinforced concrete deck slab. Prestressed girders may be pre- or post-tensioned, but with bonded tendons.

Different constructional procedures may be used and analyzed in any desirable sequence. The effects of dead and live loads, prestressing, support displacements and temperature variation as well as time-dependent effects of aging, creep, shrinkage and relaxation are taken into account. Two different methods are used for the application of external loads, by increments of load or displacement. Both the rate of creep method and the superposition method are used in estimating creep strains of concrete under variable stresses. A method similar to the rate of creep method is used in determining the relaxation of prestressing steel under varying stresses. The variation of the prestressing force due to deformation of the beam is also considered.

Different concrete materials may be used for deck slab and for girders. Either a bilinear or a Hognestad stress-strain relationship is assumed for concrete, and an elasto-plastic relationship is assumed for mild steel. For the prestressing steel, a trilinear relationship is assumed. The relationships for strength, creep and shrinkage of concrete, and relaxation of prestressing steel are based on the models proposed by the ACI Committee 209 and the PCI Committee on Prestress Losses. A smeared cracking model is used and concrete cracking is governed by a strain criterion.

The mechanical properties of the materials as well as the methods used for estimating instantaneous and time-dependent responses under
uniaxial load are thoroughly described in Chapter 2. Chapter 3 describes in detail the formulations of two finite elements as well as the fundamental concepts used as the basis for the development of the proposed model. The capability and validity of the solution are demonstrated in Chapter 4. Ten example problems are presented in which eighteen different beams are analyzed and the results are compared with the experimental data reported in the literature.

The application of the proposed solution to deck-continuous beams is treated in Chapter 5. A two-span composite beam with steel girders and a four-span composite beam with prestressed concrete girders are analyzed therein. Various loadings, support conditions and arrangements, continuity and constructional scheme are studied.

6.2 Conclusions

Based on the results obtained, the following conclusion are drawn regarding the proposed model and the behavior of deck-continuous beams.

6.2.1 Model

(1) The possibility of obtaining both instantaneous and time-dependent responses in a single analysis is extremely useful in the determination of long-term strength of beams, i.e. the strength of beams that under long-time sustained load are eventually subjected to its failure load.

(2) The use of two methods for application of loads, by increments of load and increments of displacement, has shown to be very convenient
for the study of different loading conditions. The Displacement Increment Method is capable of capturing the whole range of structural response, up to ultimate.

(3) Good agreement is found between measured and predicted results under the assumption of different stress-strain relationships for concrete. The models for aging, creep, shrinkage and steel relaxation together with the rate of creep and the superposition methods for estimating creep strains under variable stresses, have enabled reasonable predictions of time-dependent responses.

(4) The smeared cracking model for concrete presented a good performance in representing the post-cracking behavior of reinforced and prestressed concrete members.

(5) Both adopted elements have performed very adequately. The beam element with its variable nodal position enables the determination of horizontal reactions of bihinged beams under vertically applied load. Due to this important feature, it is possible to consider the actual horizontal movement of supports and its effect on the behavior of beams, especially deck-continuous beams. The performance of the connection element is yet to be validated by experimental data.

6.2.2 Deck-Continuous Beams

(1) The responses of jointless deck-continuous beams have been observed to be governed primarily by the imposed boundary conditions.

(2) When steel girders are used for which hinged supports are not a difficult task, a deck connection tensile behavior may be achieved by
having a hinge of an adequate shearing capacity at each side of the connection, i.e. for both adjacent girders. A couple will be created at this section, through the deck connection and hinges, and negative or positive moments can be resisted nearly as well as if full continuity were provided. The beam has its response enhanced remarkably and under service loads may be compared to a fully-continuous beam. Under overload conditions more deflections and less strength can be expected; however, this may be corrected by increasing the amount of mild reinforcement.

(3) More ductility may be obtained by increasing the width of the deck connections, either by widening the space between girders or by providing some unbonded length between deck and girders, at each side of the support.

(4) For girders which are simply supported on bearing pads, as are precast prestressed concrete beams, tension is obtained at the deck connections. In this case a deck-continuous beam behaves very similarly to a jointed non-continuous beam, with stiffness slightly enhanced and with the advantage of having no undesirable joints.

(5) In prestressed concrete deck-continuous beams under overload conditions, cracking will occur at the deck connection; but under unloading, cracks are likely to close completely. Should cracks remain unclosed, they should be less damaging than a built-in joint.

(6) Failure of deck-continuous beams eventually occurs by extensive yielding of the reinforcement in the connection and after that the beams behave like a jointed beam, observing all its ductility and strength
properties.

(7) Under sustained loading, due to time effects such as aging, differential creep, differential shrinkage and steel relaxation, and also under temperature variation, the behavior of a deck-continuous beam does not show significant difference from those presented by the correspondent fully-continuous and non-continuous beams. Their long-term strength is also unchanged.

6.3 Recommendations for Further Research

The study of this new type of beams deserves a more extensive investigation. The study presented herein has served only as a first step, a preliminary analysis, which must be improved and extended by other investigators. Some suggestions for further research follow.

(1) The use of prestressing in the deck slab.

(2) The use of pre- and post-tensioning in the same structure, often used for dead load carrying.

(3) The inclusion of the tension stiffening effect found in concrete under cracked conditions.

(4) The inclusion of the bending stiffness of the deck within the connection elements.

(5) The effects of differential creep, differential shrinkage, steel relaxation and temperature variation in long multi-span beams with little or no joints.
(6) The investigation of an optimum span length of deck-continuous beams.

(7) It is finally and strongly suggested that some experimental work be done for obtaining a more reliable source of information on the behavior deck-continuous beams.
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8.1 Input and Output Guide

The input data are arranged in 24 different lines or sets of lines containing all the information necessary for the analysis of one or more problems at a time. A sample input is shown in Appendix C. The details for each entering are given below together with the necessary explanation pertaining each type of information. Data are to be input following the last card of the program coding. No blank lines are accepted and free format is to be used. Any desirable physical dimensioning may be used provided it is consistent throughout each problem.

1st line

Number of problems to be analyzed.

2nd line

A sequence of eight different enterings is contained in this single line, corresponding to the following variables:

a) Total number of elements.

b) Number of connection elements.

c) Number of nodal points.

d) Number of supports.

e) Number of loading cases, according to line 17.

f) Number of layers to be used in the girders. For rectangular or steel I-girders a multiple of 6 and for concrete I-girders a multiple of 4.

g) Number of layers to be used in the deck, any number.
h) Number of elements for which a complete information is to be printed. This information is updated at every step of the solution and contains: position, area, strain, stress and instantaneous modulus of elasticity for each of the layers. For every step of the solution, and corresponding to the chosen elements, the following results will be listed at the end of the output: bending moment, curvature, deflection, fraction of the applied load, current time of loading and number of iterations in the step.

3rd line

In a single line the numbers of all the elements chosen (line 2h) are to be listed.

4th line

Stiffnesses of the bearing pads, calculated according to section 2.4.

5th line

When the Displacement Increment Method is used for the application of the live load (line 17), three different entering are necessary for establishing:

a) Displacement increment to be applied at every incremental step. This displacement may be positive or negative according to the deflection presented by the structure at the particular nodal point, under the applied live load.

b) Number of the nodal point at which the incremental displacement
(item a) is to be applied.

c) Fraction of the total live load to be applied.

6th line

Five integer values are needed for defining girder material, section type and variability, reinforcement distribution and concrete stress-strain relationship as follows.
a) Girder material: (0) steel, (1) prestressed concrete and (2) reinforced concrete.
b) Girder cross-sectional type: (0) rectangular and (1) other shapes like I, Tee, double Tee and box girders.
c) Dimensions of the girder cross-sections along the length of the: (0) constant and (1) variable.
d) Reinforcement distribution along the length of the beam: (0) constant and (1) variable.
e) Assumed stress-strain relationship for concrete: (0) Hognestad's and (1) bilinear.

7th line

For establishing the dead loads of the girders and deck the specific weights of the materials are needed. Two values are to be entered in one single line, calculated according to the chosen physical dimensions and corresponding to girders and deck respectively.

8th line

For establishing the ages of the materials and the consequent effects of creep, shrinkage and relaxation five time values, in days,
are to be input in one single line and corresponding to:

a) Time at which the curing of the girder concrete has ended.
b) Time at which the curing of the deck concrete has ended.
c) Time at which the girders have been first loaded.
d) Time at which the deck has been first loaded.
e) Time of prestress.

Lines 9 to 12 correspond to setting of material properties. All six entries for each line are to be assumed as positive and their proper signs will be assigned inside the program.

9th line

According to Fig. 2.6, the tri-linear stress-strain relationship for the prestressing steel is established by the setting of six different variables. The first three correspond to the modulii of elasticity of the three linear segments, the following two define the two yielding stresses, $f_{S1}$ and $f_{S2}$, and the last one defines its ultimate strain.

10th line

The bilinear stress-strain relationship of reinforcing steel, Fig. 2.5, is defined by the variables: initial modulus of elasticity, yielding strain and ultimate strain.

11th line

The girders may be made of steel or concrete. For steel girders the stress-strain relationship of its material is input in a similar fashion as in line 10 for steel reinforcement. For the concrete girders four variables are used in defining the stress-strain relationship of
concrete: Initial modulus of elasticity, 28-day cylinder strength, tensile rupture strain and ultimate compressive strain.

12th line

The variables used for defining the stress-strain relationship of concrete used in the deck slab are input similarly as the ones for the girder concrete, as in line 11.

13th line

As many lines as the number of elements are needed for defining the connectivity and the type of each element used. Three entering are used in each line. The first two define the number of the two nodal points and the last defines the type of the element as follows:

(0) A beam element defined at the beginning of the solution, representing the girder and deck cross-sections.

(1) A connection element defined at the time when the deck slab is cast and its stiffness is added to the girder stiffness.

(2) A beam element used for obtaining continuity for live load. The element will be defined when the deck slab is cast and continuity is obtained. Its dimensions may be different from the other beam elements for representing a diaphragm for instance.

14th line

Coordinates and nodal conditions are defined by five enterings in as many lines as there are nodal points. The first two numbers define the horizontal and vertical coordinates of each nodal point, respectively. The origin of the horizontal coordinates may be located at any desirable
position, but the vertical coordinates are referred to the bottom face of each girder.

The nodal conditions represent the restrictions applied to each nodal point in each of the three directions, i.e. horizontal, vertical and rotational directions respectively as: (0) free and (1) restricted.

15th line

When the dimensions of the cross-sections of the elements are constant throughout the length of the member, see line 6c, only one line is needed for the input of the sectional dimensions. Otherwise, as many lines as there are elements are needed. The following cases apply for each type of cross-section:

a) Steel I-girders - Eight enterings define the dimensions of the deck and girders, in the following order: deck width and thickness; girder area, depth and moment of inertia; thicknesses of the top flange, web and bottom flange.

b) Rectangular cross-section - Only four enterings are used: deck width and thickness, and girder width and thickness.

c) Girders of other shapes - Any other shape of the girders is modeled as an I cross-section. Width and thickness of the deck, top flange, web and bottom flange are input in a similar sequence.

d) Connection element - For the deck connection, and only when variable cross-sections are used, see line 6c, four enterings are needed: width and thickness of the deck and distances from the C.G.C. of the deck to the bottom faces of the two adjacent girders.
Three levels of reinforcement may be used, one in the deck slab and two in the girders. Total area and position of each level with respect to the bottom face of the girders are defined by six enterings. When constant steel distribution is used, see line 6d, only one line is needed. Otherwise, as many lines as there are elements are input. The first reinforcement level is defined as the one in the deck, the second and third following downwards, respectively.

Each loading condition, instantaneous or time-dependent, is defined by a line containing eight integer enterings:

a) A number defining the type of loading, load case, according to the following list:
   1 Girder dead load
   2 Prestressing
   3 Deck dead load
   4 Support displacements
   5 Live load
   6 Temperature
   7 Time effects

b) A variable, 0 or 1, defining whether the beam is to be assumed noncomposite or composite, respectively, for each load case.

c) Number of steps to be used in each loading. For prestressing one step is assumed and when using the Displacement Increment Method the
variable shall be assigned the value 1000.

d) A variable, 1 or 0, defining whether the results for each step shall be printed or not, respectively. For the last step of each load case the results are always printed regardless of the value assumed.

e) A variable defining which time-effects are to be analyzed.

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>None</td>
</tr>
<tr>
<td>1</td>
<td>Creep</td>
</tr>
<tr>
<td>2</td>
<td>Shrinkage</td>
</tr>
<tr>
<td>3</td>
<td>Relaxation</td>
</tr>
<tr>
<td>4</td>
<td>All</td>
</tr>
</tbody>
</table>

f) Time at which the loading case is to be initiated, in days.

g) Time at which the loading case is to be finished, in days.

h) A variable, 1 or 0, defining whether the bearing supports are to be transformed into hinges or not, respectively. This effect is obtained by assigning a very large stiffness to the bearing supports.

18th line

In as many lines as there are supporting nodes two variables shall define: the number of the supporting nodal point and magnitude of vertical displacements to be applied at the respective supports.

19th line

A temperature variation gradient is defined by three temperature values at the top of the deck slab, at the deck-girder interface and at the bottom of the girders, respectively.
20th line

In as many lines as there are nodal points three concentrated nodal loads may be defined, in the horizontal, transversal and rotational directions respectively.

21st line

Loads applied at the elements are defined by three enterings in as many lines as there are elements: A uniformly distributed load, and a concentrated load and its point of application, measured from the left nodal point, respectively.

Lines 22 to 25 regard information concerning the prestressing cables and their characteristics. They shall be omitted when the girders are not prestressed.

22nd line

One single line with six enterings is used for defining the following prestressing variables:

a) Type of prestressing, (0) post-tensioned and (1) pre-tensioned.

b) Type of cable profile, (0) straight, (1) parabolic and (2) other.

c) Number of prestressing segments, defined by beginning and end of each segment.

d) Friction coefficient.

e) Wobbling coefficient.

f) Type of steel, (0) low-relaxation and (1) stress-relieved.
In as many lines as there are prestressing segments, see line 22c, five enterings are used to define:

a) Number of the element at which the prestressing segment starts.

b) Number of the element at which the prestressing segment ends.

c) Cross-sectional area of the tendons.

d) Initial prestressing force.

e) Side of the prestressing segment at which the cable will be pulled: (1) left and (2) right, used for the calculation of friction losses.

When the assumed cable profile is either straight or parabolic, see line 22b, three nodal points are chosen to define the cable profile. In as many lines as there are prestressing segments the number of the nodal points and the distances from the cable to the bottom face of the girders, at those nodal points, are to be defined for each chosen node, respectively. When the cable profile is defined as other the vertical position of the cable shall be informed at each node, in as many lines as there are nodes.

The program output as shown in Appendix D, is divided into two sections: the first section refers to the input properties of each problem. It is repeated as many times as the number of problems analyzed and follows the same order used for data input, as explained in lines 1 through 24. The second section immediately follows, it presents all the results obtained by the finite element program and it is subdivided into
the following subsections:

1. Global displacements - For each and all nodal points a global displacement number is assigned, in crescent order and skipping the prescribed degrees of freedom for which a zero is assigned.

2. Storage needed - The total storage capacity needed for each problem is calculated. If the number is considerably different then the provided storage capacity, shown in the first output section, the latter may be changed accordingly.

3. Dead load - Dead load of the girders and deck are separately calculated and printed out for each of the elements.

4. Original section properties - Transformed section properties, centroidal position, area and moments of area, are calculated for both possible situations: noncomposite and composite sections. Those properties refer to the original uncracked sections and are printed for all elements.

5. New section properties - Updated section properties are printed at every new load case and step. Those properties reflect any changing of the material properties as well as cracking development at the sections.

6. Nodal displacements - At every increment step, displacements for the step and the total accumulated displacements are printed for all nodal points and for the three possible directions, horizontal, transversal and rotational, respectively.
(7) Element forces - The total updated forces, axial, moment and shear, are calculated and printed for all elements, at every step. Those forces reflect the action of the external loading, reactions and prestressing effects as well.

(8) Reactions - Nodal external reactions, at all nodal supports, are calculated for the three directions and printed at every loading step.

(9) Prestressing cable properties - For all the elements under prestressing, the updated forces, stresses, strains and instant elastic modulii are calculated and printed at every loading step. This section is omitted in non prestressed problems.

(10) Strains - For all elements and at every loading step the updated and the maximum strains reached at the top and bottom surfaces of the girders and deck are printed, respectively.

(11) Cracking - For all elements and at every loading step, cracking length is printed for both girders and deck, starting from the top and bottom surfaces, respectively.

(12) Cracking layout - A complete cracking layout is printed for every loading step. For all elements and layers cracking is represented by 0's and uncracked layers by 1's.

(13) Crack angles - For all elements and layers the angles formed by the cracks with the horizontal direction are printed out, approximated by the nearest integer angle.

(14) Reinforcement stresses - Stresses at the three reinforcement levels, in the deck, top flange and bottom flange of the girders,
respectively, are printed at every load step and for all elements.

(15) Yielding layout - For all three reinforcement levels and all layers of a steel girder, a complete layout of yielding is printed by representing yielded layers by 0's. Similarly as for the cracking layout this is updated at every loading step.

(16) Checking elements - For all the chosen checking elements, see line 2h, and at every layer, centroidal position of the layer, cross-sectional area of the layer, stress, strain and instant modulus of the material are printed.

(17) Moment-curvature-deflection - After all loading steps are printed, a complete list of bending moments, curvatures, deflections, fraction of the applied load, current times and number of iterations are shown for all of the loading steps and every chosen checking elements. This list is immediately followed by a message indicating the reason for which the problem has been terminated.

(18) Support reactions - A complete list of the support reactions is printed for all supports and every loading step.

(19) Plots - Plots of the moment-curvature and moment-deflection distributions are available for each and all of the chosen checking elements.
WRITE(2,1222)
READ(1,1)PREF,IPROF,PSIS,PS,F,B,IST
WRITE(2,1570)
1570 FORMAT(15,'ESTRESSING AREA AND INITIAL FORC...',/)
IF(IST.EQ.0) THEN
WRITE(2,1575)
1575 FORMAT('F',/,'STRESSED CABLE *')
IF(IST.EQ.0) THEN
WRITE(2,1577)
1577 FORMAT('F',/,'STRESS-BELIEVED STEEL *')
ELSE
WRITE(2,1578)
1578 FORMAT('F',/,'STRESS-BELIEVED STEEL *')
END IF
IF(ISIP.GT.0) THEN
WRITE(2,1579)
1579 FORMAT(15,'PRE-TENSIONED CABLE *')
IF(ISIP.EQ.0) THEN
WRITE(2,1580)
1580 FORMAT(15,'FOR ELEM K TO ELEM K+1 AREA',/)
DO 150 I=1,PS2
READ(1,1580)PSIS(I),IPROF(I),PSIS(I),IPROF(I),PSIS(I),IPROF(I),PSIS(I),IPROF(I)
WRITE(2,1581)
1581 FORMAT(15,'PRE-TENSIONED CABLE *')
150 CONTINUE
WRITE(2,1582)
1582 FORMAT('F',/,'STRESS-BELIEVED STEEL *')
IF(IPROF.EQ.0) THEN
WRITE(2,1583)
1583 FORMAT(15,'STRESS-BELIEVED STEEL *')
END IF
WRITE(2,1584)
1584 FORMAT(15,'PARABOLIC *')
IF(ISIP.EQ.O) THEN
WRITE(2,1585)
1585 FORMAT(15,'PARABOLIC *')
END IF
WRITE(2,1586)
1586 FORMAT(15,'OTHER *')
END IF
WRITE(2,1587)
1587 FORMAT(15,'OTHER *')
1555 FORMAT(15,'ANGLE A,ANG...)
C

PROCEDURE ANALYSIS

CALL ANALYSIS(LOAD, DUMP, BULK, CAMAG, CAMAD, LISTER, KOPPS, ICOST,)
*  VR00, K009, K002, K003, K005, K008, X, RC, YVAR, YVAR,
IF (ICOUNT, N, 0) THEN
  DO 60 L=1,N
  STRT(NE, L) = STRT(NE, L) + 255
60  CONTINUE
END IF
END
1010  CONTINUE
END
1020  IF (ICOM. Y, 1) THEN
  LHY = LHY + 1
  DO 70 L = LHY, NL
  STRT(NE, L) = STRT(NE, L) + 255
70  CONTINUE
END IF
END
1030  CONTINUE
RETURN
END
<table>
<thead>
<tr>
<th>STEEL GIRDER</th>
<th>2241</th>
<th>C</th>
<th>C</th>
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</thead>
<tbody>
<tr>
<td>IF (ICMAT, EQ. 0) THEN</td>
<td>2242</td>
<td>C</td>
<td>C</td>
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<tr>
<td>DO 10 1 = 1, NEL</td>
<td>2243</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>GA=DIM(1, 1)</td>
<td>2244</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>GA=QIEM(1, 1) * DIM(1, 2)</td>
<td>2245</td>
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<td>C</td>
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<tr>
<td>DELO(1, 1) = GA-CARAD</td>
<td>2246</td>
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<tr>
<td>IF (DEL(1, 1) .LE. 0.0) THEN</td>
<td>2247</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>CARAD=CARAD</td>
<td>2248</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>ELSE</td>
<td>2249</td>
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<td>C</td>
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<tr>
<td>DELO(1, 1) = 0.0</td>
<td>2250</td>
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<tr>
<td>END IF</td>
<td>2251</td>
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<td>C</td>
</tr>
<tr>
<td>10 CONTINUE</td>
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<table>
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<td>IF (ICMAT, EQ. 0) THEN</td>
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<td>C</td>
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<tr>
<td>DO 20 1 = 1, NEL</td>
<td>2255</td>
<td>C</td>
<td>C</td>
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<tr>
<td>GA=QIEM(1, 1) * DIM(1, 2) * DIM(1, 3) * DIM(1, 6)</td>
<td>2256</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>GA=QIEM(1, 1) * DIM(1, 2)</td>
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<td>C</td>
<td>C</td>
</tr>
<tr>
<td>DELO(1, 1) = GA-CARAD</td>
<td>2258</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>IF (DEL(1, 1) .LE. 0.0) THEN</td>
<td>2259</td>
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<td>C</td>
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<tr>
<td>CARAD=CARAD</td>
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<tr>
<td>ELSE</td>
<td>2261</td>
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<td>C</td>
</tr>
<tr>
<td>DELO(1, 1) = 0.0</td>
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<td>C</td>
</tr>
<tr>
<td>END IF</td>
<td>2263</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>20 CONTINUE</td>
<td>2264</td>
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**ROUTINE漸進** 

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>KAD</td>
<td>2266</td>
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</tr>
<tr>
<td>CARAD</td>
<td>2267</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>ESTEM</td>
<td>2268</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>GB = (A + 1) / (E + 1)</td>
<td>2269</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>EGB = (E + 1) / (E + 1)</td>
<td>2270</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>DO 100 1 = 1</td>
<td>2271</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>IF (KAD, 1 / KAD) THEN</td>
<td>2272</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>DO 200 1 = 1</td>
<td>2273</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>GA = DIM(1, 6)</td>
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<td>C</td>
</tr>
<tr>
<td>GA = QIEM(1, 1) / KAD</td>
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<td>C</td>
<td>C</td>
</tr>
<tr>
<td>DELO(1, 1) = GA-CARAD</td>
<td>2276</td>
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<td>C</td>
</tr>
<tr>
<td>IF (DEL(1, 1) .LE. 0.0) THEN</td>
<td>2277</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>CARAD=CARAD</td>
<td>2278</td>
<td>C</td>
<td>C</td>
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<tr>
<td>ELSE</td>
<td>2279</td>
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<td>C</td>
</tr>
<tr>
<td>DELO(1, 1) = 0.0</td>
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<td>C</td>
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<tr>
<td>END IF</td>
<td>2281</td>
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<td>C</td>
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<tr>
<td>200 CONTINUE</td>
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**SUBROUTINE漸進**

<table>
<thead>
<tr>
<th>SUBROUTINE漸進 (ICMAT, ISECT, QAMS, QAM, QAM, STEEL)</th>
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</thead>
<tbody>
<tr>
<td>THIS SUBROUTINE CALCULATES: E2, AREA, ROLE AND STRESS.</td>
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<td>OF UNCHARRACKED SECTIONS: IN STEEL AND CONCRETE, COMPOSITE OR NOT</td>
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<tr>
<td>VÀRIBLE SIGNIFICANCE: 1 AREA</td>
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<td>C</td>
</tr>
<tr>
<td>2 E2</td>
<td>2287</td>
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<td>C</td>
</tr>
<tr>
<td>3 ROLE</td>
<td>2288</td>
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<td>C</td>
</tr>
<tr>
<td>4 STRESS</td>
<td>2289</td>
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<tr>
<td>5 E2</td>
<td>2290</td>
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<td>C</td>
</tr>
<tr>
<td>6 ROLE</td>
<td>2291</td>
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<td>C</td>
</tr>
<tr>
<td>7 STRESS</td>
<td>2292</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>8 UNCHARRACKED</td>
<td>2293</td>
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</tr>
<tr>
<td>9 COMPOSITE</td>
<td>2294</td>
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</tr>
<tr>
<td>10 CONCRETE</td>
<td>2295</td>
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</tr>
<tr>
<td>11 RETURN</td>
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**ROUTINE漸進**

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<tbody>
<tr>
<td>IF (ICMAT, EQ. 0) THEN</td>
<td>2298</td>
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<td>C</td>
</tr>
<tr>
<td>DO 10 1 = 1, NEL</td>
<td>2299</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>GA = QIEM(1, 1) / KAD</td>
<td>2300</td>
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<td>C</td>
</tr>
<tr>
<td>DELO(1, 1) = GA-CARAD</td>
<td>2301</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>IF (DEL(1, 1) .LE. 0.0) THEN</td>
<td>2302</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>CARAD=CARAD</td>
<td>2303</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>ELSE</td>
<td>2304</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>DELO(1, 1) = 0.0</td>
<td>2305</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>END IF</td>
<td>2306</td>
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</tr>
<tr>
<td>10 CONTINUE</td>
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**SUBROUTINE漸進**

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<td>OF UNCHARRACKED SECTIONS: IN STEEL AND CONCRETE, COMPOSITE OR NOT</td>
<td>2310</td>
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</tr>
<tr>
<td>VÀRIBLE SIGNIFICANCE: 1 AREA</td>
<td>2311</td>
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</tr>
<tr>
<td>2 E2</td>
<td>2312</td>
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</tr>
<tr>
<td>3 ROLE</td>
<td>2313</td>
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<td>C</td>
</tr>
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<td>4 STRESS</td>
<td>2314</td>
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</tr>
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<td>5 E2</td>
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<td>6 ROLE</td>
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<td>7 STRESS</td>
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<td>C</td>
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<td>8 UNCHARRACKED</td>
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<td>9 COMPOSITE</td>
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<td>10 CONCRETE</td>
<td>2320</td>
<td>C</td>
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<tr>
<td>11 RETURN</td>
<td>2321</td>
<td>C</td>
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</table>
**SUBROUTINE TLOAD**

```
C***************************************************************
C SUBROUTINE TLOAD([X,Y,Z],[X,Y,Z],[X,Y,Z],[X,Y,Z],[X,Y,Z])
C***************************************************************
C INTEGRAL REAL*8(A,R,T)
C DIMENSION 1(6),12(6).tLOAD(6),12(6).tLOAD(6),12(6).tLOAD(6),12(6).
C REAL*8 L
C CLOAD = MODAL FORCES
C CLOAD = DISTRIBUTIVE FORCES
C DO 10 1=1,6
C C AP-POSITION OF THE VERT. FORCE IN THE ELEMENT
C AP=DLOAD(1,3)
C W1=IEL(1,1)
C W2=IEL(2,1)
C BE=W1+W2
C DO 10 J=1,2
C DLOAD(J,0)=0
C 2 CONTINUE
C 1 CONTINUE
C C DISTRIB. (1,1) AND CONG (1,2) VERTICAL FORCES ARE
C TRANSFORMED TO EQUIVALENT MODAL FORCES
C C ELOAD(1,2)=DLOAD(1,2)+C*DLOAD(2,1)+DLOAD(1,1)*W1/2
C ELOAD(1,2)=DLOAD(1,2)+C*DLOAD(2,1)+DLOAD(1,1)*W2/2
C DO 20 J=1,6
C C MODAL FORCES = MODAL LOADS + EQUIVALENT MODAL LOADS
C CLOAD[1,2]=ELOAD[1,2]+ELOAD(1,2)
C CLOAD[1,2]=ELOAD[1,2]+ELOAD(1,2)
C 20 CONTINUE
C 10 CONTINUE
C C R - STORES THE MODAL FORCES
C WRID=0
C DO 9 J=1,6
C WRID=VREAD(J,1)
C IF (VREAD(J,1).EQ.0.) THEN
C KID=2560+J
C ELOAD(KID)=0
C END IF
C 9 CONTINUE
C 1 CONTINUE
C RETURN
C END
C***************************************************************
```

**SUBROUTINE ELOAD**

```
C***************************************************************
C SUBROUTINE ELOAD([X,Y,Z],[X,Y,Z],[X,Y,Z],[X,Y,Z],[X,Y,Z])
C***************************************************************
C INTEGRAL REAL*8(A,R,T)
C REAL*8 L
C CLOAD = MODAL FORCES
C CLOAD = DISTRIBUTIVE FORCES
C DO 10 1=1,6
C C AP-POSITION OF THE VERT. FORCE IN THE ELEMENT
C AP=DLOAD(1,3)
C W1=IEL(1,1)
C W2=IEL(2,1)
C BE=W1+W2
C DO 10 J=1,2
C DLOAD(J,0)=0
C 2 CONTINUE
C 1 CONTINUE
C C DISTRIB. (1,1) AND CONG (1,2) VERTICAL FORCES ARE
C TRANSFORMED TO EQUIVALENT MODAL FORCES
C C ELOAD(1,2)=DLOAD(1,2)+C*DLOAD(2,1)+DLOAD(1,1)*W1/2
C ELOAD(1,2)=DLOAD(1,2)+C*DLOAD(2,1)+DLOAD(1,1)*W2/2
C DO 20 J=1,6
C C MODAL FORCES = MODAL LOADS + EQUIVALENT MODAL LOADS
C CLOAD[1,2]=ELOAD[1,2]+ELOAD(1,2)
C CLOAD[1,2]=ELOAD[1,2]+ELOAD(1,2)
C 20 CONTINUE
C 10 CONTINUE
C C R - STORES THE MODAL FORCES
C WRID=0
C DO 9 J=1,6
C WRID=VREAD(J,1)
C IF (VREAD(J,1).EQ.0.) THEN
C KID=2560+J
C ELOAD(KID)=0
C END IF
C 9 CONTINUE
C 1 CONTINUE
C RETURN
C END
C***************************************************************
```
WRITE(3,9999)
1000 FORMAT(15E12.5,'FACET RESULTANT FORCES',\
   1, * 25,5,'( TENSION,COMPUT,LEFT UP )',/)
   25,5,'( TOTAL )',/)
WRITE(3,9999)
1100 FORMAT(16,E12.5,'NODAL EXTERNAL REACTIONS',\
   1, * 25,5,'( TOTAL )',/)
WRITE(3,9999)
1200 CONTINUE
C WRITE(3,2222)
WRITE(3,2222)
7000 FORMAT(15,E12.5,'NO. FACET, MODAL PROP',\
   1, * 25,5,'( TOTAL )',/)
WRITE(3,9999)
7010 CONTINUE
WRITE(3,9999)
2100 IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
2100 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
2200 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
2300 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
2400 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
2450 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
2451 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
2452 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
2600 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
2800 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
2900 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
2930 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
2940 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
3100 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
3101 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
3102 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
3103 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
3104 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
3105 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
3106 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
3107 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
3108 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
3109 CONTINUE
C IF(COUNT.EQ.1) THEN
   WRITE(3,2222)
   WRITE(3,2000)
8.3 Sample Input

Input for the deck-continuous beam shown in Figs. 5.16 and 5.17 and loaded as in Fig. 5.23.

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Sample Input: 1.2.3. 4.5.6.7.8.9.10.
8.4 Sample Output

Output for the deck-continuous beam shown in Figs. 5.16 and 5.17 and loaded as in Fig. 5.23.
**ELEMENT INCIDENCE & TYPE**

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**MODAL COORDINATES**

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## Displacement Driven Method for LLI

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## RESULTS

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**ELEPHANT RESULTANT FORCES IN TRADITIONSUX00P (TOTAL)**

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**Final Cable Properties (up to this step)**

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## Tinplate, Steel Lathes

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### Diagram: Yielding Limits

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Note: The table and diagram contain specific measurements and properties relevant to the materials and techniques discussed in the context of tinplate, steel lathes.
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### Node External Reactions (Temperature) at Step 1

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The table details the crack angles for various layers in a structure, with each layer represented by a sequence of U's indicating the presence of a crack.
### Table 1: Yielding Efficiency

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### Table 2: Residual Steel Loss & Latent Heat

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**FINAL CELL PROPERTIES** (UP TO THIS STEP)

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**STRAIN AT THE BOUNDARY** (UP TO THIS STEP)
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